

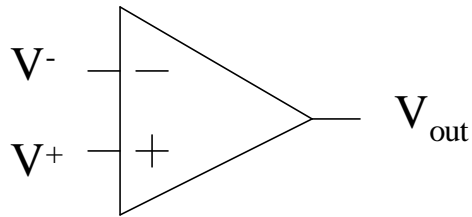
# Basics of Transconductance – Capacitance Filters

Dr. Paul Hasler

# Operational Amplifiers

OP Amp:

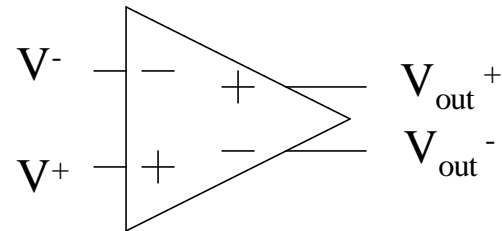
$$V_{\text{out}} = A_v (V^+ - V^-)$$



Balanced Op Amp:

$$V_{\text{out}}^+ = A_v (V^+ - V^-)$$

$$V_{\text{out}}^- = A_v (V^- - V^+)$$

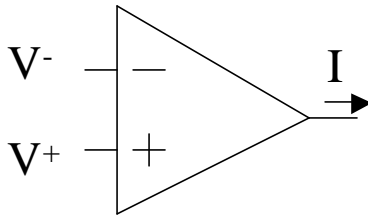


$A_v$  is frequency dependent

# Transconductance Amplifiers

OTA:

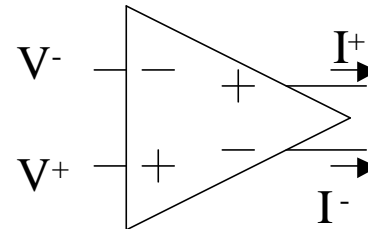
$$I = G_m (V^+ - V^-)$$



Balanced OTA:

$$I^+ = G_m (V^+ - V^-)$$

$$I^- = G_m (V^- - V^+)$$



- Most op-amps can be used as OTAs, most OTAs can be used as op-amps.  
(depends which application is being optimized)

# Major Issues for OTAs

## Major Issues for OTAs:

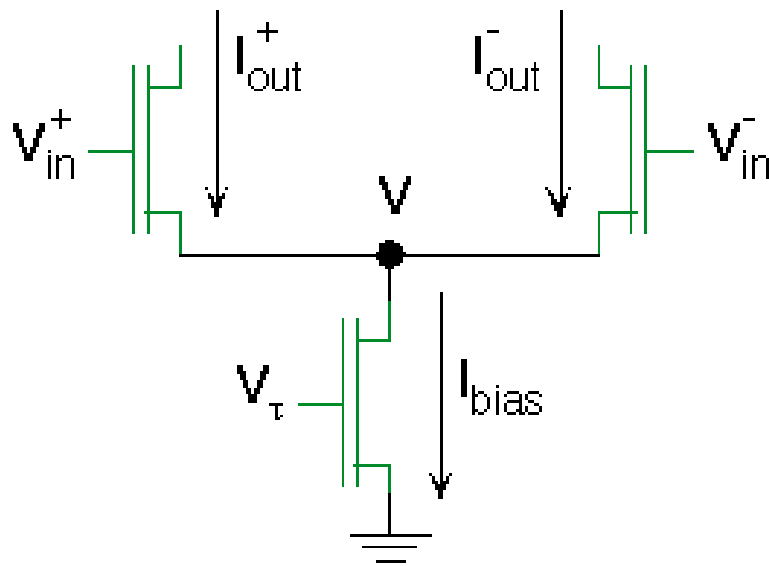
- Linearity
- Offsets
- Output Resistance (depends upon application)

# How to Build OTAs

Basic transistor differential amplifier,  
Wide-output-range differential amplifier ...

Build with cascodes or folded cascode or differential approaches.

# Analysis of Diff-Pair



$$I_{out}^+ = I_0 e^{\kappa V_{in}^+ / U_T} e^{V / U_T}$$

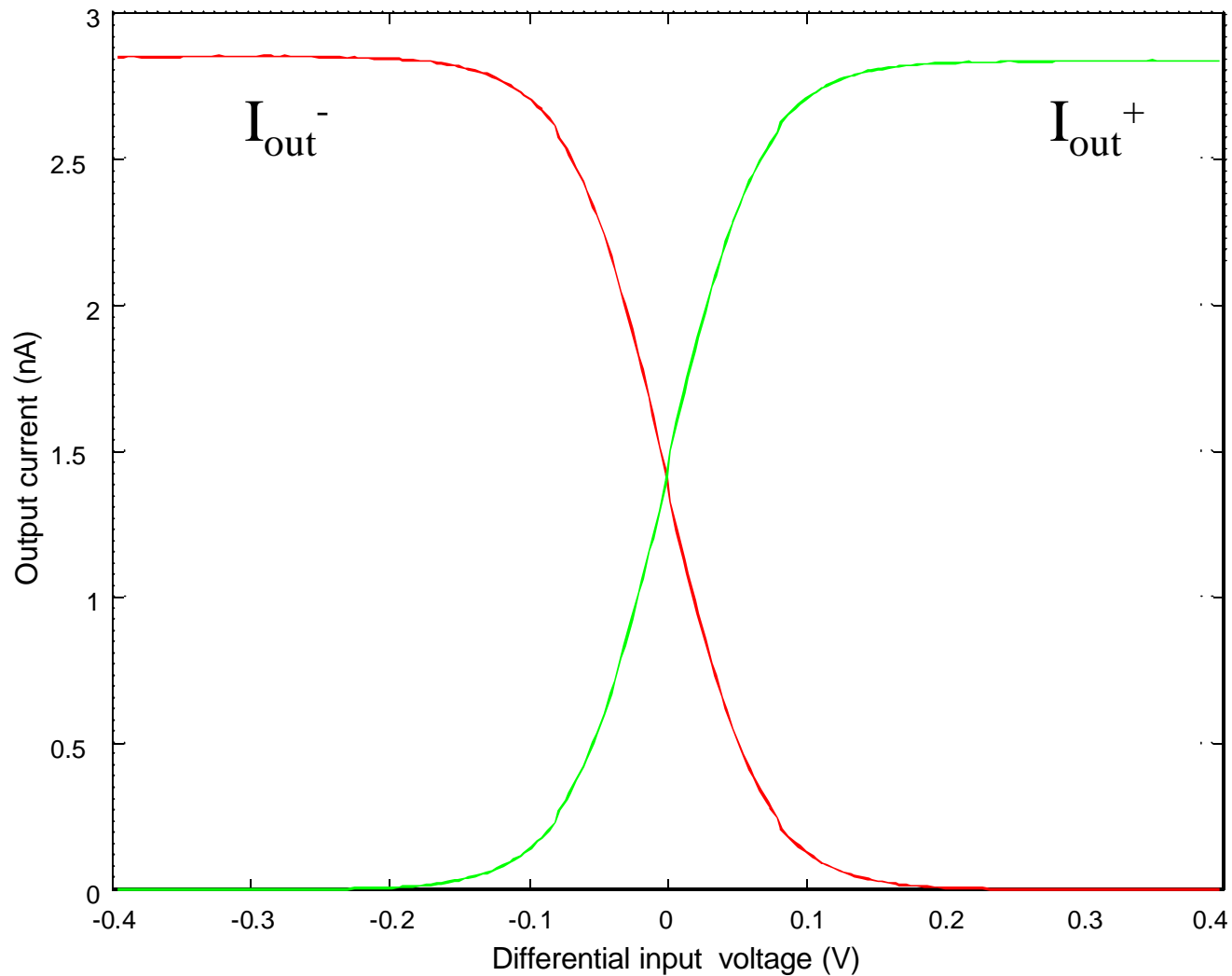
$$I_{out}^- = I_0 e^{\kappa V_{in}^- / U_T} e^{V / U_T}$$

$$I_{out}^+ = I_{bias} \frac{e^{\kappa V_{in}^+ / U_T}}{e^{\kappa V_{in}^+ / U_T} + e^{\kappa V_{in}^- / U_T}}$$

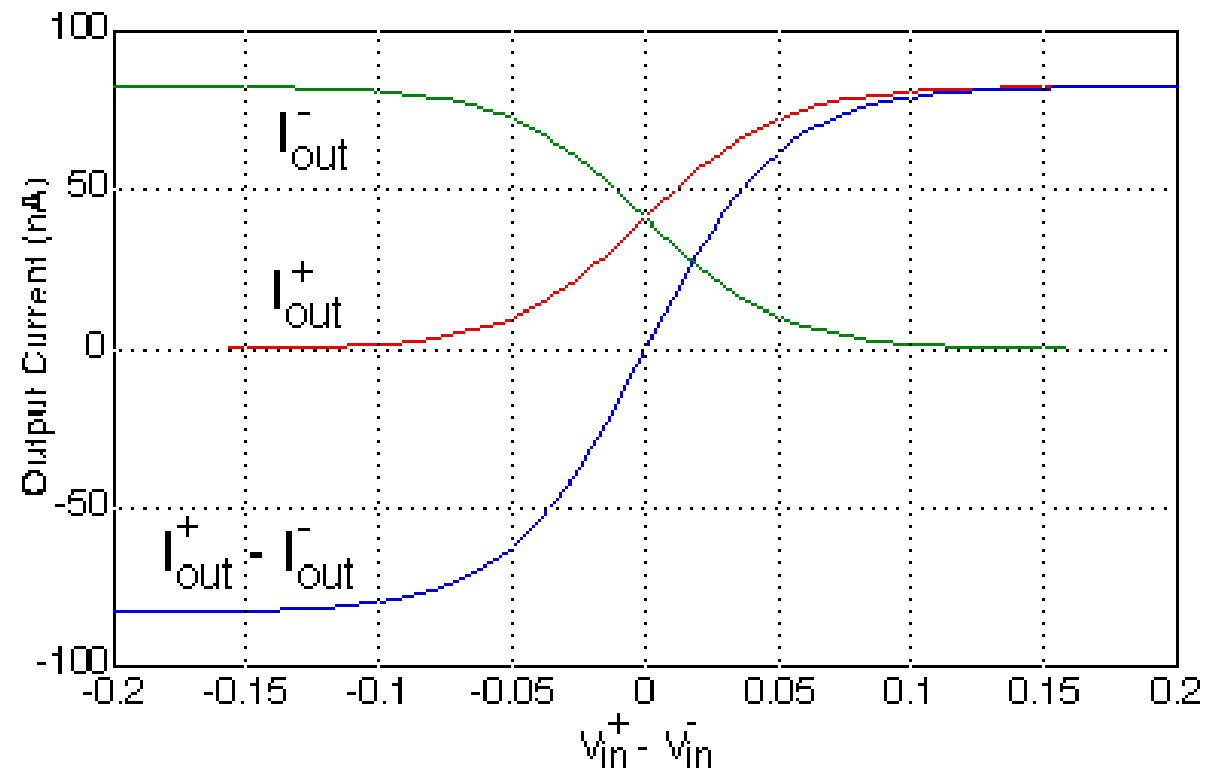
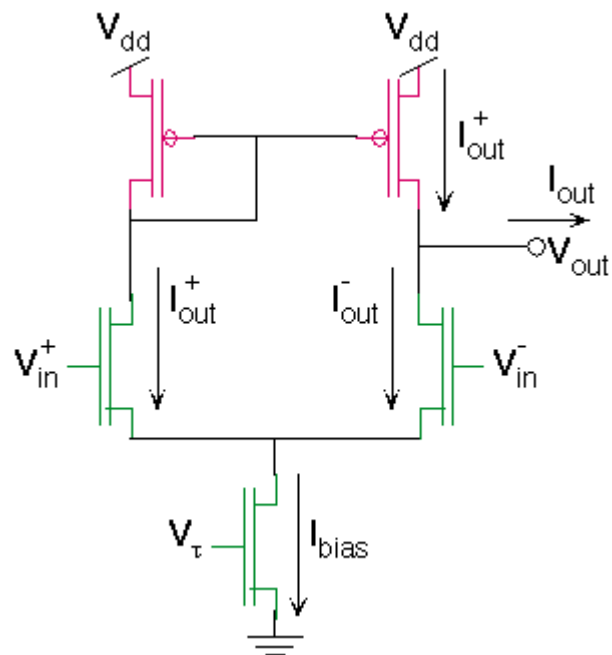
$$= I_{bias} \frac{1}{1 + e^{\kappa (V_{in}^- - V_{in}^+) / U_T}}$$

$$I_{out}^- = I_{bias} \frac{1}{1 + e^{\kappa (V_{in}^+ - V_{in}^-) / U_T}}$$

# Differential Pair Currents

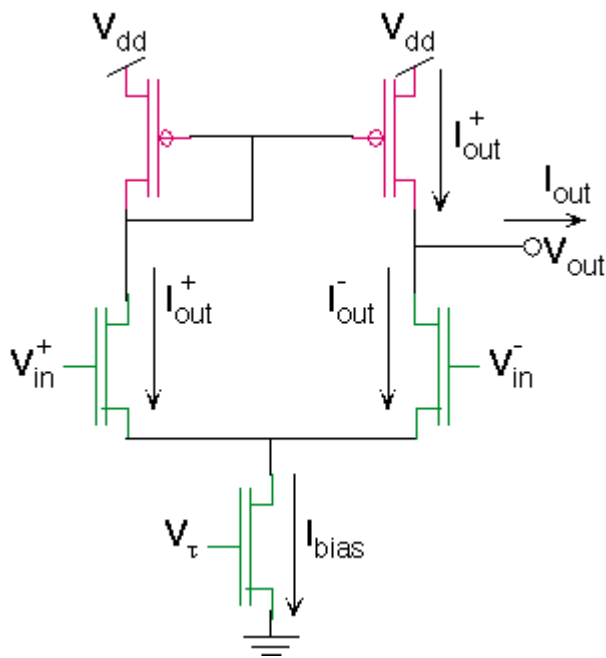


# Basic Differential Amplifier

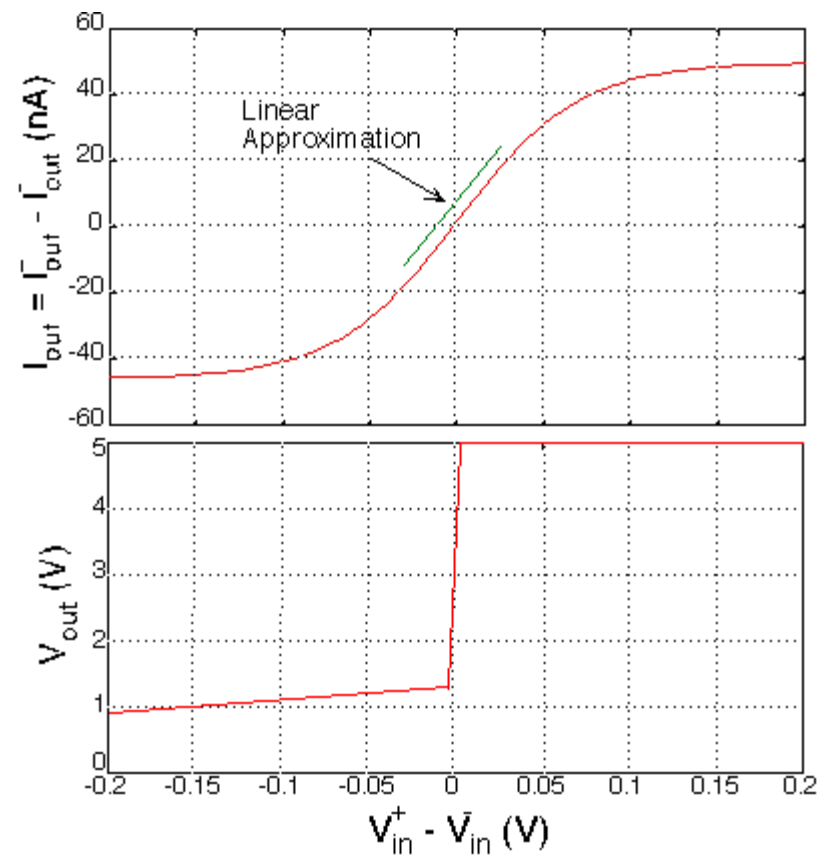




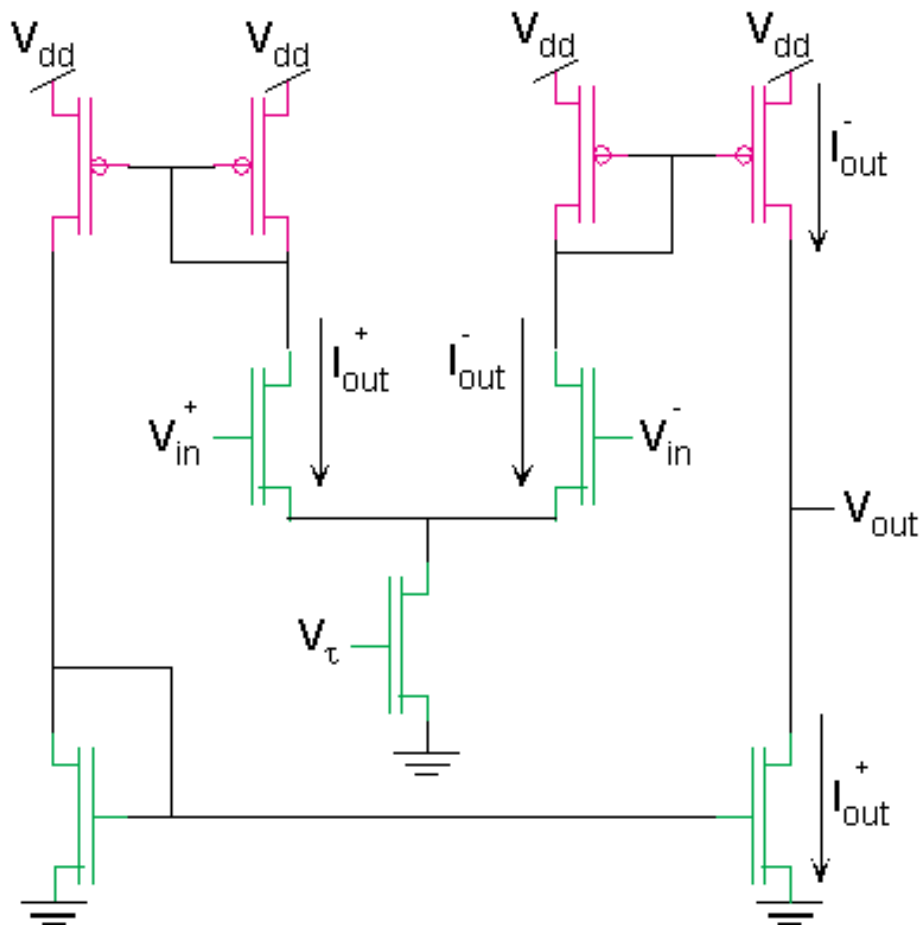
# Transfer Functions



$$I_{out}^+ - I_{out}^- = I_{bias} \tanh\left(\frac{\kappa (V_{in}^+ - V_{in}^-)}{2 U_T}\right)$$



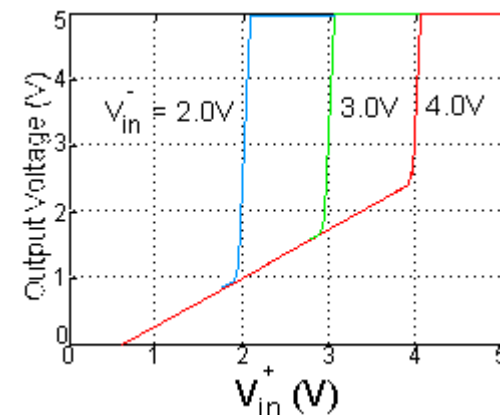
# Wide-Output Range Differential Amplifier



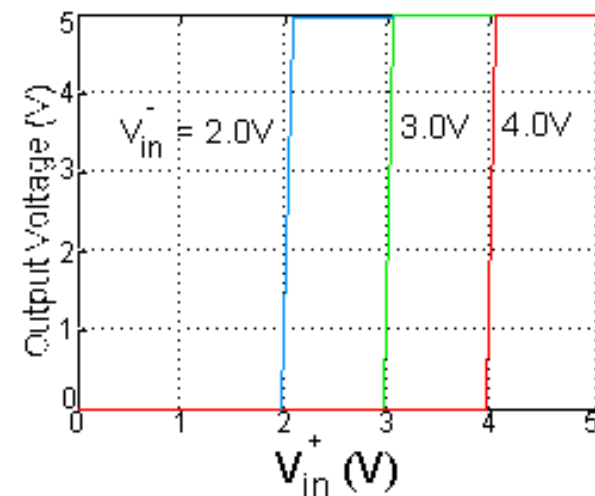
High gain region:  $V_{out} = \frac{\kappa V_0}{2 U_T} (V_{in}^+ - V_{in}^-)$

## Simple differential amplifier

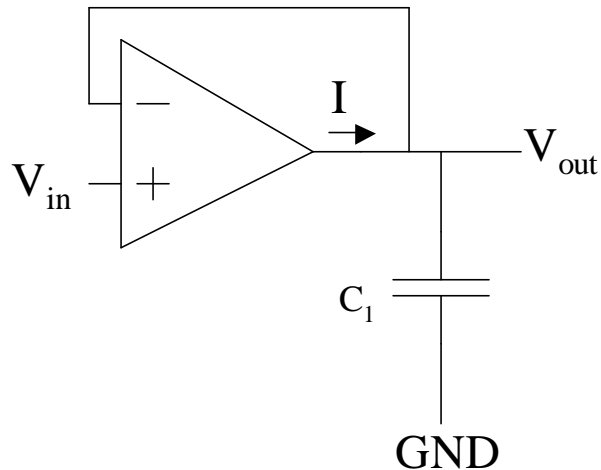
"Vmin" effect



## Wide-output range amplifier



# Simplest Gm-C Filter



If an ideal op-amp, then  $V_{in} = V_{out}$

$$\text{By KCL: } G_m(V_{in} - V_{out}) = C_1 \frac{dV_{out}(t)}{dt}$$

Defining  $\tau = C_1 / G_m$

$$V_{in} = V_{out} + \tau \frac{dV_{out}(t)}{dt}$$

We can set  $G_m$  and build  $C$  sufficiently big enough (slow down the amplifier), or set by  $C$  (smallest size to get enough SNR), and change  $G_m$ .

Also, should mention the high-pass version as well.

# Tuning Frequency using Bias Current

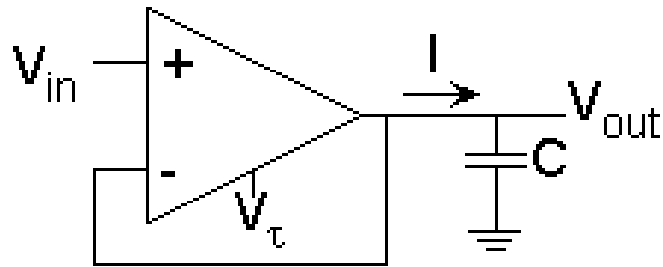
Bias Current	Transconductance	Cutoff Frequency
1mA	1/250 $\Omega$	600MHz
1 $\mu$ A	1/25k $\Omega$	6MHz
1nA	1/25M $\Omega$	6kHz
1pA	1/25G $\Omega$	6Hz

$C = 1\text{pF}$ ,  $W/L$  of input transistors = 30,  $I_{th} \sim 10\mu\text{A}$ .

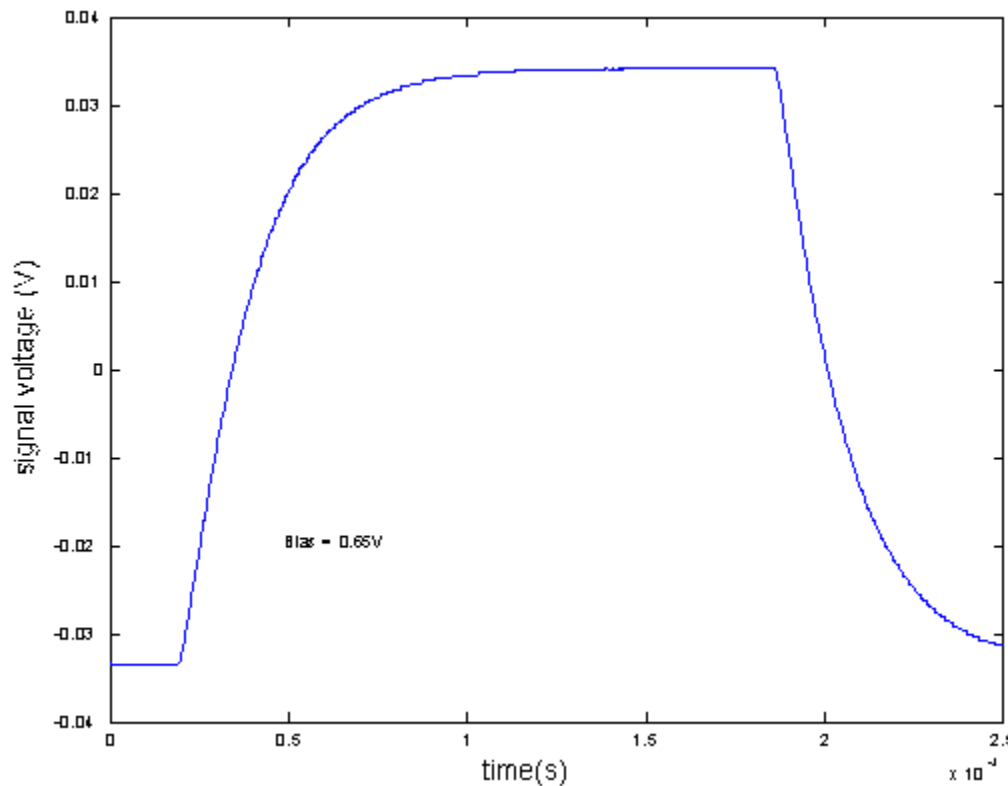
This approach is the most power-efficient approach for any filter.

All electronically tunable: Advantage: we can electronically change the corner  
Disadvantage: we need a method to set this frequency (tuning)

# A Low-Pass Filter



$$\begin{aligned}
 C (dV_{out}/dt) &= I \\
 &= I_{bias} \tanh(\kappa(V_{in} - V_{out})/(2 U_T)) \\
 &= ((\kappa I_{bias})/(2 U_T)) (V_{in} - V_{out})
 \end{aligned}$$

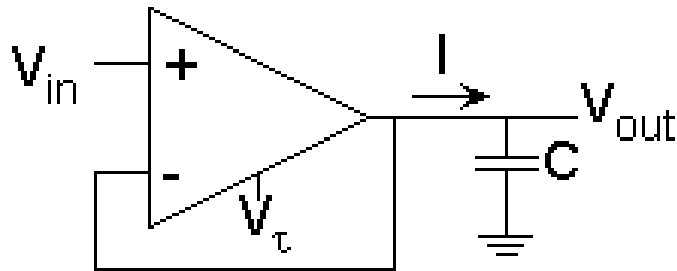


$$\tau (dV_{out}/dt) + V_{out} = V_{in}$$

$$\tau = C U_T / \kappa I_{bias}$$

Time-constant changes  
with bias

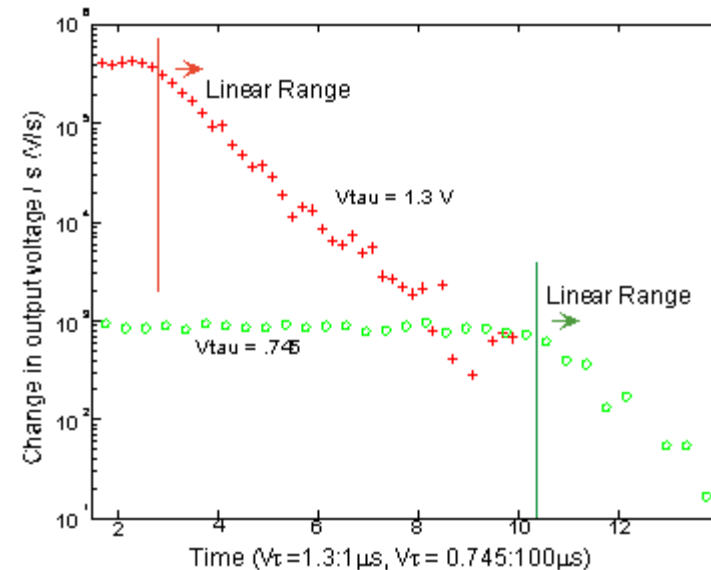
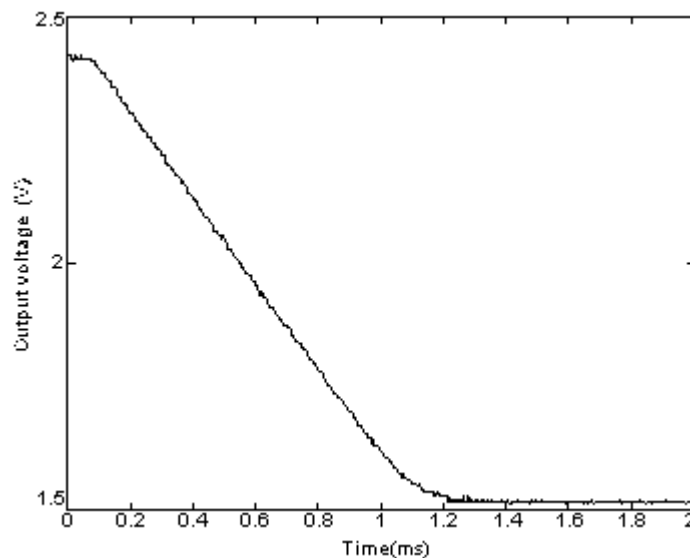
# Linear Range



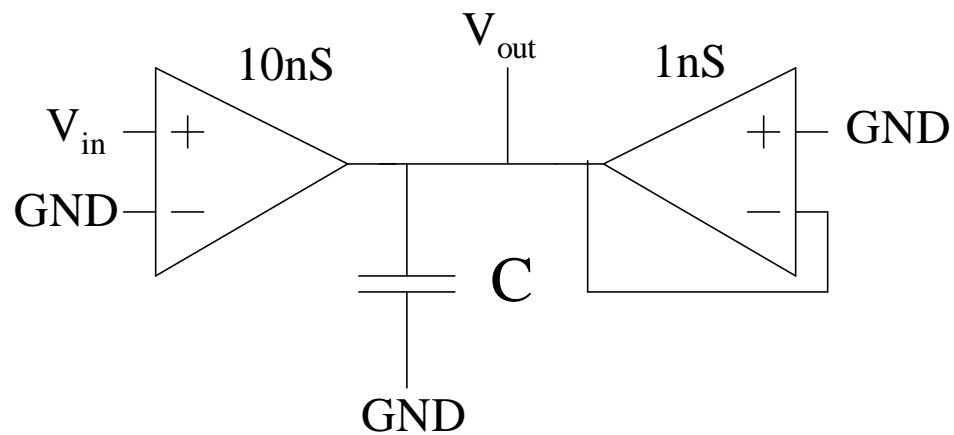
This is not linear....

“Small-signal analysis:  
How small is “small”?”

$$C (dV_{out}/dt) = I_{bias} \tanh(\kappa(V_{in} - V_{out})/(2 U_T))$$



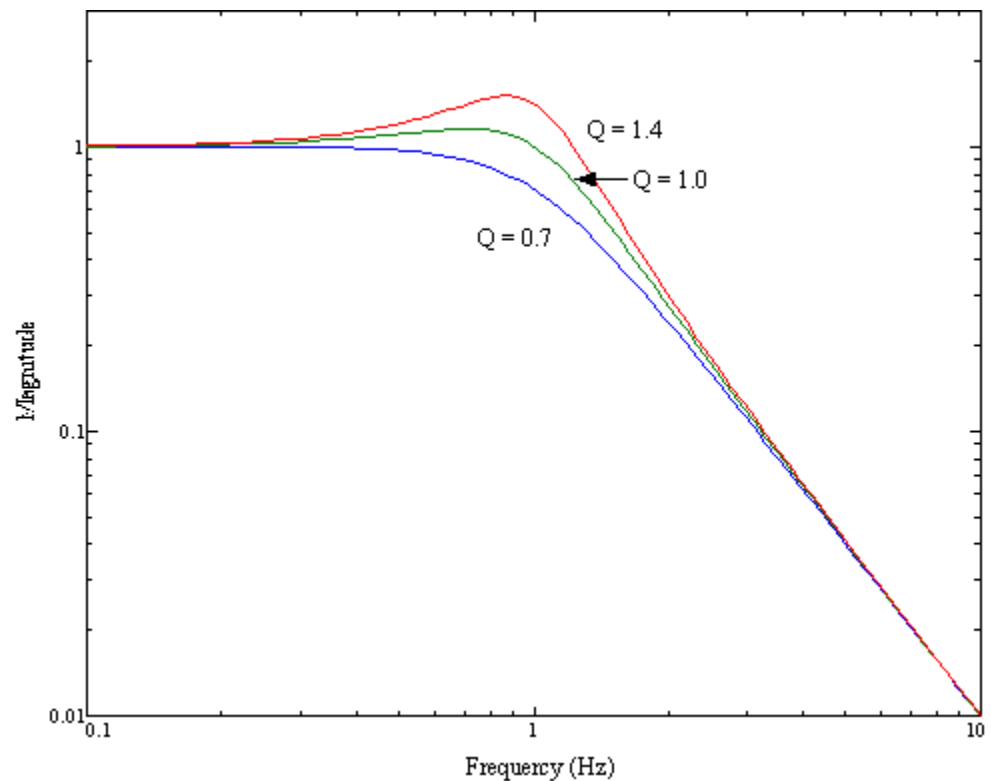
# Another First-Order Low-Pass Filter



# Second-Order Behavior

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + (\tau/Q) s + \tau^2 s^2}$$

Gain =  $Q$   
at  $\omega = 1/\tau$ ,



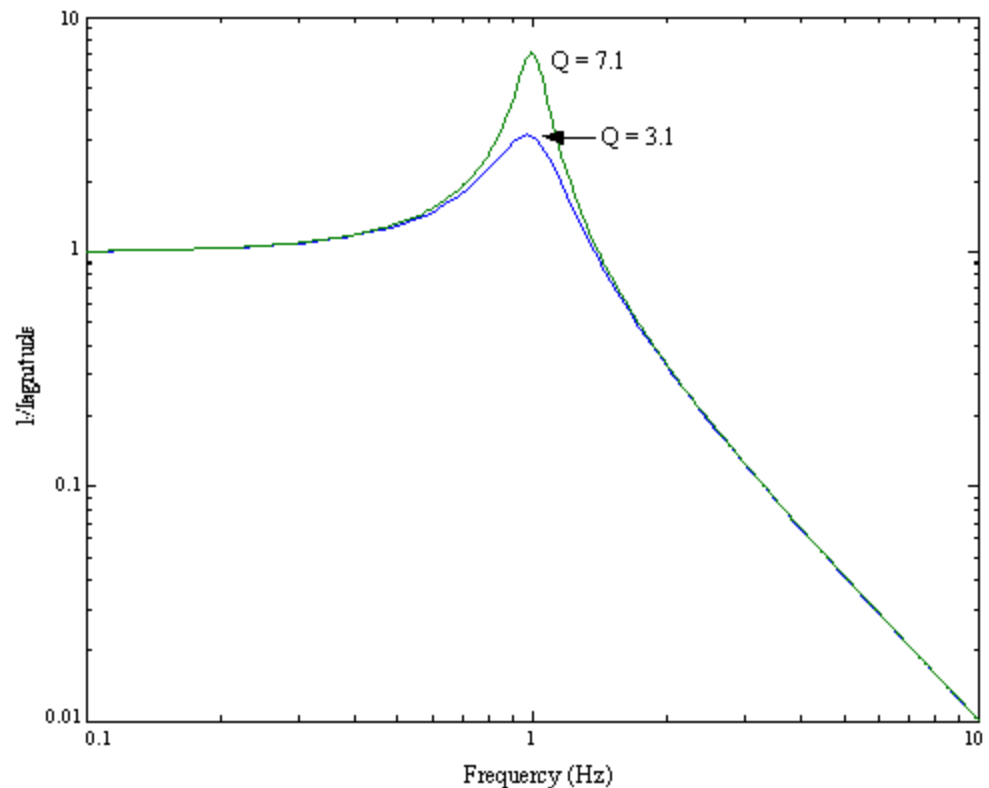


# Second-Order Behavior

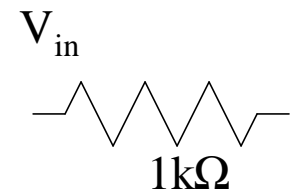
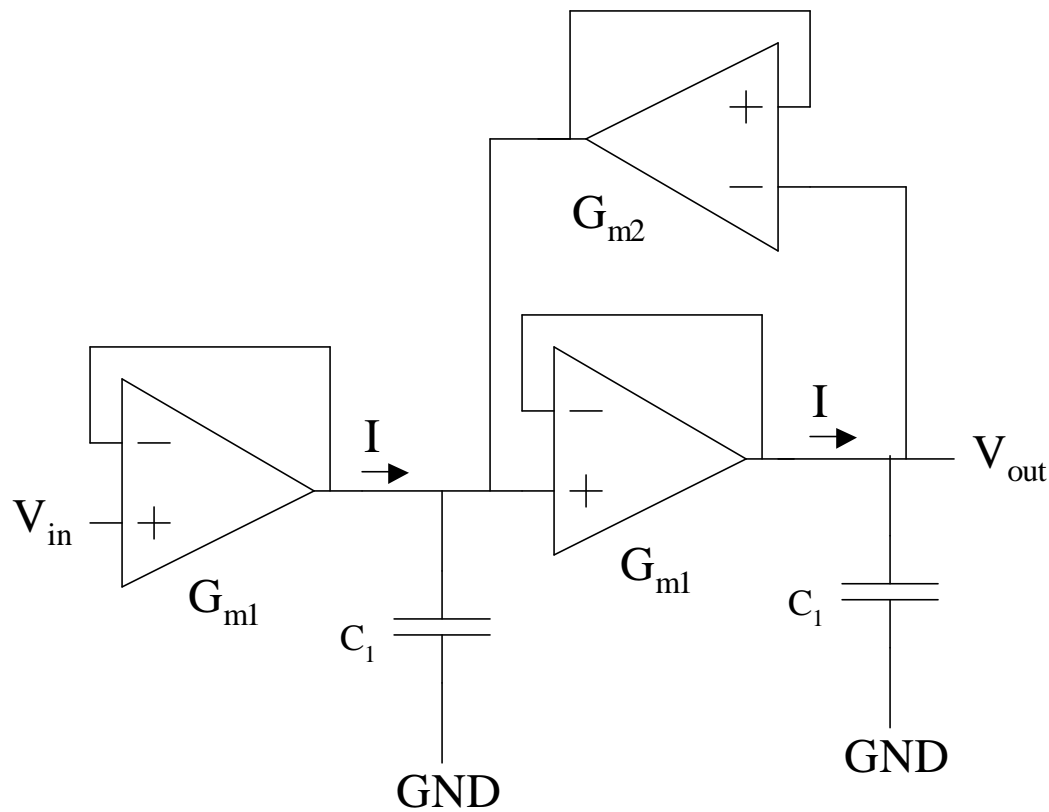
$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + (\tau/Q) s + \tau^2 s^2}$$

Gain =  $Q$   
at  $\omega = 1/\tau$ ,

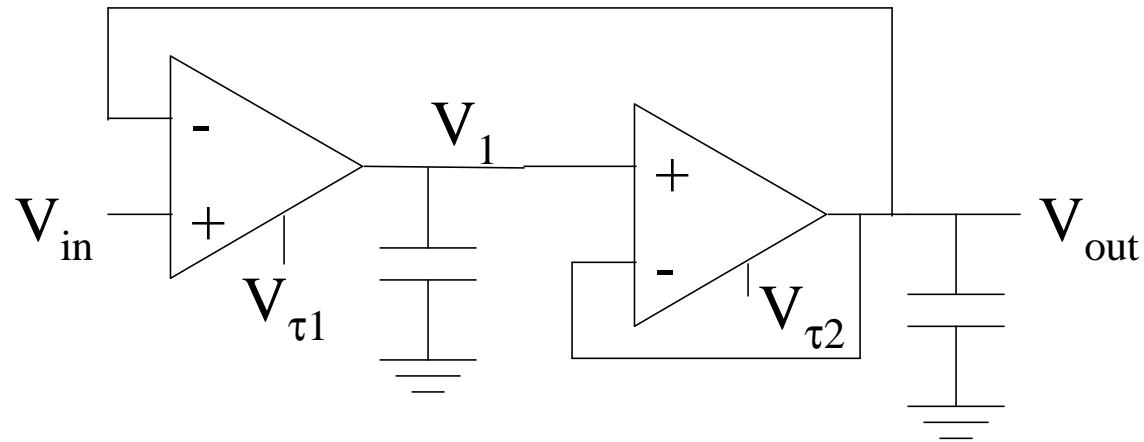
High  $Q$  =  
very narrow band  
(width  $\propto 1/Q$ )



# Basic Second-Order Sections



# Diff2 Based Second-Order Filter

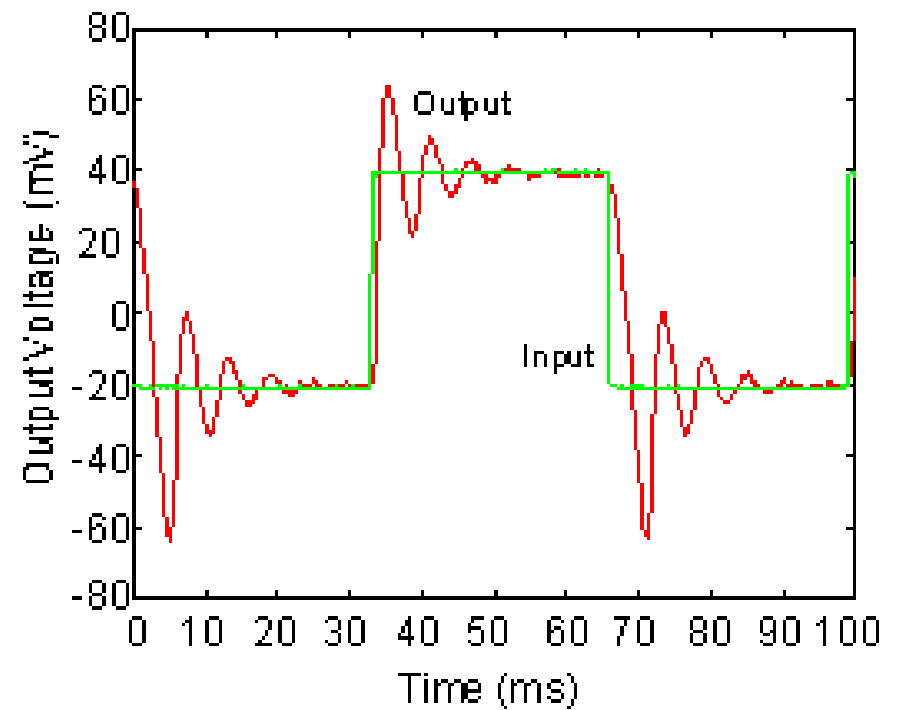
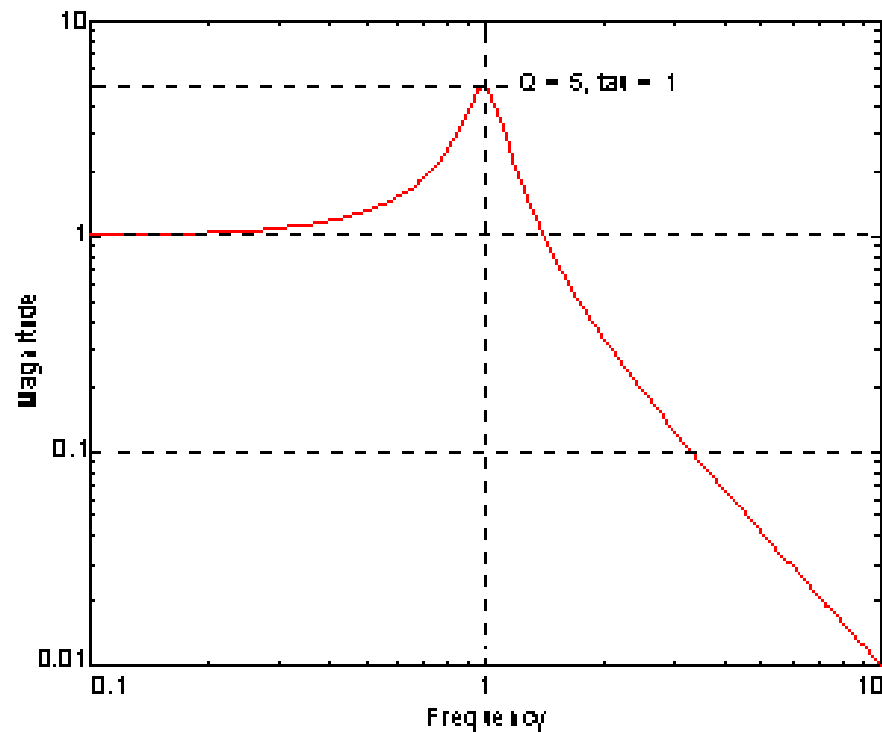
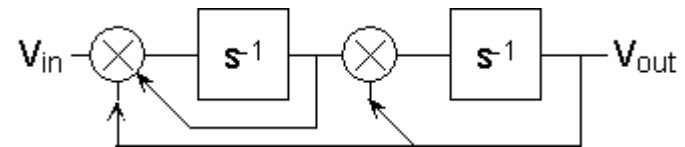
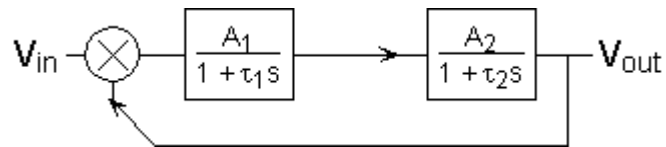


$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + (\tau/Q) s + \tau^2 s^2}$$

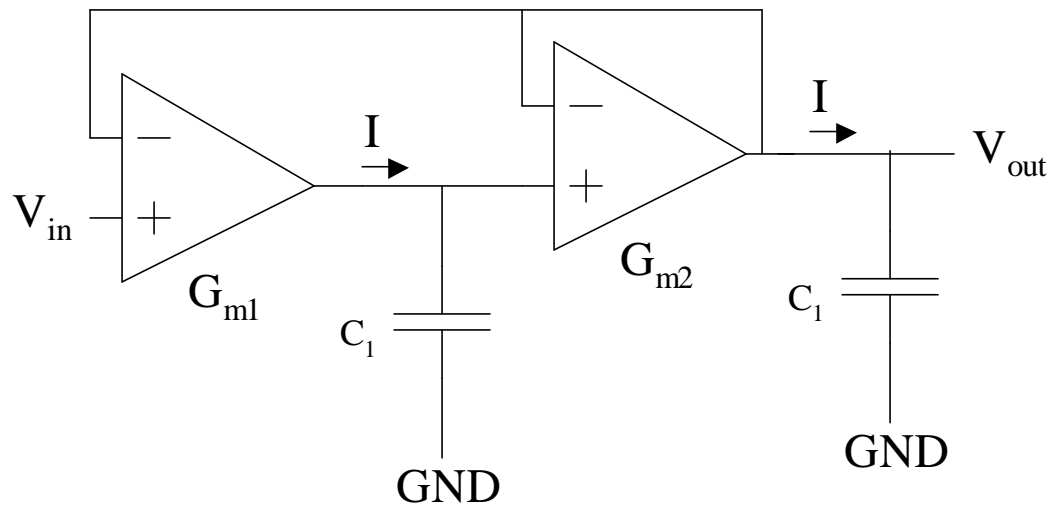
$$\tau = (\tau_1 \tau_2)^{1/2}$$

$$Q = (\tau_2/\tau_1)^{1/2}$$

# Diff2 Responses



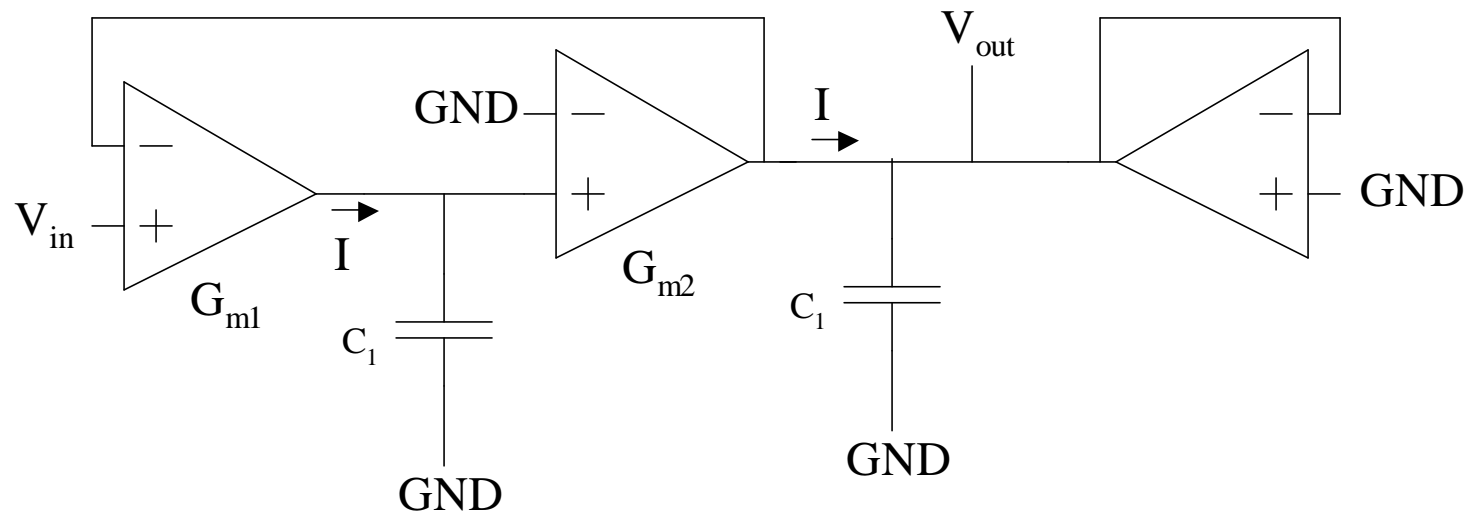
# Diff2 Second-Order Section



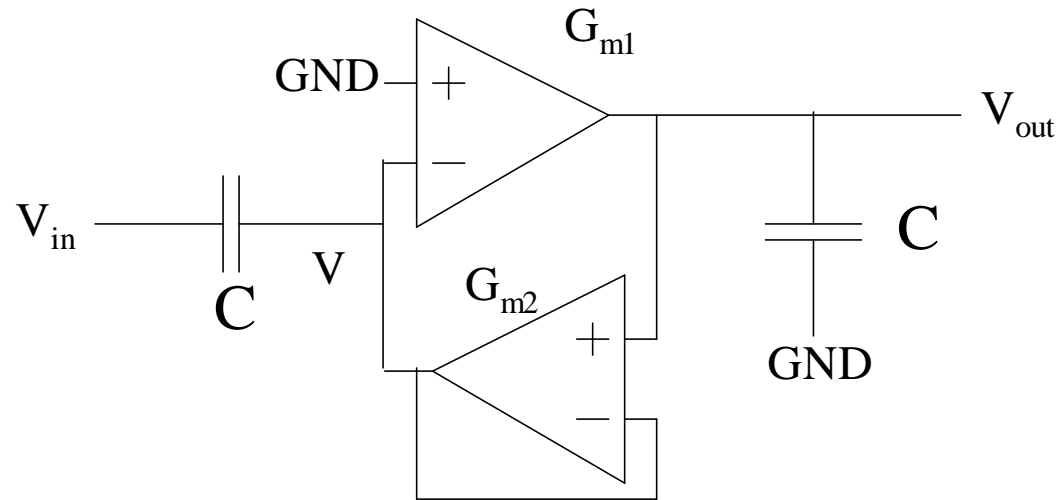
We will see this circuit again :  
sample and hold elements

Issue of feedback with an output  
buffer for an op-amp.

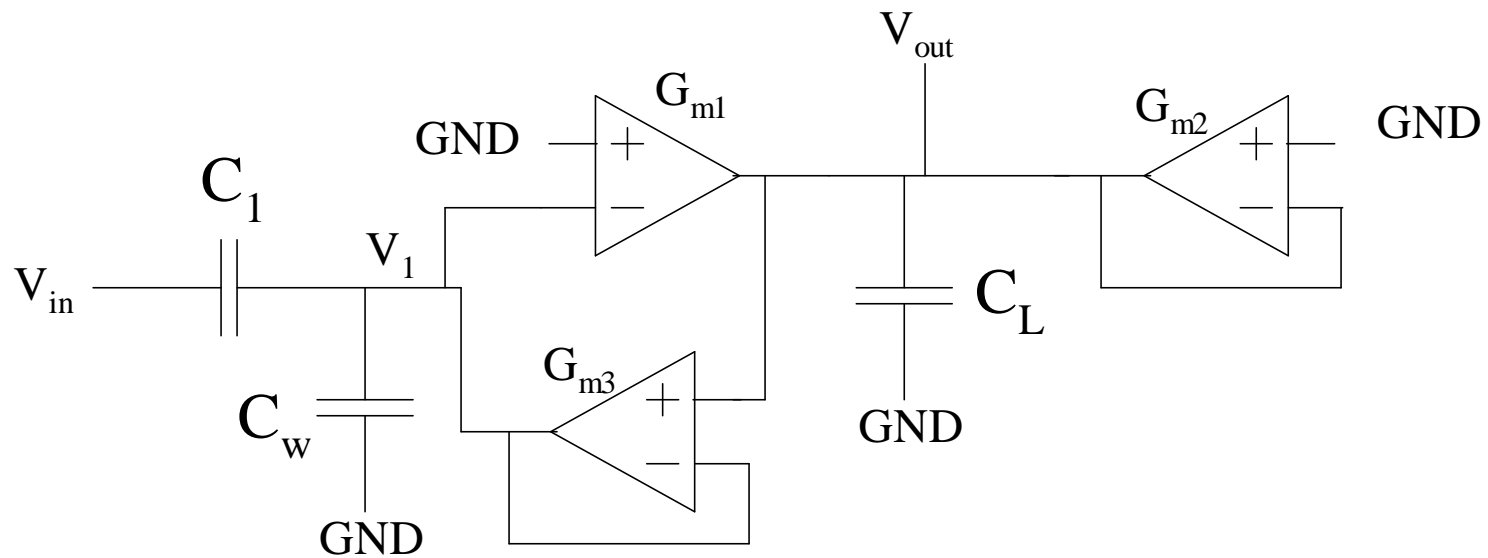
# Tow-Thomas Second-Order Section



# Bandpass Second-Order Section

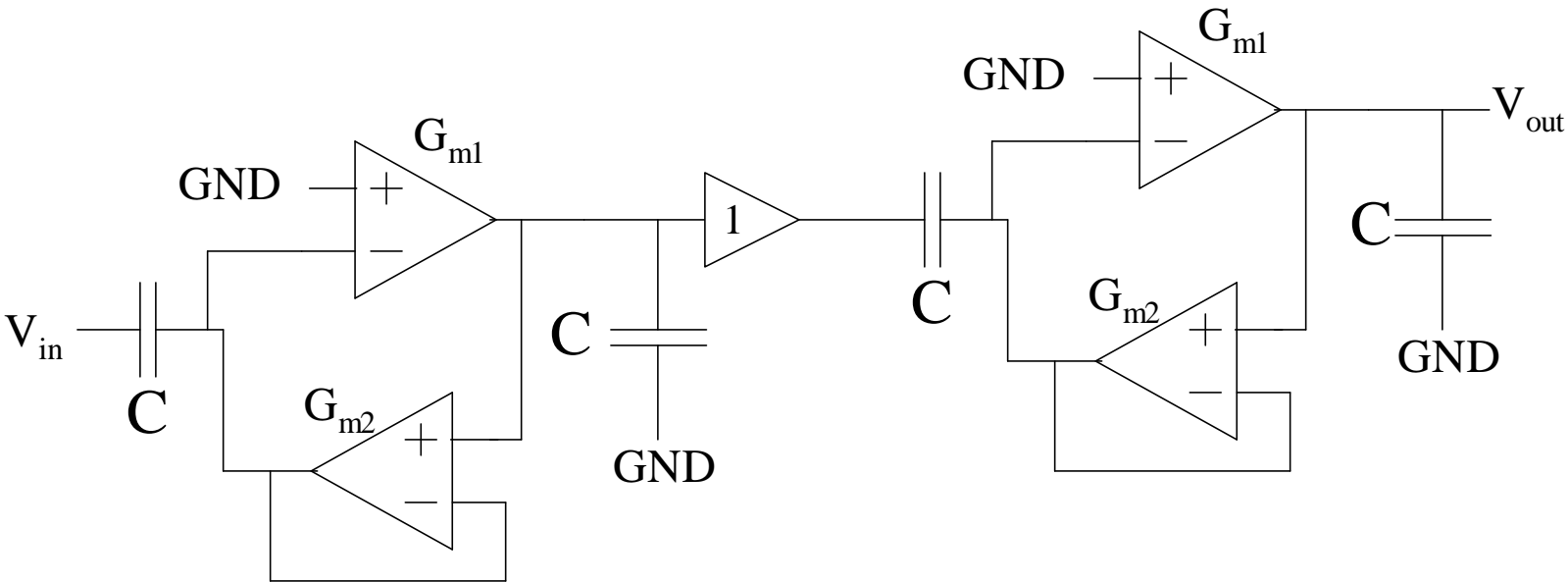


# Another Second-Order Section





# Even Bigger Gm-C Filter



# Filter Design Problem

Design example(s):

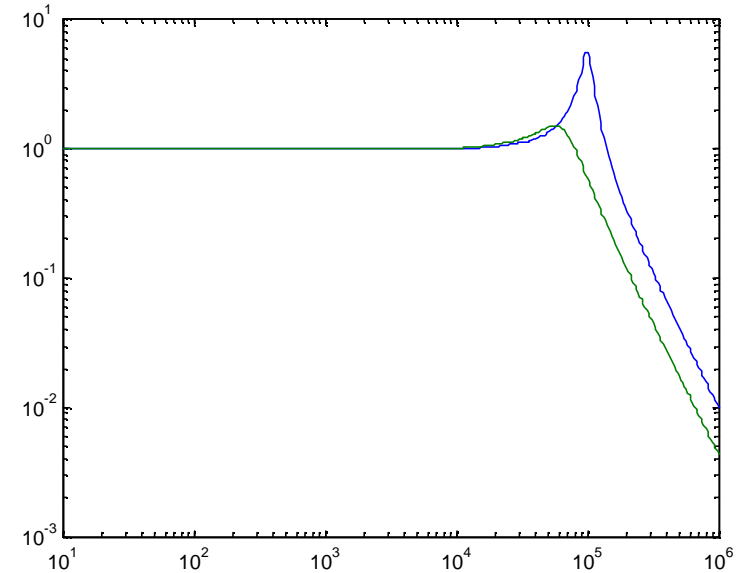
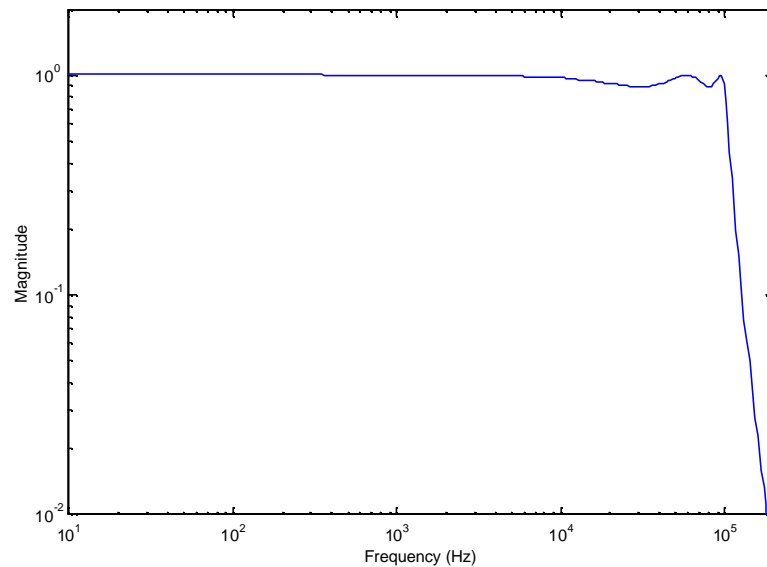
Design a Chebyshev filter with the following specs:

From our MATLAB functions, we get the following  $\tau$ 's and Q's:

Tpb	-1dB ( 0.5088)
Tsb	-25dB
fpb	100kHz
fsb	150kHz

We get the following filter:

# Filter Design Example



From MATLAB code:  $N = 5$

$Q = 5.5561\text{e}+000$ ,  $\tau = 1.6009\text{e}-006$

$Q = 1.3987\text{e}+000$ ,  $\tau = 2.4290\text{e}-006$

$\tau = 5.4974\text{e}-006$