ECE 6550 Exam 2
Fall 2022

Name _____

General Instructions:

- Exam is closed book / closed notes other than the one-page of handwritten notes.
- Choose the best possible answer available in all cases.
- Blank scratch paper is allowed

_____ Part I: Objective Questions

_____ Part II: Open Response Questions

_____ Final Score

Part I: Objective Questions

These questions have straight-forward answers. Make sure to put your answer in the line required as that is the part that will be graded for the answer given. Only the final answers, as indicated by the question, will be considered correct for each question. Each question is worth 4 points (total of 72 points)

Matching

_____ 1. $\mathcal{O}$

_____ 2. LQR

_____ 3. Parameters for LQR

_____ 4. $\mathcal{C}$

_____ 5. LQE

_____ 6. Algebraic Riccatti Equation

a. $\mathbf{Q}$ & $\mathbf{R}$

b. $\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0$

c. $\begin{pmatrix}\mathbf{B} & \mathbf{BA} & \mathbf{BA}^2 & \ldots & \mathbf{BA}^{m-1}\end{pmatrix}$

d. $\mathbf{P}_f\mathbf{C}^T\mathbf{Q}_v^{-1}$

e. $\begin{pmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \ldots & \mathbf{A}^{m-1}\mathbf{B}\end{pmatrix}$

f. $\begin{pmatrix}\mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^{m-1}\end{pmatrix}$

g. $\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$

h. $\begin{pmatrix}\mathbf{C} \\ \mathbf{AC} \\ \mathbf{A}^2\mathbf{C} \\ \mathbf{A}^{m-1}\mathbf{C}\end{pmatrix}$

i. $\mathbf{Q}_n$ & $\mathbf{Q}_d$

7. (True / False) One can find a good margin for LQG controllers assuming the system is controllable and observable.

For the following A, B, and C, is the resulting system controllable and observable?

_____ 8. Controllable?

_____ 9. Observable?

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.2 & 2.5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 20 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1/4 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

_____ 10. Controllable?

_____ 11. Observable?

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.2 & 2.5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 20 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$$

_____ 12. Controllable?

_____ 13. Observable?

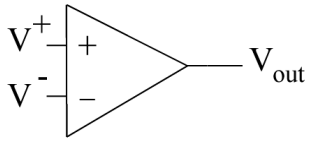
$$\begin{pmatrix} -0.25 & 2 & 0 \\ 0 & 0 & 1 \\ -1 & 10 & 0 \end{pmatrix} \begin{pmatrix} 0.17 \\ 0 \\ 1.1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

_____ 14. Controllable?

_____ 15. Observable?

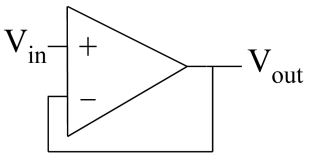
$$\begin{pmatrix} -0.25 & 2 & 0 \\ 0 & 0 & 1 \\ -1 & 10 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

Assume we have an op-amp with the following transfer function



$$H(s) = \frac{100000}{1 + s(10ms) + 0.0005(ms)^2 s^2}$$

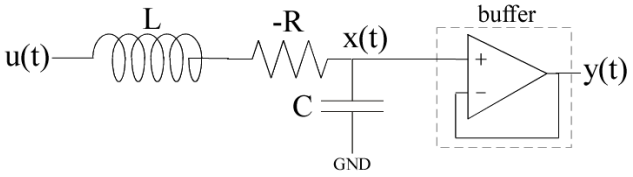
We build a feedback system



For this closed loop system, sketch L(ω), S(ω), T(ω) (3 questions: 16, 17, 18).

Part II: Open Response Question (28 points)

Consider the following linear circuit that arrives from a linearized model



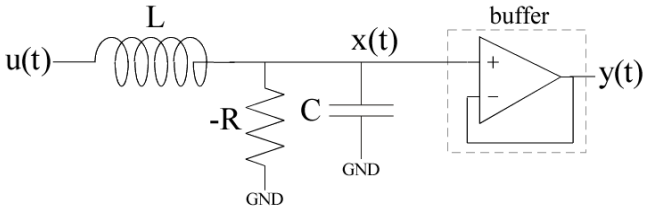
Assume the buffer circuit is ideal with a gain of 1.

a) Create the normalized linearized state equation model for this circuit system. C = 1nF. L = 1mH. R = 1kΩ.

b) Show the stability of this system through the eigenvalues of the **A** matrix.

c) Show that this system from u(t), and through y(t), is controllable and observable, respectively. Show the relative ability to control or observe along fundamental directions (e.g. eigenvalues)

d) Assuming $\mathbf{Q} = \mathbf{R} = \mathbf{I}$, no disturbance noise and noise variance σ (noise power σ^2), set up and reduce the resulting two Riccatti equations for the controller and estimator blocks.



e)