# Switched Capacitor Filters (SCF)

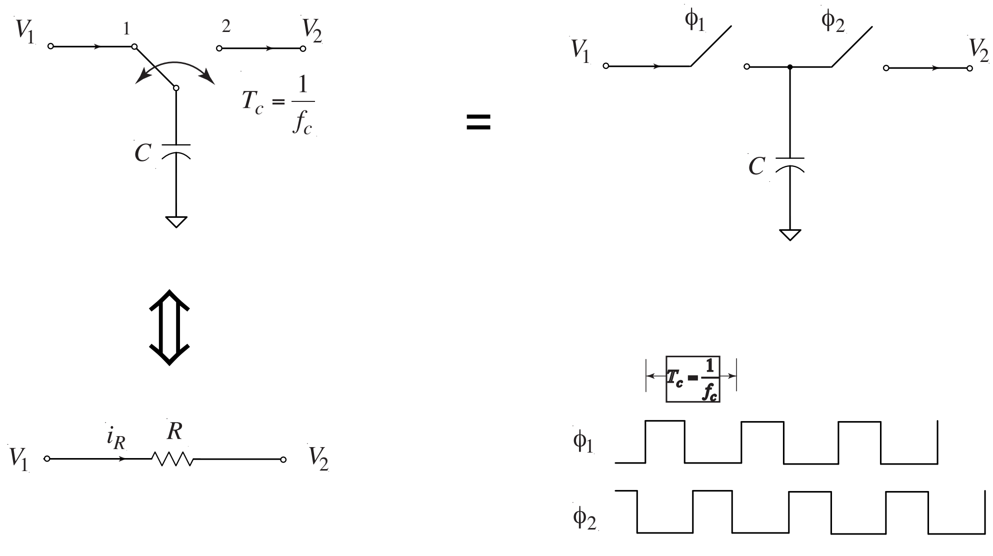
- Passive Filters
  - Components are R, L, C
  - Big, Heavy, discrete
  - Inductors are limited in quality
  - Designed in s-domain
- Active RC filters
  - Components are Opamps, OTAs, R's and C's
  - Can be integrated on the same chip
  - Inaccurate RC in ICs
  - Designed in s-domain
- Switched Capacitor Filters
  - Idea well known for over 80 years
  - Accurate RC, where R is realized using switches and capacitors
  - Clock noise and noise alias
  - Small chip area
  - Designed in z-domain
  - Two most useful filter realizations
    - Cascade
    - Ladder Realization



# SCF Design

- Three basic methods
  1. Resistor substitution
     - Replace the resistors in a continuous time active RC filter, with switched capacitor circuits
  2. Uses switched capacitor integrators to simulate passive RLC prototype circuits, RLC ladder networks, for a desired filter realization
  3. Uses a direct building block approach in the Z- domain of which there are two approaches:
     a. Convert the transfer function into a signal flow diagram consisting of amplifiers, delays and summers
     b. Break the transfer function into products of first and second order terms.

# SC Resistor Simulation




- $\phi_1, \phi_2$ is a two phase nonoverlapping clock



# SC Resistor Simulation (Cont'd)

At position 1 the charge on $C$ at steady state is $q_1 = CV_1$

At position 2 the charge on $C$ at steady state is $q_2 = CV_2$

Assuming $V_1 > V_2$, the charge $\Delta q$ transferred is

$$\Delta q = q_1 - q_2 = C(V_1 - V_2)$$

The current flowing will be on average

$$i(t) = \frac{\delta q(t)}{\delta t} = \lim_{\Delta T_c \to 0} \frac{\Delta q}{\Delta T_c} \cong \frac{C(V_1 - V_2)}{T_c} \qquad (1)$$

Consider now a resistance of value $R$ connected to the same two sources $V_1 \& V_2$. Then

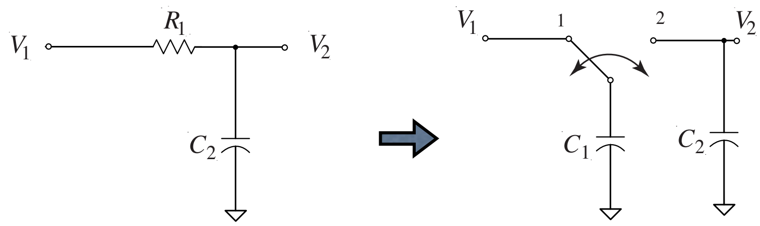
$$i_R(t) = \frac{(V_1 - V_2)}{R} \qquad (2)$$

Equating (1) and (2) yields $R = \frac{T_c}{C} = \frac{1}{f_c C}$

Ex. Using $C = 1pF, f_c = 100kHz \to R = 10M\Omega$

# Conditions for SC-R Approximation

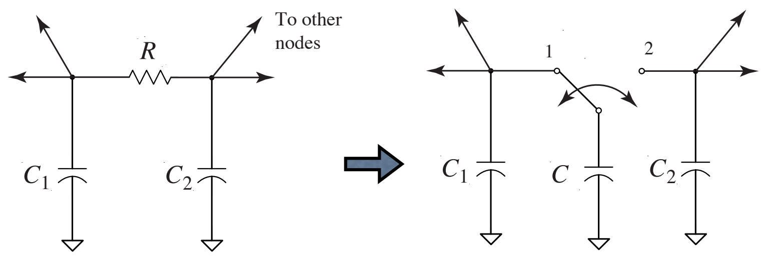
- At least one of the sources must be a voltage source




$$R_1 = T_c\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

- If $C_2 >> C_1$, $R_1 = \frac{T_c}{C_1}$

Note if neither $V_1$ nor $V_2$ are voltage sources then



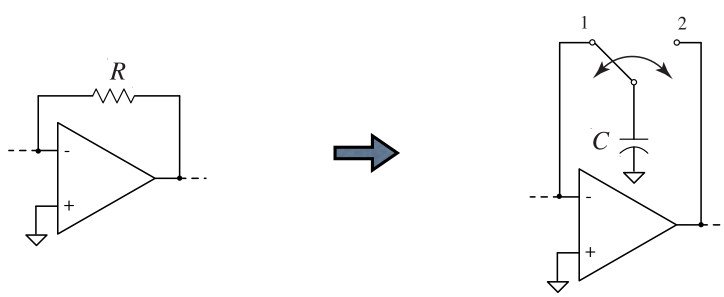

Show that

$$R = \left[\frac{C_2C_1 + (C_2 + C_1)C + C^2}{C_2C_1C}\right]T_c$$

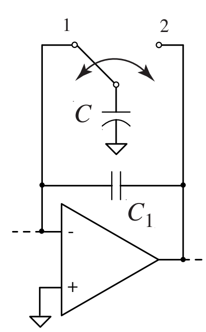


# Conditions for SC-R Approximation (Cont'd)

- SC Resistors cannot close an opamp feedback path



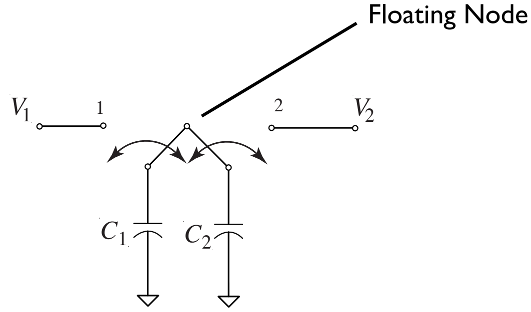

Not allowed



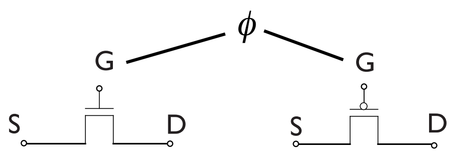

Allowed

# Conditions for SC-R Approximation (Cont'd)

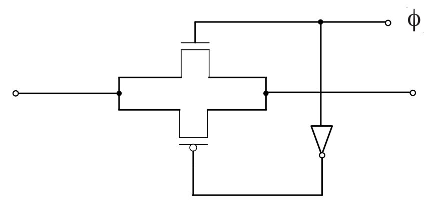
- No floating nodes allowed




- Switches




NMOS PMOS




Complementary Switch

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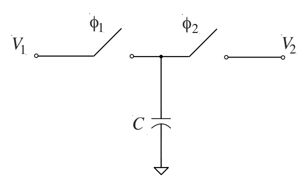
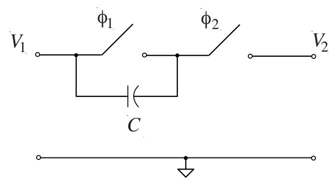
# Summary of Approximated Resistance of Four Switched Capacitor Resistor Circuits

Parallel



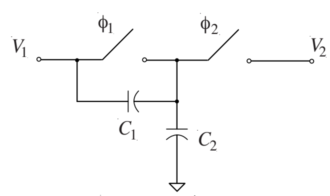

$$R = \frac{T}{C}$$

Series



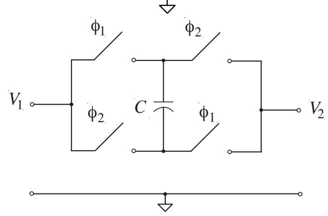

$$R = \frac{T}{C}$$

Series-Parallel




$$R = \frac{T}{C_1 + C_2}$$

Bilinear




$$R = \frac{T}{4C}$$

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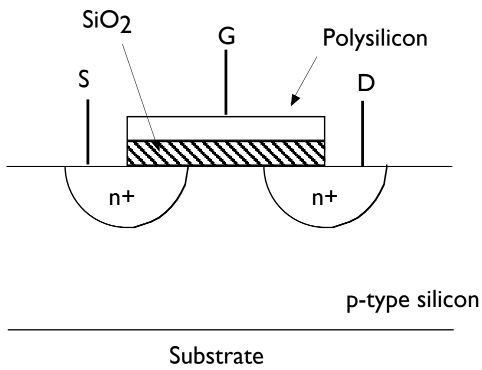
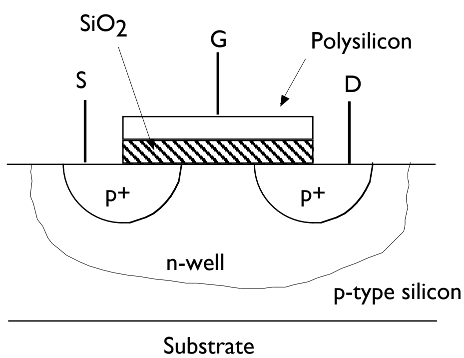
# MOS Switch Realization




NMOS Switch

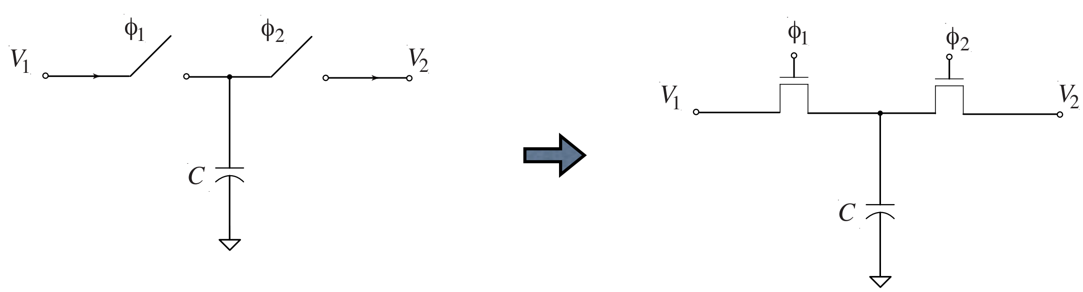


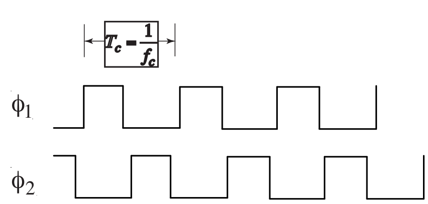

PMOS Switch



# Switches Continued

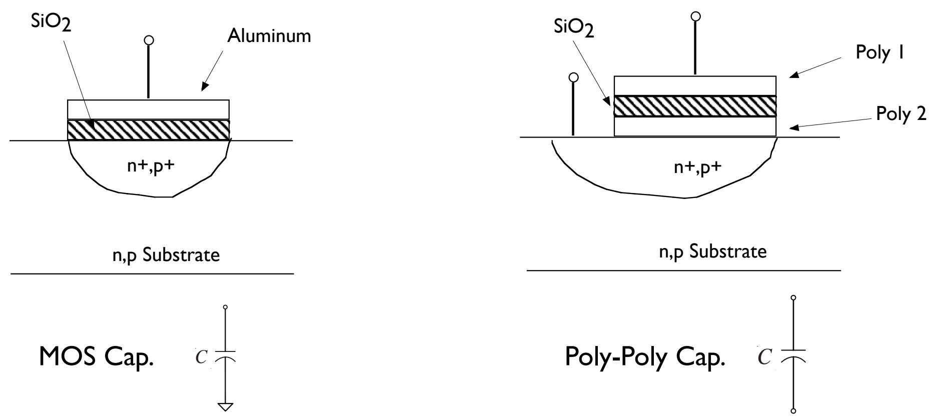
- Switch turns on when $V_G > V_{TH}$ and turns off when $V_G < V_{TH}$
- Typically $R_{on} \cong 1k\Omega - 10k\Omega$, and $R_{off} \cong 100M\Omega - 1G\Omega$ for single switches. For complementary MOS switches $R_{on}$ is lower and in the order of $R_{on} \cong 10\Omega - 1k\Omega$



# Capacitor Realization

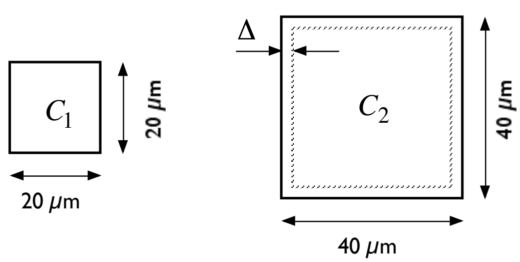
- Basically two types of capacitors
  1. Metal Oxide Crystalline Silicon Capacitor (Grounded capacitor)
     - Best suited for metal to gate CMOS and MOS processes which do not use self alignment procedures.
  2. Polysilicon-Oxide-Polysilicon Capacitor (Floating Capacitor)
     - Found in many processes today
     - Requires little self alignment
- Selling feature of SCs is the capacitor ratio accuracy which can reach 0.1% in some cases. Note transistors can also be used as capacitors!





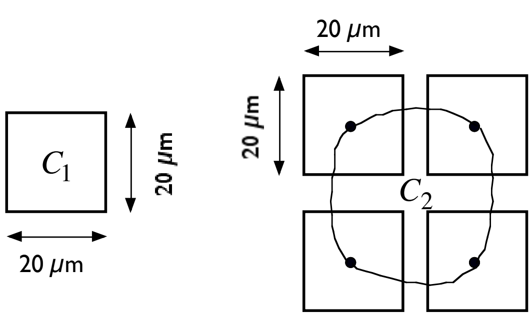

# Capacitance Mismatch

- Area Inaccuracy & Oxide Thickness Variation
- Keep Capacitance Ratio Constant



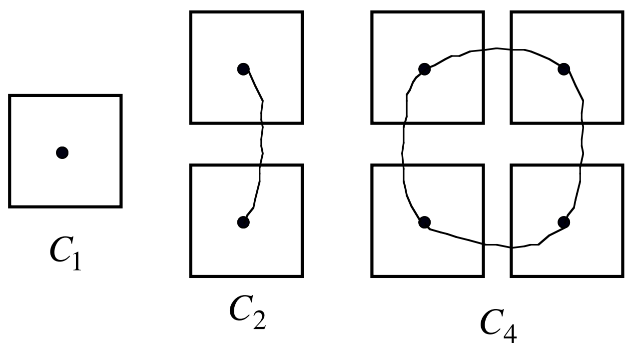

$$\frac{C_2}{C_1} = \frac{(40-0.5)\cdot(40-0.5)}{(20-0.5)\cdot(20-0.5)} = 4.103$$

$\rightarrow$ 2.6% error

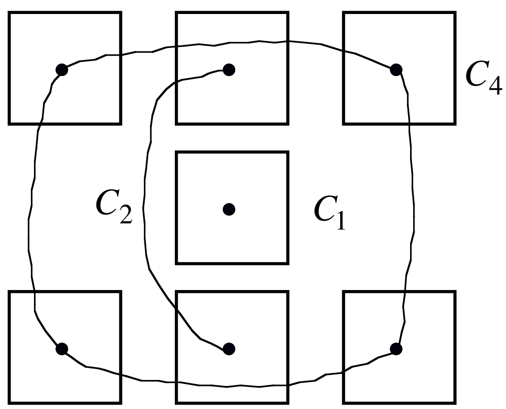



$$\frac{C_2}{C_1} = \frac{4(20-0.5)\cdot(20-0.5)}{(20-0.5)\cdot(20-0.5)} = 4$$

# Capacitance Layout




Non optimal approach



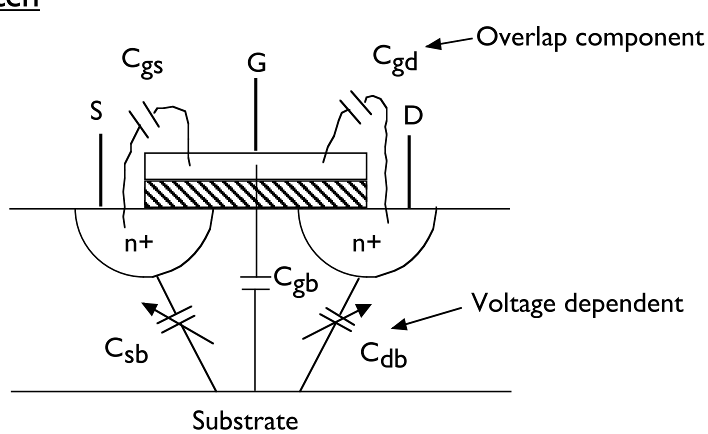

- Optimal layout is the Centroid Approach
- Ground shielding should also be added

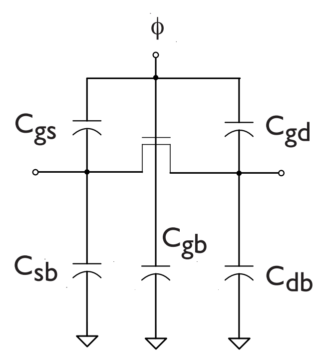


# Parasitic Capacitances

Parasitic capacitances exist in switches and in capacitors

Switch

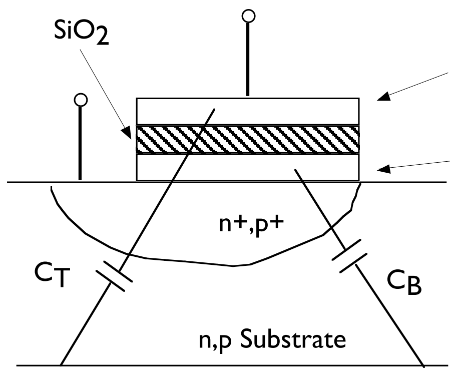


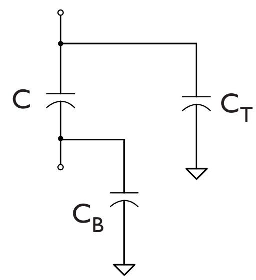




$$C_{gs} = C_{gd} \cong 0.005\,pF$$
$$C_{sb} = C_{db} \cong 0.02\,pF$$

# Parasitic Capacitances Continued

## Poly-poly Capacitor



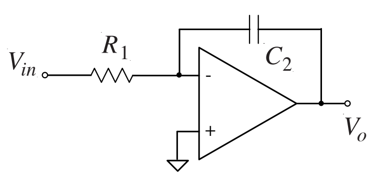




$C_T \cong 0.1\% - 1\%$ of the desired MOS capacitance $C$

$C_B \cong 5\% - 20\%$ of the desired MOS capacitance $C$

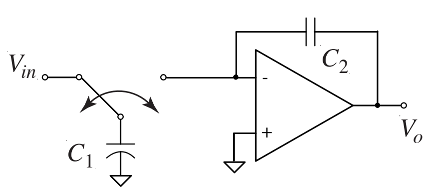


# A Switched Capacitor Integrator




- s – domain "Miller" Integrator

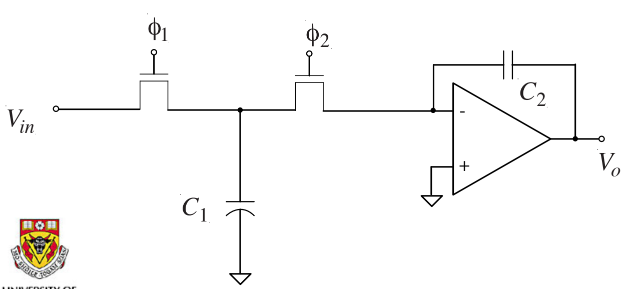
$$\frac{V_o}{V_{in}} = -\frac{1}{R_1 C_2 s}$$



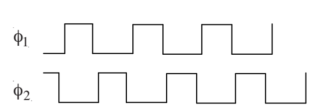

- SC Integrator obtained by replacing "$R$" with a switched capacitor

$$\frac{V_o}{V_{in}} = -f_c\left(\frac{C_1}{C_2}\right)\frac{1}{s}$$

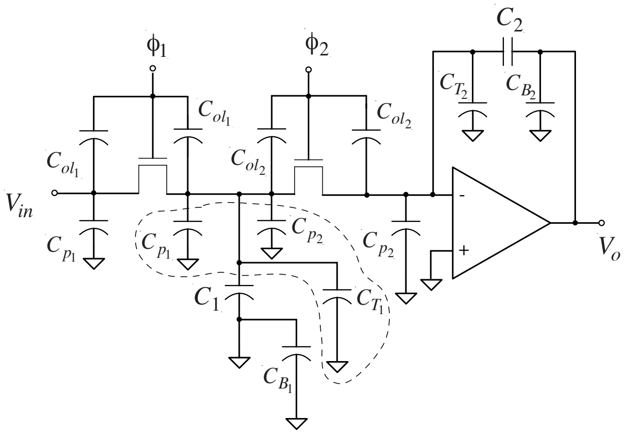
$$C_1 \cong \frac{1}{f_c R_1}$$




- Implementation



# Effects of Parasitics



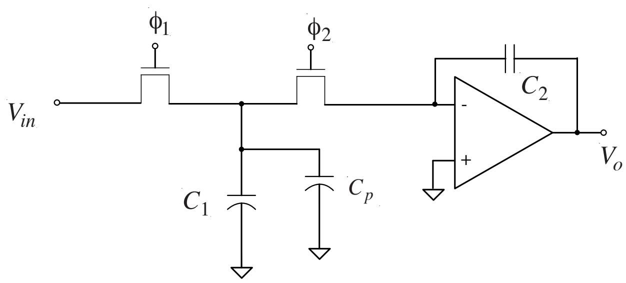

$C_p$'s are $C_{sb}$ and $C_{db}$

$C_{ol}$'s are $C_{gs}$ and $C_{gd}$

- $C_{p_1}$ is always charging from $V_{in}$ and has no effect
- $C_{B_1}$ is shorted
- $C_{p_2}$ and $C_{T_2}$ have zero voltage across them. This has no effect
- $C_{B_2}$ is always charging to $V_o$
- $C_{ol_1}$ and $C_{ol_2}$ only affect the output by feeding the clock signal through the integrator. This is known as clockfeedthrough. It can be minimized but never eliminated.
- Thus we are left with $C_p = C_{p_1} + C_{p_2} + C_{T_1}$



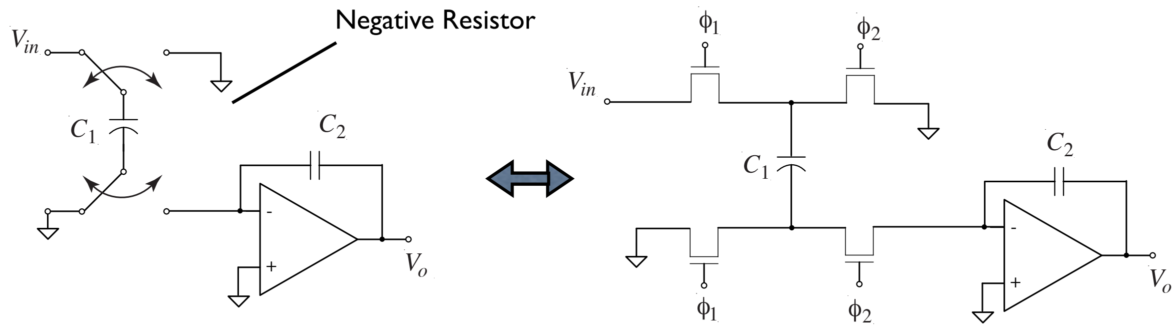
# Effects of Parasitics (Cont'd)



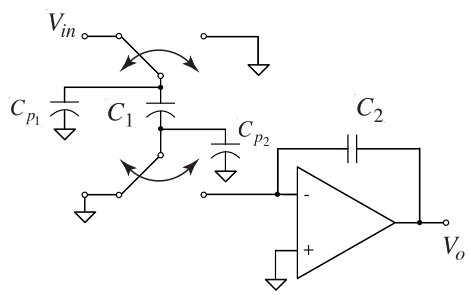

$$\frac{V_o}{V_{in}} = -f_c\left(\frac{C_1 + C_p}{C_2}\right)\frac{1}{s}$$

- The above integrator is said to be parasitic sensitive
- Not desireable if precision is required
- Parasitic insensitive integrators exist.

# A Noninverting (Parasitic Insensitive) Integrator




Examining parasitics

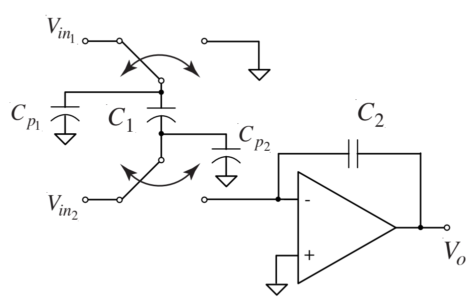



$$\frac{V_o}{V_{in}}(s) = f_c\left(\frac{C_1}{C_2}\right)\frac{1}{s}$$

- $C_{p_2}$ always has a zero potential
- $C_{p_1}$ charges to $V_{in}$ during one cycle and discharges to zero in the next. It has no effect.



# A Differential (Parasitic Sensitive) Integrator




- By superposition

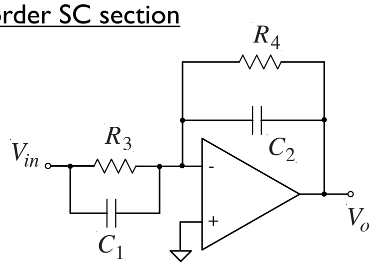
$$V_o(s) = \frac{f_c}{s}\left(\frac{C_1}{C_2}\right)\left[V_{in_1} - \left(1 + \frac{C_{p_2}}{C_1}\right)V_{in_2}\right]$$

If $C_{p_2} << C_1$

$$V_o(s) = \frac{f_c}{s}\left(\frac{C_1}{C_2}\right)\left[V_{in_1} - V_{in_2}\right]$$

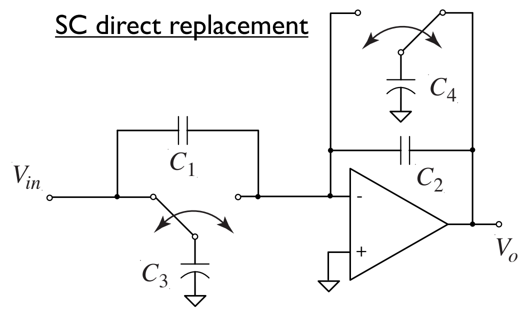
# Sample Circuits

First order SC section




$$\frac{V_o}{V_{in}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{s+\frac{1}{R_3C_1}}{s+\frac{1}{R_4C_2}}\right)$$

SC direct replacement



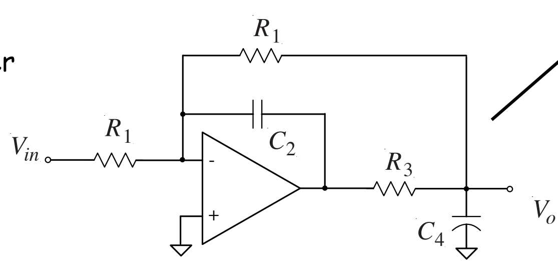

$$\frac{V_o}{V_{in}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{s+f_c\frac{C_3}{C_1}}{s+f_c\frac{C_4}{C_2}}\right)$$



# Sample Circuits (Cont'd)

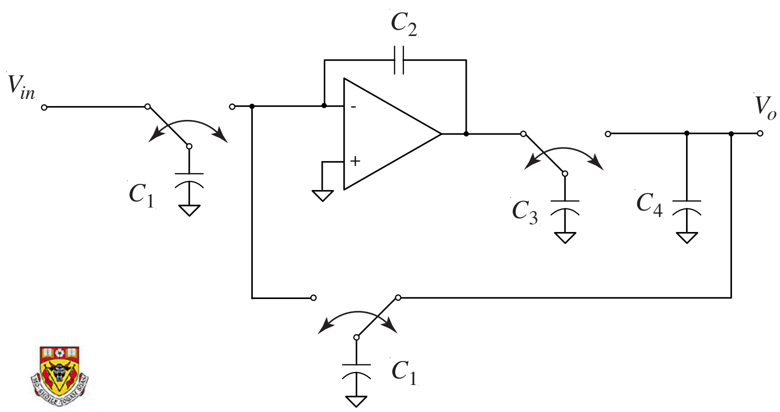
Second order SC section

LP Filter




RC opamp realization

$$\frac{V_o}{V_{in}}(s) = -\frac{\frac{1}{R_1R_3C_2C_4}}{s^2+\frac{1}{R_3C_4}s+\frac{1}{R_1R_3C_2C_4}}$$




$$\frac{V_o}{V_{in}}(s) = -\frac{\frac{C_1C_3}{C_2C_4}f_c^2}{s^2+f_c\frac{C_3}{C_4}s+\frac{C_1C_3}{C_2C_4}f_c^2}$$

$$= -\frac{\omega_o^2}{s^2+\frac{\omega_o}{Q}s+\omega_o^2}$$

# Sample Circuits (Cont'd)

Ex. 1

- Design a LP filter with $f_o = 1.59kHz$ and $Q = 5$ . The DC gain is to be unity.

Solution

$$\frac{V_o}{V_{in}}(s) = -\frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} = -\frac{10^8}{s^2 + 2000s + 10^8}$$

Using the previous circuit the design equations are

$$\frac{C_1}{C_2}\frac{C_3}{C_4}f_c^2 = 10^8 \quad \text{and} \quad \frac{C_3}{C_4}f_c = 2000.$$

Choosing $f_c = 16kHz$ ($\approx 10$ times $f_o$)

$$\therefore \frac{C_1}{C_2} = 3.125 \text{ and } \frac{C_3}{C_4} = 0.125.$$

Let $C_2 = 1pF$ and $C_4 = 10pF$ yields $C_1 = 3.125pF$ and $C_3 = 1.25pF$

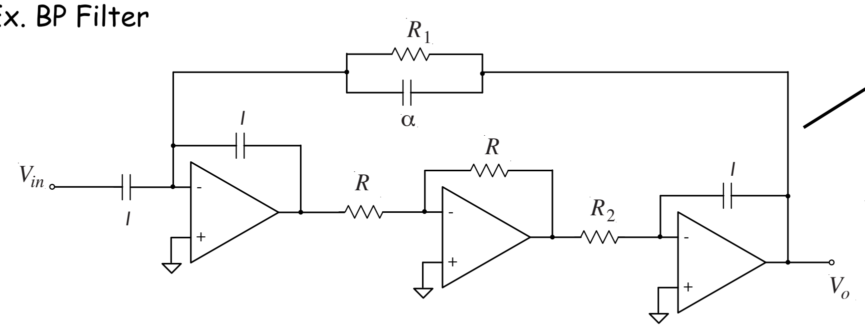
Note the maximum capacitance ratio is 10



# Sample Circuits (Cont'd)

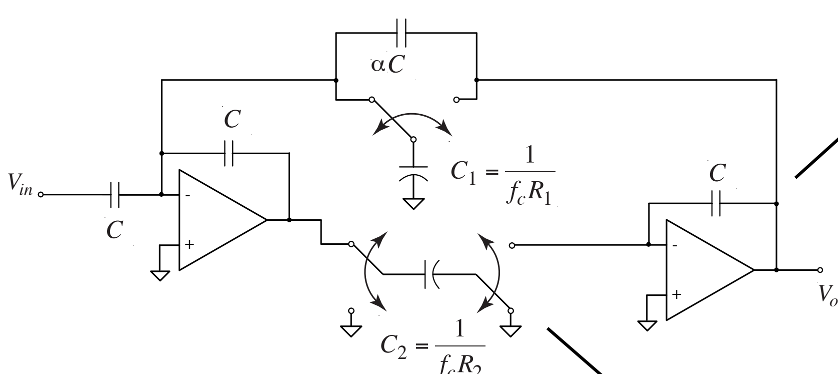
Ex. BP Filter

RC opamp realization




$$\frac{V_o}{V_{in}}(s) = -\frac{\frac{1}{R_2}s}{s^2 + \frac{\alpha}{R_2}s + \frac{1}{R_1 R_2}}$$

SC opamp realization




$$\frac{V_o}{V_{in}}(s) = -\frac{C_2 f_c s}{s^2 + \alpha C_2 f_c s + C_1 C_2 f_c^2}$$

Behaves as a negative resistor

Ex. 2 • Realize a SC BPF with $f_o = 1.59kHz$ and $Q = 5$. The center frequency gain is to be 2.5.

Soln. • Using the previous SC BPF we obtain the following design equations:

$$\omega_o^2 = C_1 C_2 f_c^2$$

$$Q = \frac{1}{\alpha}\sqrt{\frac{C_1}{C_2}}$$

$$CFG = \frac{1}{\alpha}$$

Letting $f_c = 16kHz(\approx 10 f_o)$ and solving the above equations, yields

$$\alpha = 0.4$$

$$C_1 = 1.25$$

$$C_2 = 0.3125$$

We may scale by $10^{12}$ to yield $C_1 = 1.25\,pF, C_2 = 0.3125\,pF, C = 1\,pF$ and $\alpha C = 0.4\,pF$. Note the maximum capacitance ratio is 4.0, which is practical.

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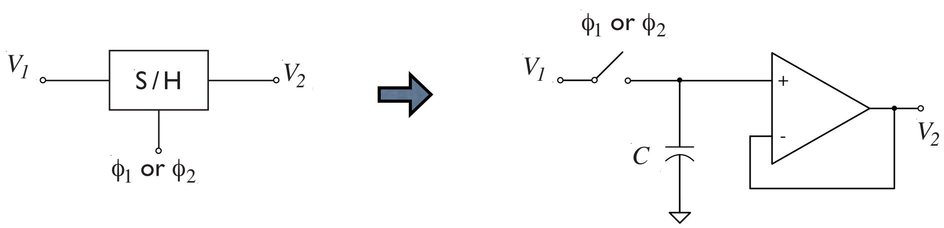
# A Useful Circuit - The Sample & Hold

- The sample and hold provides a means for delaying the input or the output of a circuit by half a clock delay in a two phase clock system.
- The simplest sample and hold is an opamp configured as a buffer with a switch clocked on either of the two clock phases and a holding capacitor. Other more elaborate schemes exist.




• In the z-domain $V_2(z) = z^{-\frac{1}{2}} V_1(z)$ regardless of the clock phasing.

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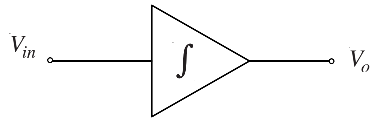
# Numerical Integration and the s-to-z transformation




$$v_o(t) = \int_{-\infty}^{t} v_{in}(t) = \int_{-\infty}^{t_o} v_{in}(t) + \int_{t_o}^{t} v_{in}(t)$$

$$= v_o(t_o) + \text{Area under the } v_{in}(t) \text{ curve from } t_o \rightarrow t$$

- In the frequency domain this can be expressed as,

$$V_o(s) = \frac{1}{s} V_{in}(s) \quad \text{or} \quad \frac{V_o(s)}{V_{in}(s)} = \frac{1}{s}$$

Numerically we can calculate $v_o(t)$ by

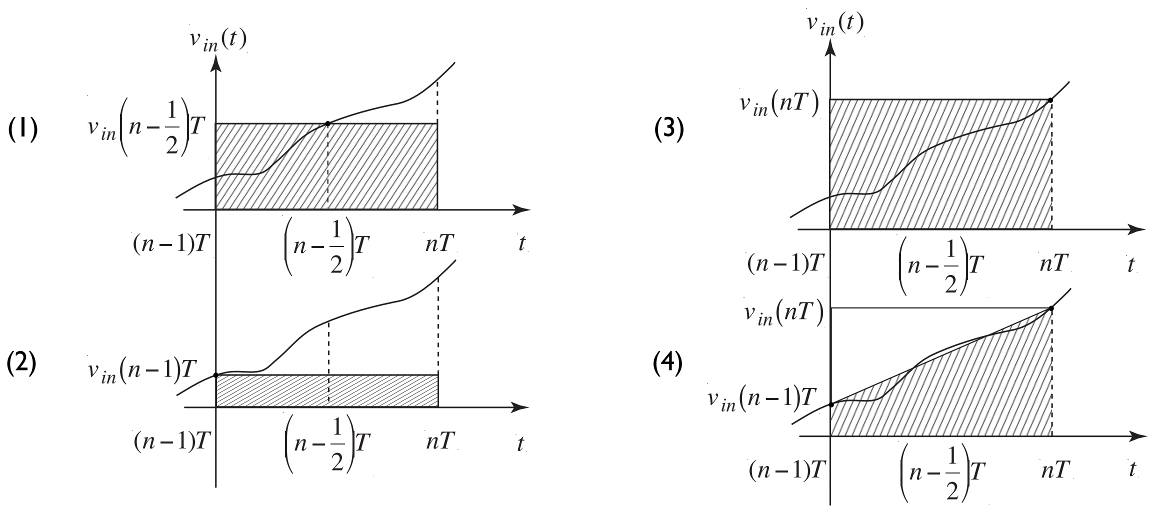
$$v_o(nT) = v_o(nT - T) + \text{Area}$$

- There are four well known methods of area calculation

Note others exist!

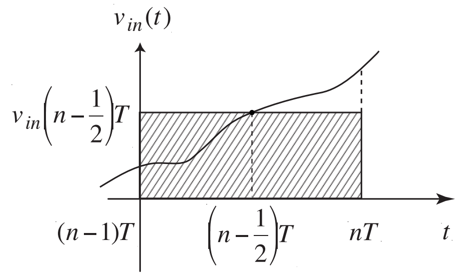


# Four methods of discrete integration




1. Middle Point Integrator or Lossless Discrete Integration (LDI)
2. Forward Euler Discrete Integration (FEDI
3. Backward Euler Discrete Integration (BEDI)
4. Bilinear Discrete Integration (BDI) or Trapeziodal Integration Rule

# Middle Point Integrator




- Area $= Tv_{in}\left(nT - \frac{T}{2}\right)$. Thus $v_o(nT) = v_o(nT - T) + Tv_{in}\left(nT - \frac{T}{2}\right)$

  Taking z - transforms

$$V_o(z) = z^{-1}V_o(z) + Tz^{-\frac{1}{2}}V_{in}(z)$$

$$\therefore \quad \frac{V_o}{V_{in}}(z) = T\,\frac{z^{-\frac{1}{2}}}{1-z^{-1}} \quad \Leftrightarrow \quad \frac{V_o}{V_{in}}(s) = \frac{1}{s}$$

The corresponding s - to - z transform is then obtained as

$$s = \frac{1}{T}\frac{\left(1-z^{-1}\right)}{z^{-\frac{1}{2}}} = \frac{1}{T}\left(z^{\frac{1}{2}} - z^{-\frac{1}{2}}\right)$$

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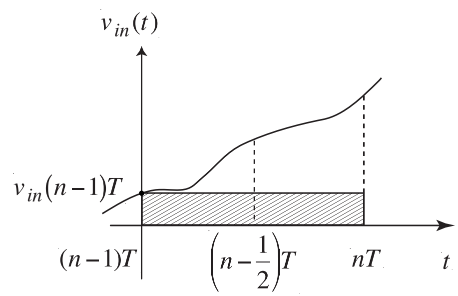
# Forward Euler Integrator




- Area $= Tv_{in}(nT - T)$. Thus $v_o(nT) = v_o(nT - T) + Tv_{in}(nT - T)$

  Taking z - transforms

$$V_o(z) = z^{-1}V_o(z) + Tz^{-1}V_{in}(z)$$

$$\therefore \quad \frac{V_o}{V_{in}}(z) = T\,\frac{z^{-1}}{1-z^{-1}} \quad \Leftrightarrow \quad \frac{V_o}{V_{in}}(s) = \frac{1}{s}$$

The corresponding s - to - z transform is then obtained as

$$s = \frac{1}{T}\frac{\left(1-z^{-1}\right)}{z^{-1}} = \frac{1}{T}(z-1)$$

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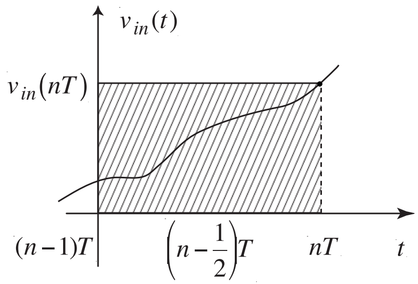
# Backward Euler Integrator




- Area $= Tv_{in}(nT)$. Thus $v_o(nT) = v_o(nT - T) + Tv_{in}(nT)$

  Taking z-transforms

$$V_o(z) = z^{-1}V_o(z) + TV_{in}(z)$$

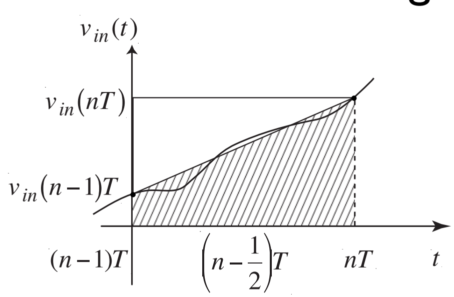
$$\therefore \quad \frac{V_o}{V_{in}}(z) = T\frac{1}{1-z^{-1}} \quad \Leftrightarrow \quad \frac{V_o}{V_{in}}(s) = \frac{1}{s}$$

The corresponding s-to-z transform is then obtained as

$$s = \frac{1}{T}\left(1 - z^{-1}\right)$$



# Bilinear Discrete Integrator




- Area of the trapezoid $= \frac{T}{2}\left[v_{in}(nT - T) + v_{in}(nT)\right]$.

  Thus $v_o(nT) = v_o(nT - T) + \frac{T}{2}\left[v_{in}(nT - T) + v_{in}(nT)\right]$

  Taking z-transforms

$$V_o(z) = z^{-1}V_o(z) + \frac{T}{2}\left[V_{in}(z) + z^{-1}V_{in}(z)\right]$$

$$\therefore \quad \frac{V_o}{V_{in}}(z) = \frac{T}{2}\cdot\frac{1+z^{-1}}{1-z^{-1}} \quad \Leftrightarrow \quad \frac{V_o}{V_{in}}(s) = \frac{1}{s}$$

The corresponding s-to-z transform is then obtained as

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

# Bilinear Discrete Integrator (Cont'd)

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \quad \Leftrightarrow \quad z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}}$$

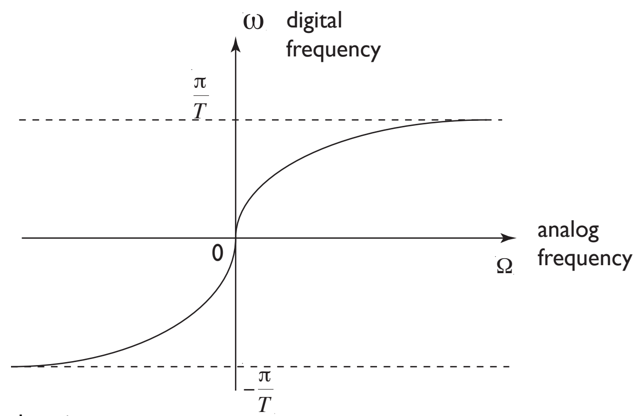
For $s = j\Omega$ and $z = re^{j\omega T}$

$$re^{j\omega T} = \frac{1+\frac{j\Omega T}{2}}{1-\frac{j\Omega T}{2}}$$

$$\therefore \;\; r = 1, \text{ and } \omega = \frac{2}{T}\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

If $s = -\sigma \pm j\Omega \quad \rightarrow \quad |r| < 1$

If $s = +\sigma \pm j\Omega \quad \rightarrow \quad |r| > 1$




- We have distortion in the digital frequency domain
- Prewarping the digital frequency can compensate for this



# Another Transform and Summary

- There exists the impulse - invariance transform which is popular due to its simplicity.
- $z = e^{sT}$ or $s = \frac{1}{T}\ln(z)$

## Summary

| | |
|---|---|
| $z_1 = \frac{1}{1-sT}$, Backward Euler | For high sampling rates $sT < 1$ |
| $z_2 = 1 + sT$, Forward Euler | $z_1 = 1 + sT + (sT)^2 + (sT)^3 + \ldots \cong 1 + sT$ |
| | $z_2 = 1 + sT$, |
| $z_3 = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}}$, Bilinear $\Rightarrow$ | $z_3 = 1 + sT + \frac{(sT)^2}{2} + \frac{(sT)^3}{4} + \ldots \cong 1 + sT$ |
| $z_4 = e^{sT}$, Impulse Invariance | $z_4 = 1 + sT + \frac{(sT)^2}{2} + \frac{(sT)^3}{6} + \ldots \cong 1 + sT$ |

## Errors associated with the Discrete Integrators

- Errors exist with each integrator
- Ideal integrator should have zero magnitude and phase error

$$H_I(\omega) = H(s)\big|_{s=j\omega} = \frac{1}{j\omega}$$

- Let us assume each integrator has a magnitude error $\varepsilon$ and phase error $\theta$ asscociated with it. That is,

$$H_{NI}(\omega) = \frac{1}{j\omega}(1+\varepsilon)e^{j\theta} \qquad (3)$$

For the LDI

$$H(z) = T\frac{z^{-\frac{1}{2}}}{1-z^{-1}} = \frac{T}{z^{\frac{1}{2}} - z^{-\frac{1}{2}}}$$

$$H(e^{j\omega T}) = \frac{T}{e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}}} \qquad (4)$$

Equating (3) and (4)

$$\frac{1}{j\omega}(1+\varepsilon)e^{j\theta} = \frac{T}{e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}}}$$

$$= \frac{1}{j\omega}\left(\frac{\omega T}{2} Co\sec\left(\frac{\omega T}{2}\right)\right)$$

$$\therefore \quad \varepsilon = \frac{\omega T}{2} Co\sec\left(\frac{\omega T}{2}\right) - 1$$

$$\theta = 0$$

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## Errors associated with the Discrete Integrators (Cont'd)

| | Magnitude Error $(\varepsilon)$ | Phase Error $(\theta)$ |
|---|---|---|
| LDI | $\frac{\omega T}{2} Co\sec\left(\frac{\omega T}{2}\right) - 1$ | 0 |
| FEDI | $\frac{\omega T}{2} Co\sec\left(\frac{\omega T}{2}\right) - 1$ | $-\frac{\omega T}{2}$ |
| BEDI | $\frac{\omega T}{2} Co\sec\left(\frac{\omega T}{2}\right) - 1$ | $+\frac{\omega T}{2}$ |
| BDI | $\frac{\omega T}{2} Cot\left(\frac{\omega T}{2}\right) - 1$ | 0 |

- Because $\theta = 0$ (for LDI) Dr. Bruton called this integrator the Lossless Discrete Integrator. Note the LDI has no real part.
- For high sampling rates all errors are low.

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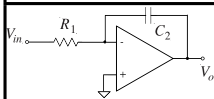
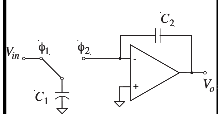
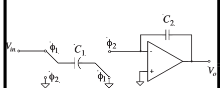
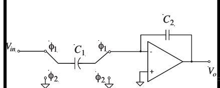
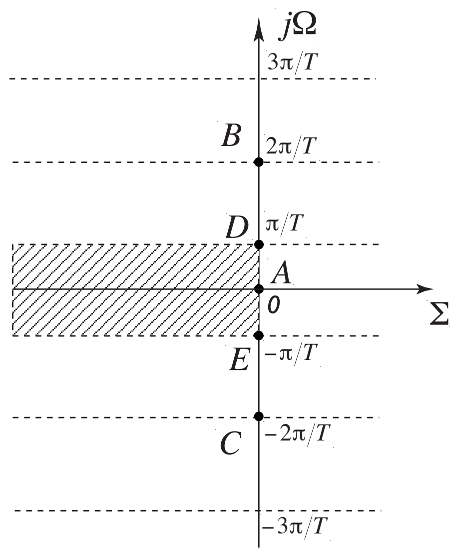
# Summary

| Type of Integrator | Magnitude $\lvert H(e^{j\omega T})\rvert$ | Phase $Arg(H(e^{j\omega T}))$ | Mapping (Equivalent) | Transfer Function |
|---|---|---|---|---|
|  | $\frac{\omega_o}{\omega}$ | $\frac{\pi}{2}$ | In the s-plane | $H(s) = -\frac{1}{sR_1C_2} = -\frac{\omega_o}{s}$ |
|  Inverting Stray-sensitive (Forward) | For $V_o$ at $\phi_2$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $\frac{\pi}{2}$ | LDI | $H(z) = -\frac{C_1}{C_2}\frac{z^{-1/2}}{1-z^{-1}}$ |
| | For $V_o$ at $\phi_1$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $\frac{\pi}{2} - \frac{\omega T}{2}$ | Forward | $H(z) = -\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}$ |
|  Non-Inverting | For $V_o$ at $\phi_2$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $-\frac{\pi}{2}$ | LDI | $H(z) = \frac{C_1}{C_2}\frac{z^{-1/2}}{1-z^{-1}}$ |
| | For $V_o$ at $\phi_1$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $-\frac{\pi}{2} - \frac{\omega T}{2}$ | Forward | $H(z) = \frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}}$ |
|  Inverting (Backward) | For $V_o$ at $\phi_1$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $\frac{\pi}{2} + \frac{\omega T}{2}$ | Backward | $H(z) = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}}$ |
| | For $V_o$ at $\phi_2$ $\frac{\omega_o}{\omega}\left(\frac{\omega T/2}{\sin(\omega T/2)}\right)$ | $\frac{\pi}{2}$ | LDI | $H(z) = -\frac{C_1}{C_2}\frac{z^{-1/2}}{1-z^{-1}}$ |

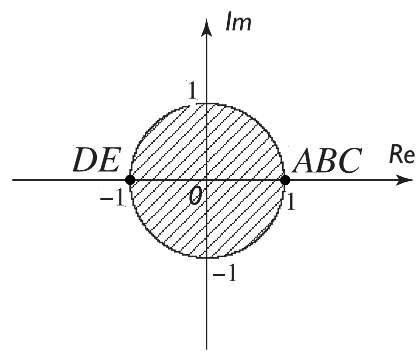


# Impulse Invariance Mapping

s-plane

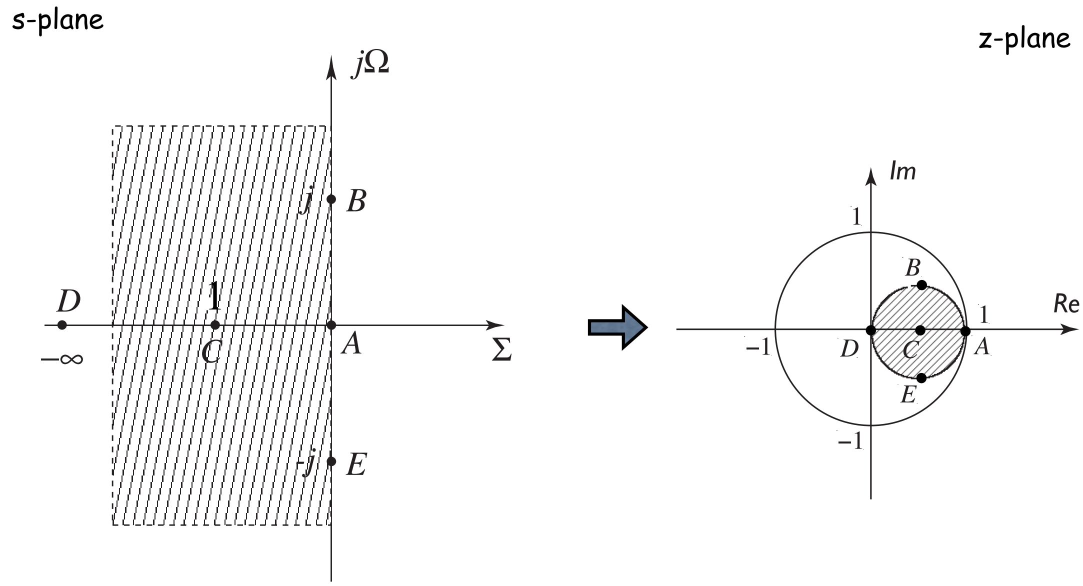



z-plane




- A strip of length $2\pi/T$ is wrapped once around the unit circle
- Method is only applicable to filters with a bandlimited analog frequency response that satisfies $\lvert H(j\Omega)\rvert \cong 0$ for $\lvert\Omega\rvert > \Omega_B$

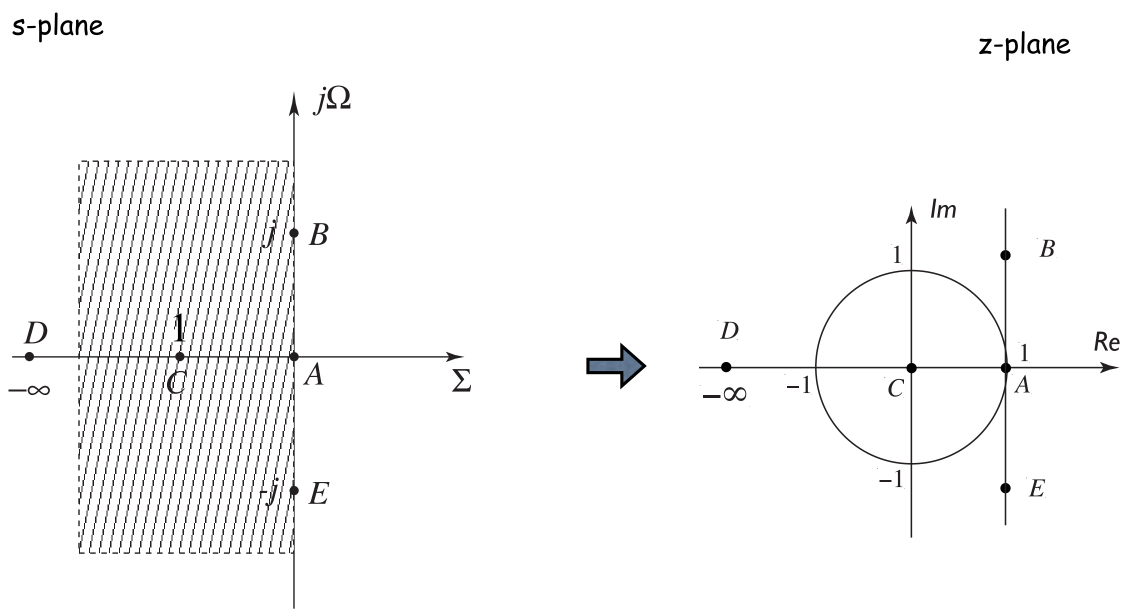
# Backward Euler Mapping




- Poles in the LHP map to a circle of center 1/ 2 and radius 1/ 2
- Integrator has good performance for low frequency signals. That is signals with discrete frequency close to 1.
- For high frequencies it performs poorly.

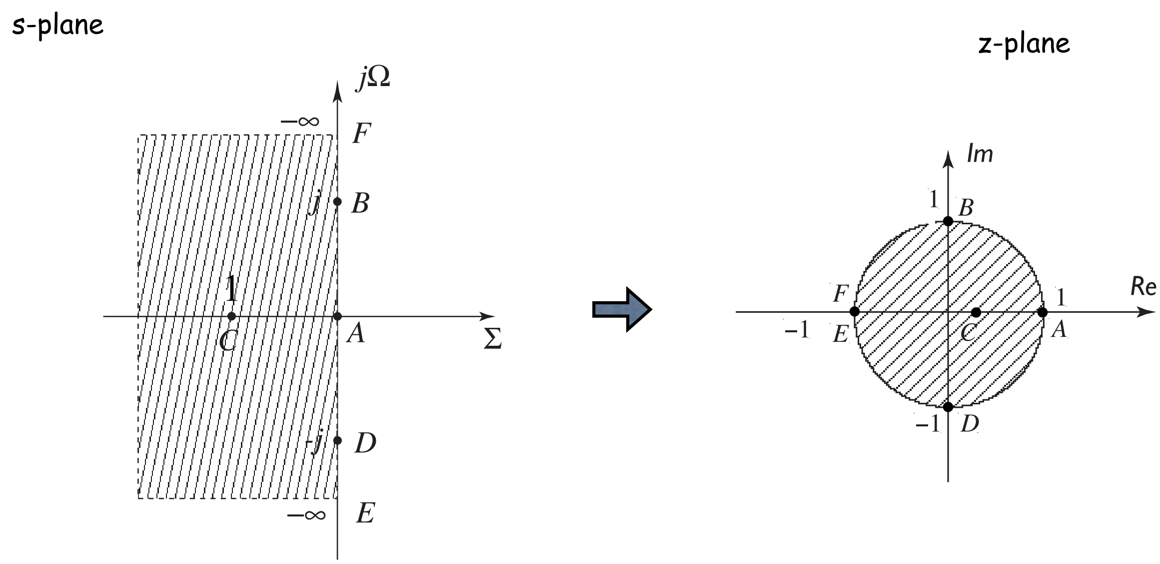


# Forward Euler Mapping




- Like the BEDI this integrator works well with low frequency signals.
- It is possible for points to be mapped outside the unit circle yielding and unstable system.

# Bilinear Mapping



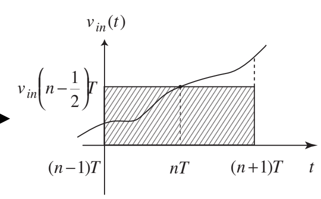

- Points all along the $j\omega$ axis are mapped to the unit circle.



# Midpoint Integration

LDI Transformation $$s_a = \frac{z^2 - 1}{2Tz} = \frac{1}{2}\left(\frac{z-1}{T} + \frac{z-1}{Tz}\right) \longleftrightarrow$$

Forward Euler    Backward Euler



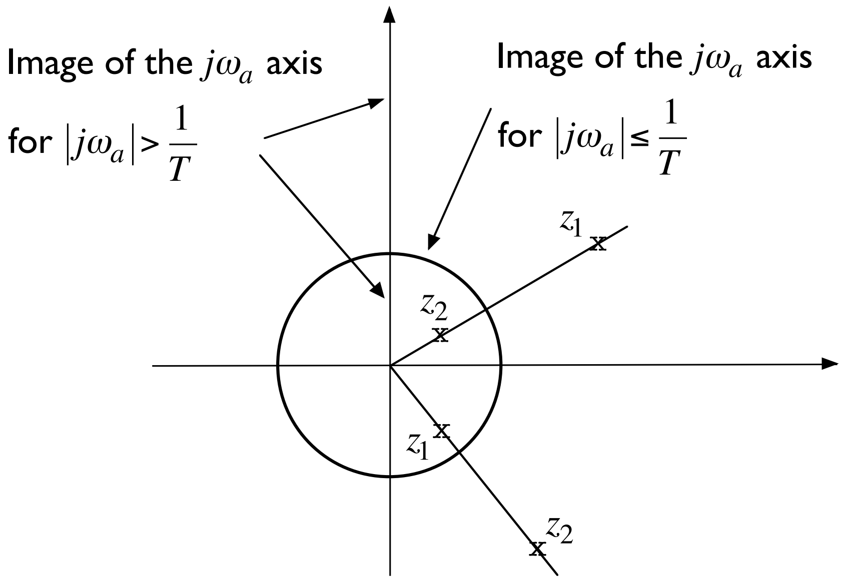

$$z_{1,2} = s_a T \pm \sqrt{(s_a T)^2 + 1}$$

$$\Rightarrow z_1 \cdot z_2 = \left(s_a T + \sqrt{(s_a T)^2 + 1}\right) \cdot \left(s_a T - \sqrt{(s_a T)^2 + 1}\right) = -1$$

$$\Rightarrow z_1 = -\frac{1}{z_2}$$

- If $|z_2| > 1$ then $|z_1| < 1$ and vice versa
- A stable pole $s_a$ will transform into one stable and one unstable pole in the $z$ - plane
- This implies $H_a(s_a)$ will not transform into a stable $H(z)$
- However, the LDI is still useful in understanding the operation of SCFs based on the LDI transformation

# Midpoint Integration Cont'd

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# SC Analysis using Charge Equations

The priniciple of charge conservation can be used to analyze SC circuits.

- Useful guidelines
  1. Charge at the present time= Charge at the instant before + Charge Injected.
  2. If the output is to be evaluated at time nT-T/2 draw the circuit at time (nT-T/2) and at (nT-T). Similarly if the output is to be evaluated at time (nT) draw the circuit at time (nT) and at (nT-T/2).
  3. If the polarity of the capacitor injecting charge into another has the same polarity as the one receiving the charge use a -ve sign in the charge equation otherwise use a +ve sign.

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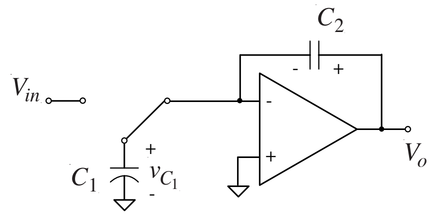
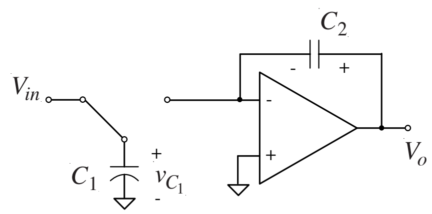
# The Middle Point Integrator (Parasitic sensitive)

- We wish to evaluate $\frac{V_o}{V_{in}}(z)$ at an instant we designate as $nT$




$\boxed{nT}$




$\boxed{nT - \frac{T}{2}}$

- Assuming a + → - current flow

$\boxed{nT - \frac{T}{2}}$ $C_2 v_o\left(nT - \frac{T}{2}\right) = C_2 v_o(nT - T) + 0$ (Zero charge Injected) (5)



# The Middle Point Integrator (Cont'd)

$\boxed{nT}$ $$C_2 v_o(nT) = C_2 v_o\left(nT - \frac{T}{2}\right) + C_1\left[v_{C_1}(nT) - v_{C_1}\left(nT - \frac{T}{2}\right)\right]$$

but $v_{C_1}(nT) = 0$ and $v_{C_1}\left(nT - \frac{T}{2}\right) = v_{in}\left(nT - \frac{T}{2}\right)$

$$\Rightarrow C_2 v_o(nT) = C_2 v_o\left(nT - \frac{T}{2}\right) - C_1 v_{in}\left(nT - \frac{T}{2}\right) \qquad (6)$$

Substuting (5) in (6) yields

$$C_2 v_o(nT) = C_2 v_o(nT - T) - C_1 v_{in}\left(nT - \frac{T}{2}\right)$$

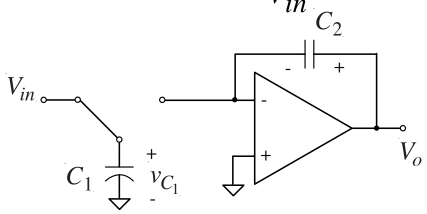
and taking z - transforms

$$C_2 V_o(z) = C_2 V_o(z) z^{-1} - C_1 V_{in}(z) z^{-\frac{1}{2}}$$

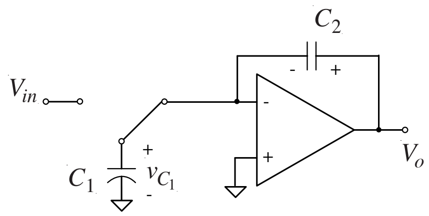
$$\Rightarrow \frac{V_o}{V_{in}}(z) = -\frac{C_1}{C_2} \cdot \frac{z^{-\frac{1}{2}}}{1 - z^{-1}}$$

# A Forward Euler Discrete Integrator (PS)

- We wish to evaluate $\frac{V_o}{V_{in}}(z)$ at an instant we designate as $nT$




$\boxed{nT}$




$\boxed{nT-\frac{T}{2}}$

- Assuming a + → - current flow

$$\boxed{nT} \quad C_2 v_o(nT) = C_2 v_o\left(nT - \frac{T}{2}\right) + 0 \text{ (Zero charge Injected)} \qquad (7)$$

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# A Forward Euler Discrete Integrator (PS)-Cont'd

$$C_2 v_o\left(nT - \frac{T}{2}\right) = C_2 v_o(nT - T) + C_1\left[v_{C_1}\left(nT - \frac{T}{2}\right) - v_{C_1}(nT - T)\right]$$

but $v_{C_1}\left(nT - \frac{T}{2}\right) = 0$ and $v_{C_1}(nT - T) = v_{in}(nT - T)$

$$\Rightarrow C_2 v_o\left(nT - \frac{T}{2}\right) = C_2 v_o(nT - T) - C_1 v_{in}(nT - T) \qquad (8)$$

Substituting (7) in (8) yields

$$C_2 v_o(nT) = C_2 v_o(nT - T) - C_1 v_{in}(nT - T)$$

and taking z - transforms

$$C_2 V_o(z) = C_2 V_o(z) z^{-1} - C_1 V_{in}(z) z^{-1}$$

$$\Rightarrow \frac{V_o}{V_{in}}(z) = -\frac{C_1}{C_2} \cdot \frac{z^{-1}}{1 - z^{-1}}$$

- Depending on which clock phase the output is sampled the outcome is a parasitic sensitive LDI or FEDI integrator.
- The two outcomes are separated only by half a clock delay.

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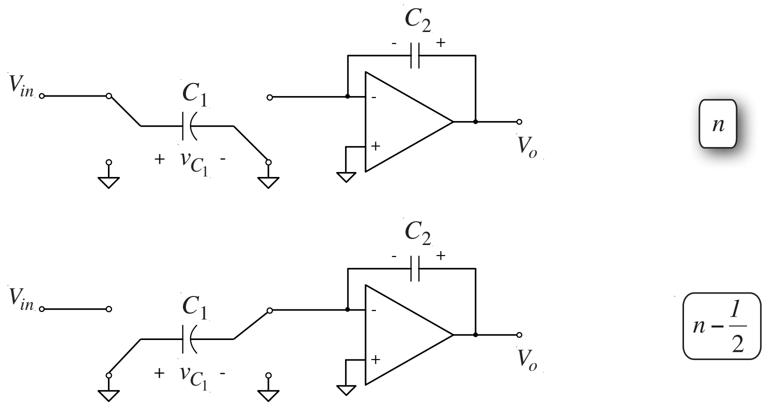
# A (Stray Insensitive) Forward Euler Discrete Integrator




$\boxed{n}$ $$C_2v_o(n)=C_2v_o\left(n-\frac{1}{2}\right)-\ 0$$

$\boxed{n-\frac{1}{2}}$ $$C_2v_o\left(n-\frac{1}{2}\right)=C_2v_o(n-1)-C_1\left[v_{C_1}\left(n-\frac{1}{2}\right)-v_{C_1}(n-1)\right]$$

but $v_{C_1}\left(n-\frac{1}{2}\right)=0$ and $v_{C_1}(n-1)=v_{in}(n-1)$

$$\Rightarrow C_2v_o(n)=C_2v_o(n-1)+C_1v_{in}(n-1)$$

$$\Rightarrow \frac{V_o}{V_{in}}(z)=\frac{C_1}{C_2}\cdot\frac{z^{-1}}{1-z^{-1}}$$

## Notes

- Output is to be evaluated at time *n*.
- *T* can be omitted since it conveys no useful information in the charge equations
- If the output was to be evaluated at time *n-1/2* instead, the numerator of the answer would be $z^{-1/2}$ instead of $z^{-1}$. That is the circuit would be non inverting LDI (Stray insensitive).

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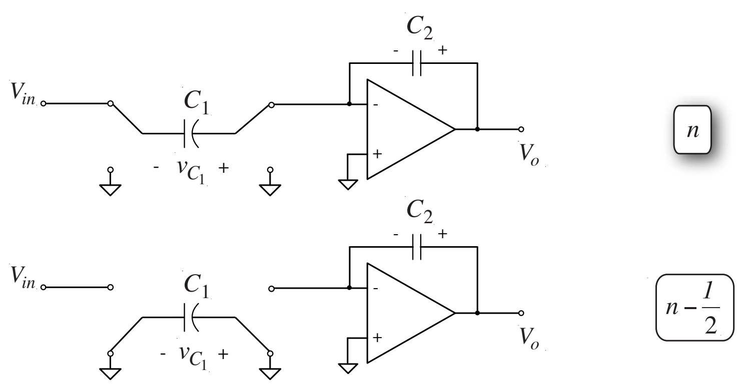
# A (Stray Insensitive) Backward Euler Discrete Integrator




$\boxed{n}$ $$C_2v_o(n)=C_2v_o\left(n-\frac{1}{2}\right)+C_1\left[v_{C_1}(n)-v_{C_1}\left(n-\frac{1}{2}\right)\right]$$

with $v_{C_1}(n)=-v_{in}(n)$ and $v_{C_1}\left(n-\frac{1}{2}\right)=0$

$\boxed{n-\frac{1}{2}}$ $$C_2v_o\left(n-\frac{1}{2}\right)=C_2v_o(n-1)+0$$

$$\Rightarrow C_2v_o(n)=C_2v_o(n-1)-C_1v_{in}(n)$$

$$\therefore \frac{V_o}{V_{in}}(z)=-\frac{C_1}{C_2}\cdot\frac{1}{1-z^{-1}}$$

## Notes

- Output is to be evaluated at time *n*.
- If the output was to be evaluated at time *n-1/2* instead, the numerator of the answer would be $z^{-1/2}$ instead of *1*. That is the circuit would be an inverting LDI (Stray insensitive).

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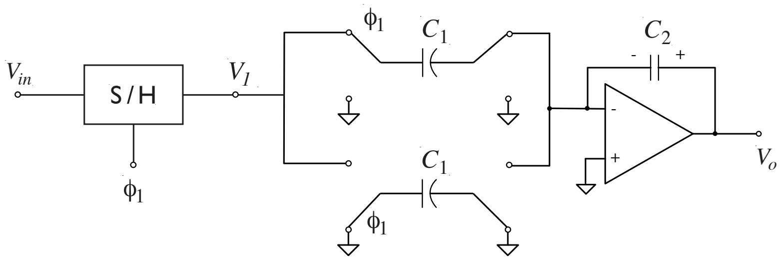
# A (Stray Insensitive) Bilinear Discrete Integrator




- By superposition the output can be computed by recognizing the presence of a BEDI and a LDI. Loosely speaking therefore,

$$V_o(z) = \{BEDI + LDI\} \cdot V_1(z)$$

$$V_o(z) = \left[ -\frac{C_1}{C_2} \cdot \frac{1}{1 - z^{-1}} - \frac{C_1}{C_2} \cdot \frac{z^{-\frac{1}{2}}}{1 - z^{-1}} \right] \cdot V_1(z)$$

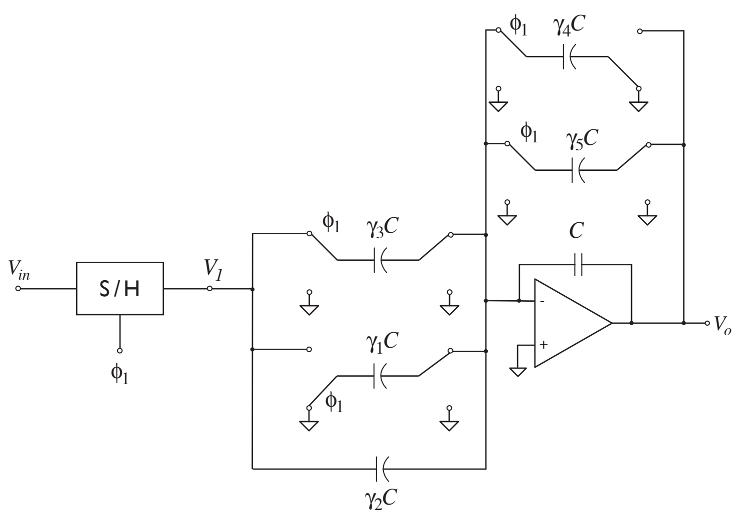
$$= -\frac{C_1}{C_2} \cdot \left[ \frac{V_1(z)}{1 - z^{-1}} + \frac{V_1(z) z^{-\frac{1}{2}}}{1 - z^{-1}} \right]$$

but $V_1(z) = z^{-\frac{1}{2}} V_{in}(z) \Rightarrow V_1(z) z^{-\frac{1}{2}} = z^{-1} V_{in}(z)$

$$\therefore \frac{V_o(z)}{V_{in}(z)} = -\frac{C_1}{C_2} \cdot \left[ \frac{1 + z^{-1}}{1 - z^{-1}} \right]$$



# A First Order Building Block




### Notes

- All switches discharge into C simultaneously. This eases how fast the the opamp has to work.

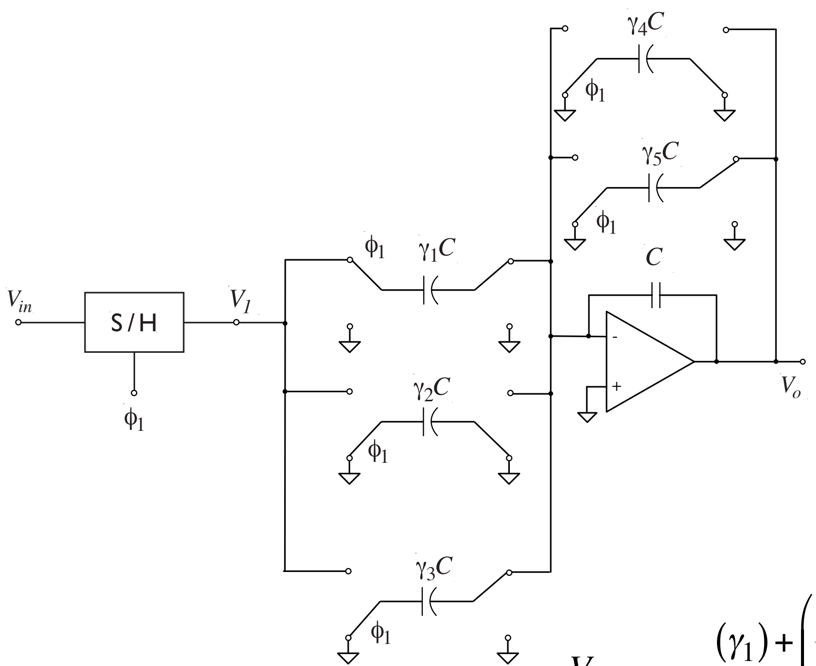
$$V_o(z) = \left( \frac{\gamma_1 z^{-1}}{1 - z^{-1}} - \gamma_2 - \frac{\gamma_3}{1 - z^{-1}} \right) V_{in}(z) + \left( \frac{\gamma_4 z^{-1}}{1 - z^{-1}} - \frac{\gamma_5}{1 - z^{-1}} \right) V_o(z)$$

$$V_o(z)\left[ 1 - z^{-1} + \gamma_5 - \gamma_4 z^{-1} \right] = V_{in}(z)\left[ \gamma_1 z^{-1} - \gamma_2 + \gamma_2 z^{-1} - \gamma_3 \right]$$

$$\frac{V_o}{V_{in}}(z) = \frac{-(\gamma_2 + \gamma_3) + (\gamma_2 + \gamma_1) z^{-1}}{(1 + \gamma_5) - (1 + \gamma_4) z^{-1}}$$

$$= \frac{\alpha_o + \alpha_1 z^{-1}}{\beta_o + \beta_1 z^{-1}}$$

# Another First Order Building Block




## Notes

- Opamp has to work twice as hard.
- Output is available only on one phase

$$\frac{V_o}{V_{in}}(z) = -\frac{(\gamma_1) + \left(\frac{\gamma_2}{1+\gamma_4} - \gamma_3\right)z^{-1}}{1 - \left(\frac{1+\gamma_5}{1+\gamma_4}\right)z^{-1}}$$

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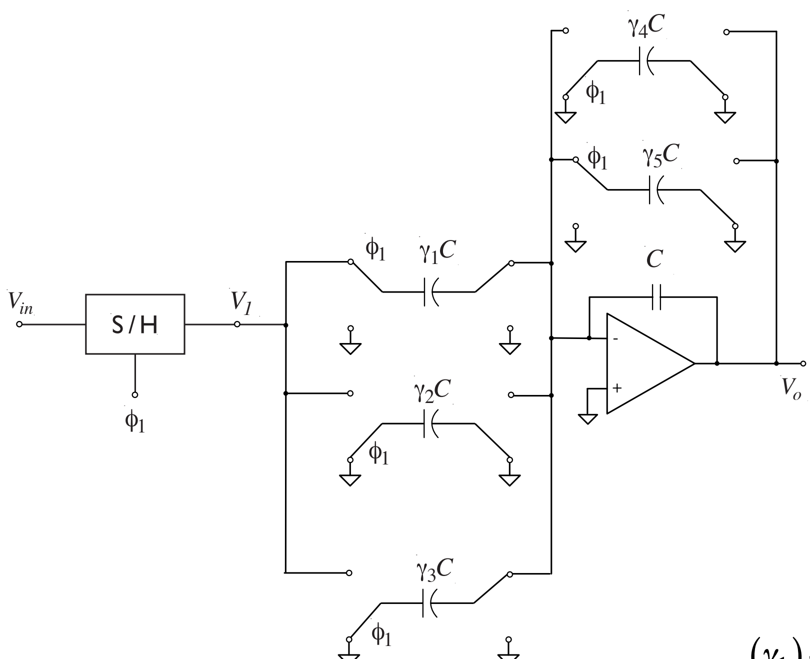
# Yet another First Order Building Block




## Notes

- Opamp has to work twice as hard.
- Output is available only on one phase

$$\frac{V_o}{V_{in}}(z) = -\frac{(\gamma_1) + \left(\gamma_2\left(\frac{1+\gamma_5}{1+\gamma_4}\right) - \gamma_3\right)z^{-1}}{1 - \left(\frac{1+\gamma_5}{1+\gamma_4}\right)z^{-1}}$$

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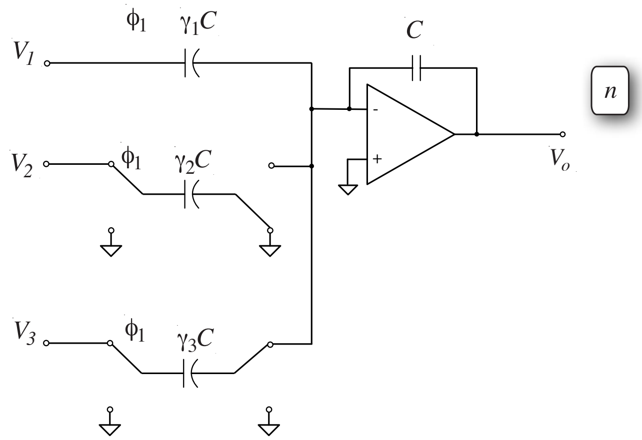
# Signal Flow Graph Analysis

$$\frac{V_o}{V_1}(z) = -\gamma_1$$

$$\frac{V_o}{V_2}(z) = \frac{\gamma_2 z^{-1}}{1 - z^{-1}}$$

$$\frac{V_o}{V_3}(z) = -\frac{\gamma_3}{1 - z^{-1}}$$



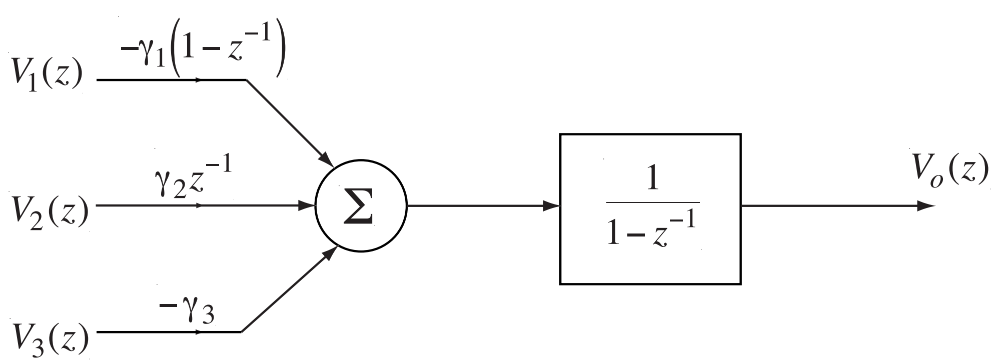

By superposition therefore

$$V_o(z) = \frac{1}{1 - z^{-1}}\left[-\gamma_1\left(1 - z^{-1}\right)V_1(z) + \gamma_2 z^{-1} V_2(z) - \gamma_3 V_3(z)\right]$$



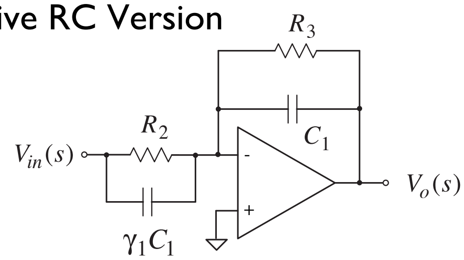
# Signal Flow Graph Analysis (Cont'd)

- Equivalent signal flow graph

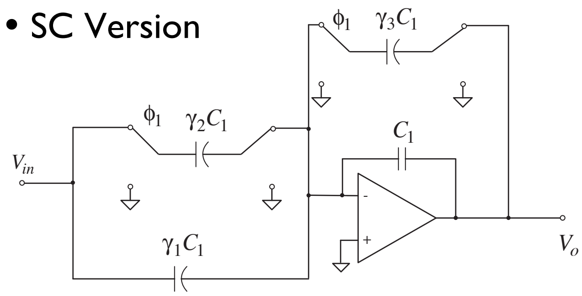


# Signal Flow Graph - 1st Order Example

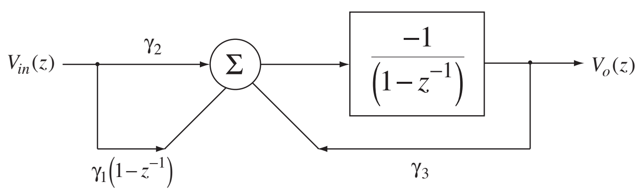
- Active RC Version




- SC Version




- Signal Flow Graph




$$\frac{V_o}{V_{in}}(z) = -\frac{\gamma_1 + \gamma_2 - \gamma_1 z^{-1}}{1 + \gamma_3 - z^{-1}}$$

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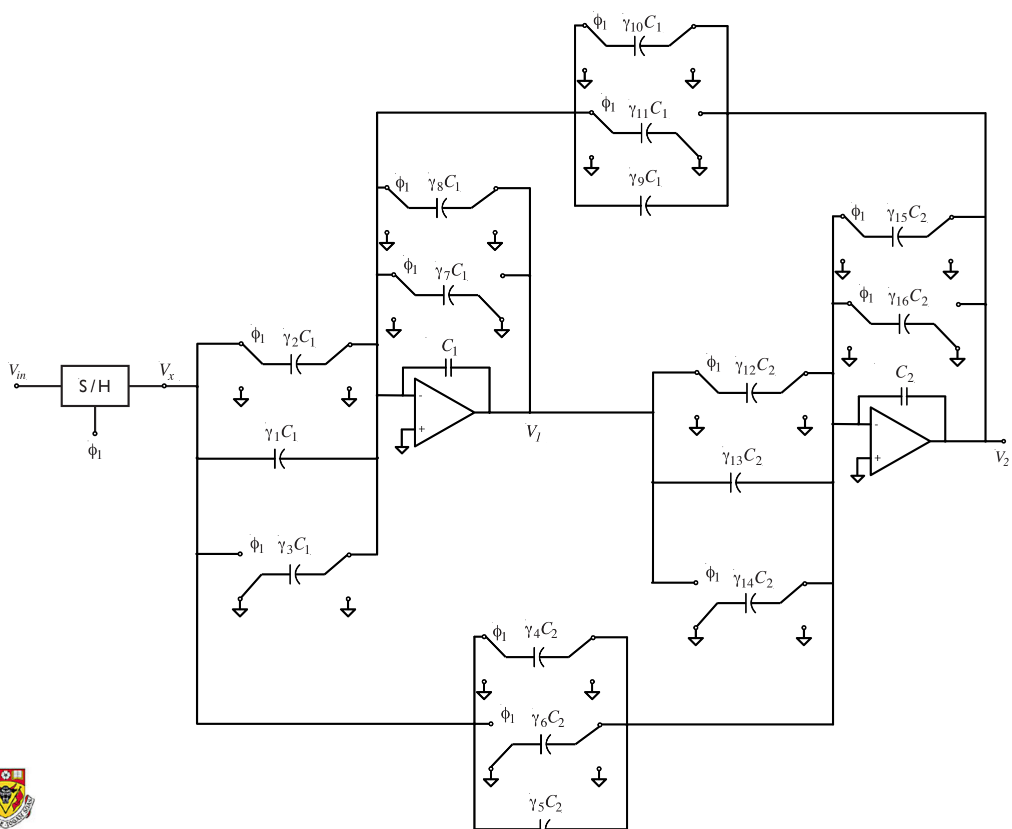
# General Stray-Insensitive SC Biquad (Dr. El-Masry)

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# General Stray-Insensitive SC Biquad (Cont'd)

A general SC Biquad can be used to realize the biquadratic transfer function

$$T(z) = \frac{\alpha_o + \alpha_1 z^{-1} + \alpha_2 z^{-2}}{\beta_o + \beta_1 z^{-1} + \beta_2 z^{-2}}$$

using two of the SC 1st order building blocks in a feedforward / feedback manner.

For the circuit shown in the figure the equations are:

$$V_1(z) = \left[\frac{\gamma_3 z^{-1}}{1-z^{-1}} - \frac{\gamma_2}{1-z^{-1}} - \gamma_1\right]V_{in}(z) + \left[\frac{\gamma_7 z^{-1}}{1-z^{-1}} - \frac{\gamma_8}{1-z^{-1}}\right]V_1(z) + \left[\frac{\gamma_{11} z^{-1}}{1-z^{-1}} - \frac{\gamma_{10}}{1-z^{-1}} - \gamma_9\right]V_2(z)$$

and

$$V_2(z) = \left[\frac{\gamma_{14} z^{-1}}{1-z^{-1}} - \frac{\gamma_{12}}{1-z^{-1}} - \gamma_{13}\right]V_1(z) + \left[\frac{\gamma_{16} z^{-1}}{1-z^{-1}} - \frac{\gamma_{15}}{1-z^{-1}}\right]V_2(z) + \left[\frac{\gamma_6 z^{-1}}{1-z^{-1}} - \frac{\gamma_4}{1-z^{-1}} - \gamma_5\right]V_{in}(z)$$

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# General Stray-Insensitive SC Biquad (Cont'd)

$$T_1(z) = \frac{V_1}{V_{in}}(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{c_0 + c_1 z^{-1} + c_2 z^{-2}} \text{ and } T_2(z) = \frac{V_2}{V_{in}}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{c_0 + c_1 z^{-1} + c_2 z^{-2}}$$

where

$$a_0 = (\gamma_4+\gamma_5)(\gamma_9+\gamma_{10}) - (\gamma_1+\gamma_2)(1+\gamma_{15})$$
$$a_1 = (\gamma_1+\gamma_2)(1+\gamma_{16}) + (\gamma_1+\gamma_3)(1+\gamma_{15}) - (\gamma_4+\gamma_5)(\gamma_9+\gamma_{11}) - (\gamma_5+\gamma_6)(\gamma_9+\gamma_{10})$$
$$a_2 = (\gamma_5+\gamma_6)(\gamma_9+\gamma_{11}) - (\gamma_1+\gamma_3)(1+\gamma_{16})$$
$$b_0 = (\gamma_1+\gamma_2)(\gamma_{12}+\gamma_{13}) - (\gamma_4+\gamma_5)(1+\gamma_8)$$
$$b_1 = (\gamma_4+\gamma_5)(1+\gamma_7) + (\gamma_5+\gamma_6)(1+\gamma_8) - (\gamma_1+\gamma_2)(\gamma_{13}+\gamma_{14}) - (\gamma_1+\gamma_3)(\gamma_{12}+\gamma_{13})$$
$$b_2 = (\gamma_1+\gamma_3)(\gamma_{13}+\gamma_{14}) - (\gamma_5+\gamma_6)(1+\gamma_7)$$
$$c_0 = (1+\gamma_8)(1+\gamma_{15}) - (\gamma_9+\gamma_{10})(\gamma_{12}+\gamma_{13})$$
$$c_1 = (\gamma_9+\gamma_{11})(\gamma_{12}+\gamma_{13}) + (\gamma_9+\gamma_{10})(\gamma_{13}+\gamma_{14}) - (1+\gamma_7)(1+\gamma_{15}) - (1+\gamma_8)(1+\gamma_{16})$$
$$c_2 = (1+\gamma_7)(1+\gamma_{16}) - (\gamma_9+\gamma_{11})(\gamma_{13}+\gamma_{14})$$

The equivalent analog pole frequency $\omega_o$ and pole $Q$ are given by

$$\omega_o = 2 f_c \sqrt{\frac{c_0+c_1+c_2}{c_0-c_1-c_2}} \quad \text{and} \quad Q = \frac{\sqrt{(c_0+c_1+c_2)(c_0-c_1+c_2)}}{2(c_0-c_1)}$$

where $f_c$ is the clock frequency.

- Note $\omega_o$ and $Q$ are obtained by using the bilinear z - transform

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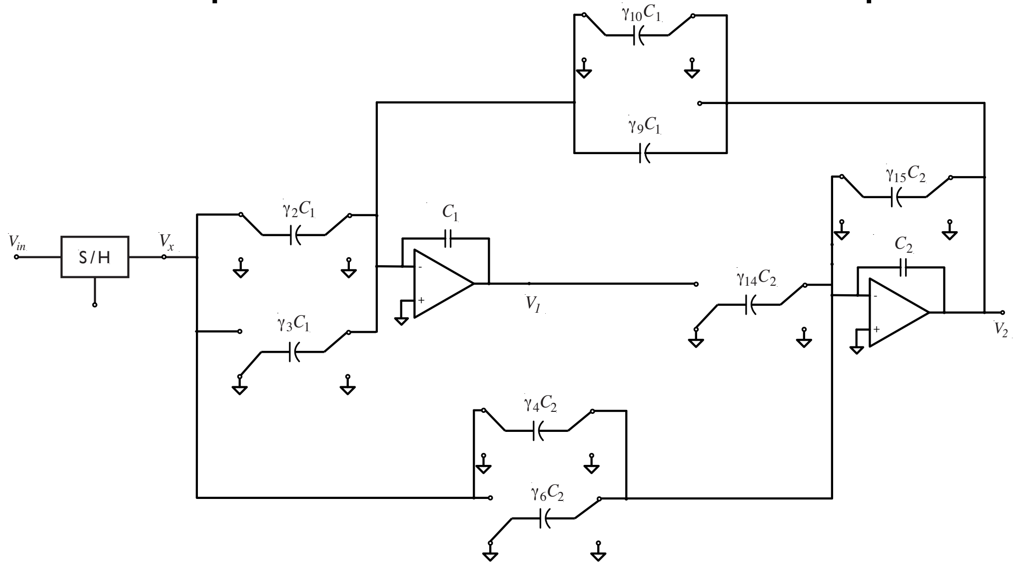
# Simplified version of the General Biquad




Let us set $\gamma_1 = \gamma_5 = \gamma_7 = \gamma_8 = \gamma_{11} = \gamma_{12} = \gamma_{13} = \gamma_{16} = 0$. Then

$$T_1(z) = \frac{V_1}{V_{in}}(z) = \frac{[\gamma_4(\gamma_9+\gamma_{10}) - \gamma_2(1+\gamma_{15})] + [\gamma_2 + \gamma_3(1+\gamma_{15}) - \gamma_6\gamma_{10} - \gamma_9(\gamma_4+\gamma_6)]z^{-1} + [\gamma_6\gamma_9 - \gamma_3]z^{-2}}{D(z)}$$

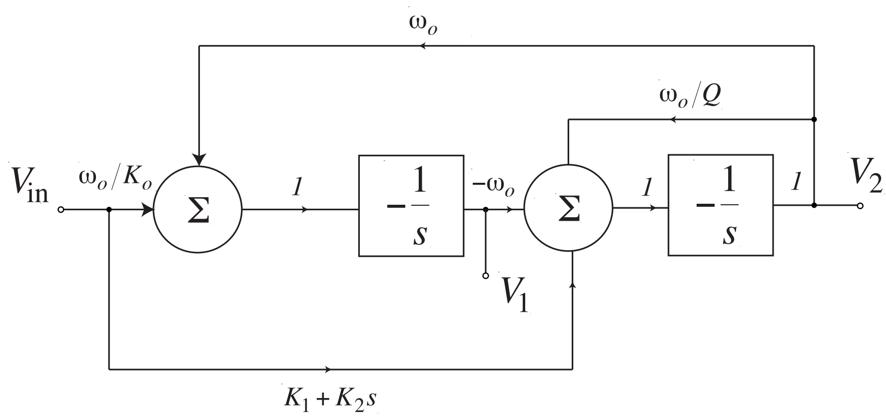
$$T_2(z) = \frac{V_2}{V_{in}}(z) = \frac{-\gamma_4 + [\gamma_2\gamma_{14} - \gamma_4 - \gamma_6]z^{-1} + [-\gamma_3\gamma_{14} + \gamma_6]z^{-2}}{D(z)}$$

where

$$D(z) = (1+\gamma_{15}) + [\gamma_{14}(\gamma_9+\gamma_{10}) - \gamma_{15} - 2]z^{-1} + [1 - \gamma_9\gamma_{14}]z^{-2}$$



# A SC Low Q Biquad (Q<1)

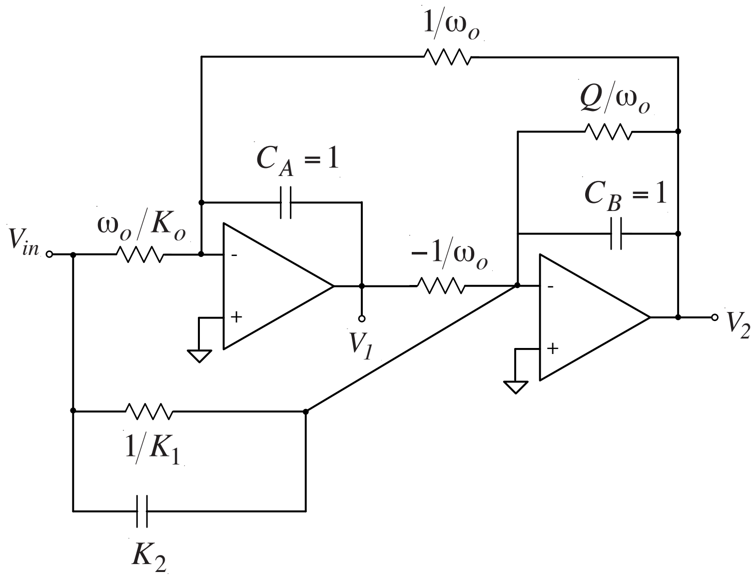



$$V_2(s) = -\frac{1}{s}\left[(k_1 + k_2 s)V_{in}(s) + \frac{\omega_o}{Q}V_2(s) - \omega_o V_1\right]$$

and

$$V_1(s) = -\frac{1}{s}\left[\frac{k_0}{\omega_o}V_{in}(s) + \omega_o V_2\right]$$

# A SC Low Q Biquad (Q<1)-Cont'd

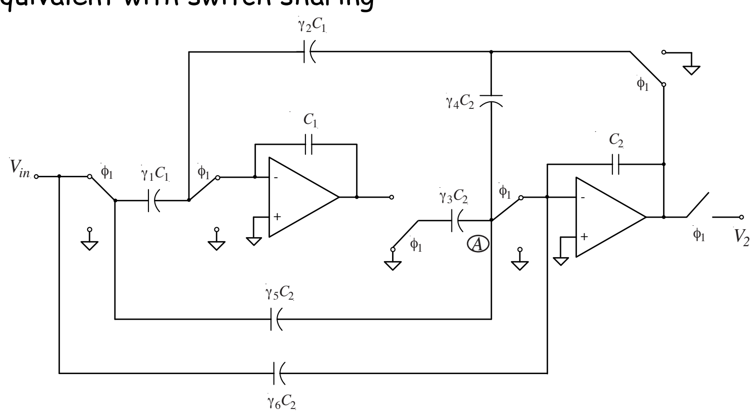



- Note the presence of the negative resistor $-1/\omega_o$ making it not practical as an active RC filter.



# A SC Low Q Biquad (Q<1)-Cont'd

SC Equivalent with switch sharing




$$\gamma_1 \approx \frac{TK_o}{\omega_o} = |H_a(0)|\omega_o T$$

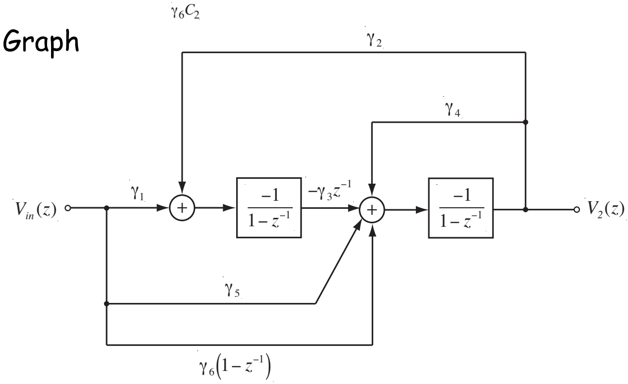
$$\gamma_2 \approx \gamma_3 \approx \omega_o T$$

$$\gamma_4 \approx \frac{\omega_o T}{Q}$$

$$\gamma_5 \approx K_1 T$$

$$\gamma_6 \approx K_2$$

Signal Flow Graph



# A SC Low Q Biquad (Q<1)-Cont'd

- Note for low $Q$ the capacitance spread $\frac{C_{\max}}{C_{\min}} = \frac{1}{\omega_o T} = \frac{1}{\gamma_2}$

  and for frequencies of interest $\omega << \omega_c \Rightarrow |\omega_o T| << 1$

  so that the capacitance spread will be large.

- The direct $z$ - transfer function of the circuit is given by

$$H(z) = \frac{(\gamma_5+\gamma_6)z^2 + (\gamma_1\gamma_3 - \gamma_5 - 2\gamma_6)z + \gamma_6}{(1+\gamma_4)z^2 + (\gamma_2\gamma_3 - \gamma_4 - 2)z + 1}$$

$$= \frac{a_2 z^2 + a_1 z + a_0}{b_2 z^2 + b_1 z + b_0}$$

$$\Rightarrow \gamma_6 = a_0$$

$$\gamma_5 = a_2 - \gamma_6 = a_2 - a_0$$

$$\gamma_1 = \left(\frac{1}{\gamma_3}\right)(a_1 + \gamma_5 + 2\gamma_6) = \frac{a_0 + a_1 + a_2}{\gamma_3}$$

$$\gamma_4 = b_2 - 1$$

$$\gamma_2\gamma_3 = b_1 + 2 + \gamma_4 = b_1 + b_2 + 1$$

6 unknown values and 5 relations. One minimization constraint is to choose

$$\gamma_2 = \gamma_3 = \sqrt{b_1 + b_2 + 1}$$

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# A SC Low Q Biquad (Q<1)-Cont'd

Possible Realizations of the low-Q Biquad

| | |
|---|---|
| Lowpass | $\gamma_5 = \gamma_6 = 0$ |
| Highpass | $\gamma_1 = \gamma_5 = 0$ |
| Bandpass | $\gamma_5 = 0$ |
| Notch | $\gamma_1 = \gamma_6 = 0$ |

Allpass realizations are possible if an inverting SC branch is added between the input terminal and node A with $\gamma_5$ removed.

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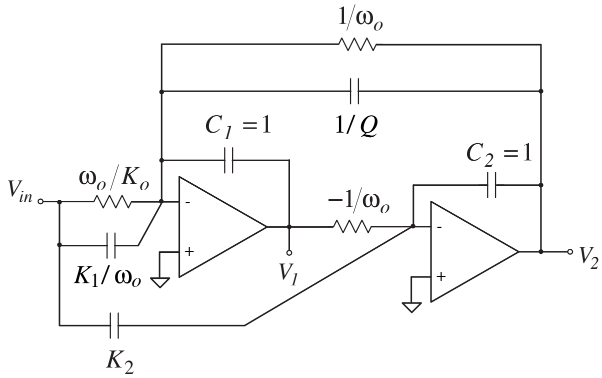
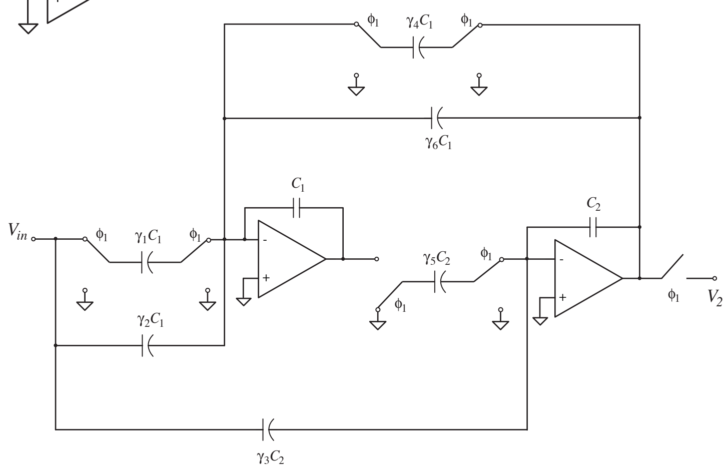
# A SC High Q Biquad

- Active RC Version




$$H(s) = \frac{V_2}{V_{in}}(s) = -\frac{K_2 s^2 + K_1 s + K_o}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

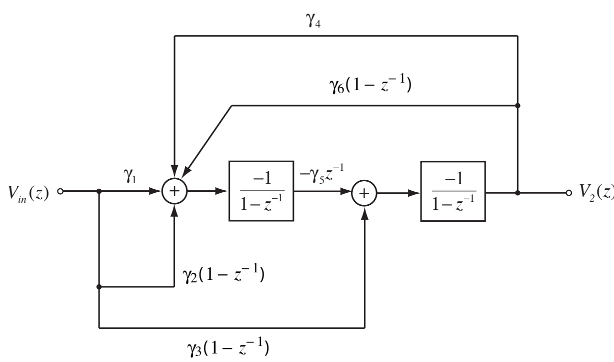
- SC Version






# A SC High Q Biquad (Cont'd)

- Signal Flow Graph




$$H(z) = \frac{V_2}{V_{in}}(z) = -\frac{\gamma_3 z^2 + (\gamma_1\gamma_5 + \gamma_2\gamma_5 - 2\gamma_3)z + (\gamma_3 - \gamma_2\gamma_5)}{z^2 + (\gamma_4\gamma_5 + \gamma_5\gamma_6 - 2)z + (1 - \gamma_5\gamma_6)} = -\frac{a_2 z^2 + a_1 z + a_o}{z^2 + b_1 z + b_o}$$

$$\gamma_3 = a_2$$

$$\gamma_2\gamma_5 = a_2 - a_o$$

$$\gamma_1\gamma_5 = a_o + a_1 + a_2$$

$$\gamma_5\gamma_6 = 1 - b_o$$

$$\gamma_4\gamma_5 = 1 + b_1 + b_o$$

# SC Filter Design Strategy using the Bilinear z-transform

Given the specifications of the SC filter in terms of the discrete frequency $(\omega)$ the following is a step by step design strategy:

- Prewarp the specs using the equation $\Omega = 2f_s \tan\left(\frac{\omega}{2f_s}\right)$ to obtain the continous time specs.
- Using available tables or formula for the continous time filter (e.g. Butterworth Chebychev, Cauer, etc) obtain $H(s)$ that meets the analog specs
- Apply the Bilinear z-transform $s = 2f_s \cdot \frac{1-z^{-1}}{1+z^{-1}}$ to the analog transfer function $H(s)$ to obtain the discrete time function $H(z)$,

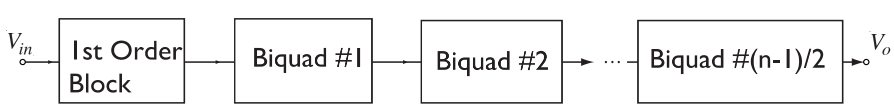
$$H(z) = H(s)\big|_{s=2f_s \cdot \frac{1-z^{-1}}{1+z^{-1}}}$$

- $H(z)$ can be realized using the cascade approach.

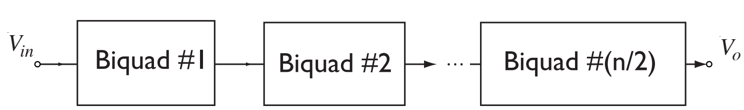


# Cascade Realization of SC Filters

- $H(z)$ is realized as a cascade of first and second order SC building blocks.
- For $n$ odd, $H(z) = \frac{a_0 + a_1 z^{-1}}{b_0 + b_1 z^{-1}} \cdot \prod_{k=1}^{\frac{n-1}{2}} \frac{\alpha_{0k} + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}}{\beta_{0k} + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}$
- For $n$ even, $H(z) = \prod_{k=1}^{\frac{n}{2}} \frac{\alpha_{0k} + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}}{\beta_{0k} + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}$




n-odd




n-even

# Bilinear z transform of the general biquadratic transfer function

Given $H(s) = \dfrac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2}$ then

$$H(z) = H(s)\Big|_{s=2f_s\frac{z-1}{z+1}}$$

$$= \frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2}}$$

where

$$\alpha_0 = a_0 + 2f_s a_1 + (2f_s)^2 a_2$$

$$\alpha_1 = 2a_0 - 2a_2(2f_s)^2$$

$$\alpha_2 = a_0 - 2f_s a_1 + (2f_s)^2 a_2$$

and

$$\beta_0 = b_0 + 2f_s b_1 + (2f_s)^2 b_2$$

$$\beta_1 = 2b_0 - 2b_2(2f_s)^2$$

$$\beta_2 = b_0 - 2f_s b_1 + (2f_s)^2 b_2$$

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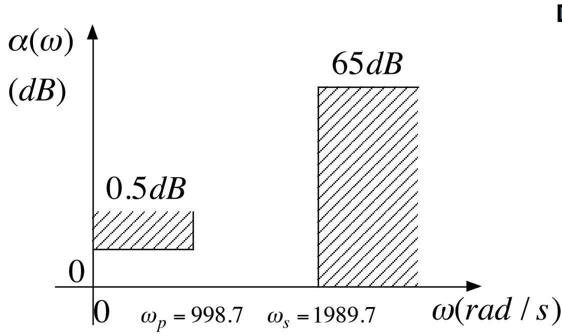
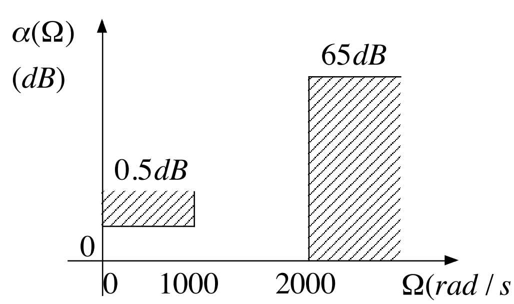
# SC Design Example using the Cascade approach

Ex. 3 • Design a SC LP filter with equiripple response in both the passband and the stopband, that meets the attentuation specifications shown below. In your design use a sampling frequency $f_s = 8kHz$.

Soln.

Prewarping the discrete time specs

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## SC Design Example using the Cascade approach (Cont'd)

- From Cauer filter tables the desired order is 5th and the normalized transfer function is given by,

$$H(s)_{norm} = \frac{0.0046205}{s+0.392612} \cdot \frac{s^2+4.36495}{s^2+0.19255s+1.03402} \cdot \frac{s^2+10.56773}{s^2+0.58054s+0.525}$$

$$\Rightarrow H(s) = H(s)_{norm}\Big|_{s=\frac{s}{1000}}$$

$$\therefore H(s) = \frac{4.6205}{s+3926.12} \cdot \frac{s^2+4.36495\times10^6}{s^2+192.55s+1.03402\times10^6} \cdot \frac{s^2+10.56773\times10^6}{s^2+580.54s+5.25\times10^6}$$

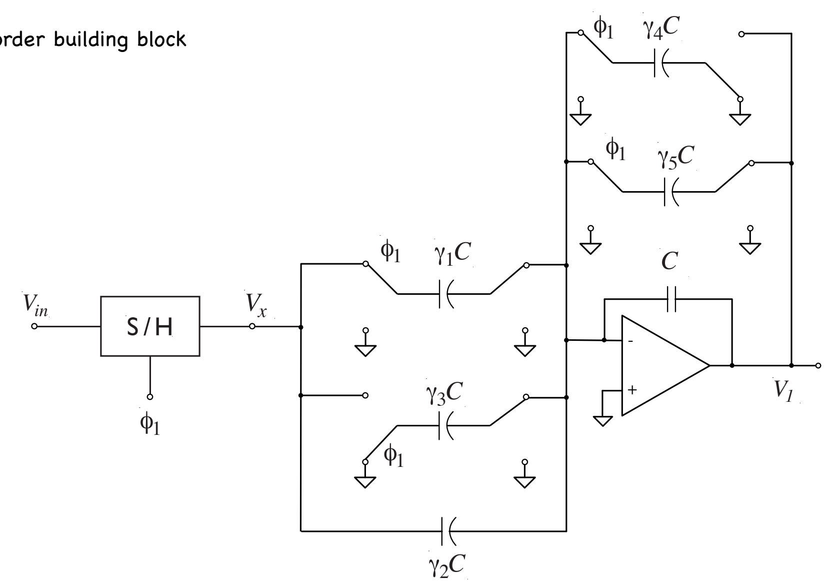
The discrete time transfer function is therefore

$$H(z) = \frac{2\times10^{-4}\left(1-z^{-1}\right)}{1-0.6059z^{-1}} \cdot \frac{1-1.9348z^{-1}+z^{-2}}{1-1.9604z^{-1}+0.9763z^{-2}} \cdot \frac{0.8388-1.5446z^{-1}+0.8388z^{-2}}{1-1.2807z^{-1}+0.9415z^{-2}}$$
$$= H_1(z)\cdot H_2(z)\cdot H_3(z)$$



## SC Design Example using the Cascade approach (Cont'd)

The first order building block




$$\frac{V_1}{V_{in}}(z) = H_1(z) = \frac{-\left(\frac{\gamma_2+\gamma_3}{1+\gamma_5}\right)+\left(\frac{\gamma_1+\gamma_2}{1+\gamma_5}\right)z^{-1}}{1-\left(\frac{1+\gamma_4}{1+\gamma_5}\right)z^{-1}} = -\frac{2\times10^{-4}\left(1-z^{-1}\right)}{1-0.6059z^{-1}}$$

# SC Design Example(Cont'd)

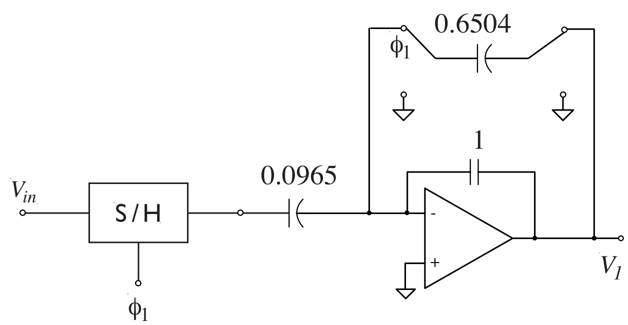
Let us choose $\gamma_1 = \gamma_3 = \gamma_4 = 0$

$\Rightarrow \dfrac{\gamma_2}{1+\gamma_5} = 2\times 10^{-4}$ and $\gamma_5 = 1/0.6059 - 1 = 0.6504 .\therefore \gamma_2 = 3.3\times 10^{-4}$

This design yields a high capacitance ratio $\left(\dfrac{1}{3.3\times 10^{-4}} = 3030\right)$ which is not practical.

A simple solution is to distribute the gain evenly over each section. Hence each stage would have a gain of $\left(2\times 10^{-4}\right)^{\frac{1}{3}} = 0.058\left(= \dfrac{1}{17}\right)$, which is more practical. For the new design then $\gamma_2 = 0.058/0.6059 = 0.0965$.

Note: The output is valid for both phases






# SC Design Example (Cont'd)

Biquad #1

$H_2(z) = -0.058\dfrac{1-1.9348z^{-1}+z^{-2}}{1-1.9604z^{-1}+0.9763z^{-2}}$. Using the simplified version of the general biquad with the output taken from the second opamp yields.

$$H_2(z) = -\frac{\gamma_4 + [\gamma_2\gamma_{14} - \gamma_4 - \gamma_6]z^{-1} + [\gamma_6 - \gamma_3\gamma_{14}]z^{-2}}{1+\gamma_{15} + [\gamma_{14}(\gamma_9+\gamma_{10}) - \gamma_{15} - 2]z^{-1} + [1-\gamma_9\gamma_{14}]z^{-2}} = -0.058\frac{1-1.9348z^{-1}+z^{-2}}{1-1.9604z^{-1}+0.9763z^{-2}}$$

The design equations are:

$$\gamma_4 = 0.058$$
$$-\gamma_2\gamma_{14} + \gamma_4 + \gamma_6 = 0.058\times 1.9348 = 0.11222$$
$$\gamma_6 - \gamma_3\gamma_{14} = 0.058$$
$$1+\gamma_{15} = 1$$
$$-\gamma_{14}(\gamma_9+\gamma_{10}) + \gamma_{15} + 2 = 1.9604$$
$$1-\gamma_9\gamma_{14} = 0.9763$$

There are 6 equations and 8 unknowns. We may therefore set $\gamma_3 = 0$ and $\gamma_{14} = 1$. Solving yields

$\gamma_4 = \gamma_6 = 0.058, \gamma_9 = 0.0237, \gamma_2 = 0.105, \gamma_{10} = 0.0159$ and $\gamma_{15} = 0$

# SC Design Example (Cont'd)

Biquad #2

$H_3(z) = -0.058\dfrac{0.8388 - 1.5446z^{-1} + 0.8388z^{-2}}{1 - 1.2807z^{-1} + 0.9415z^{-2}}$. Using the simplified version of the general biquad with the output taken from the first opamp yields.

$$H_3(z) = -\frac{\left[\gamma_4(\gamma_9+\gamma_{10}) - \gamma_2(1+\gamma_{15})\right] + \left[\gamma_2 + \gamma_3(1+\gamma_{15}) - \gamma_6\gamma_{10} - \gamma_9(\gamma_4+\gamma_6)\right]z^{-1} + \left[\gamma_6\gamma_9 - \gamma_3\right]z^{-2}}{1+\gamma_{15} + \left[\gamma_{14}(\gamma_9+\gamma_{10}) - \gamma_{15} - 2\right]z^{-1} + \left[1 - \gamma_9\gamma_{14}\right]z^{-2}}$$

The design equations are:

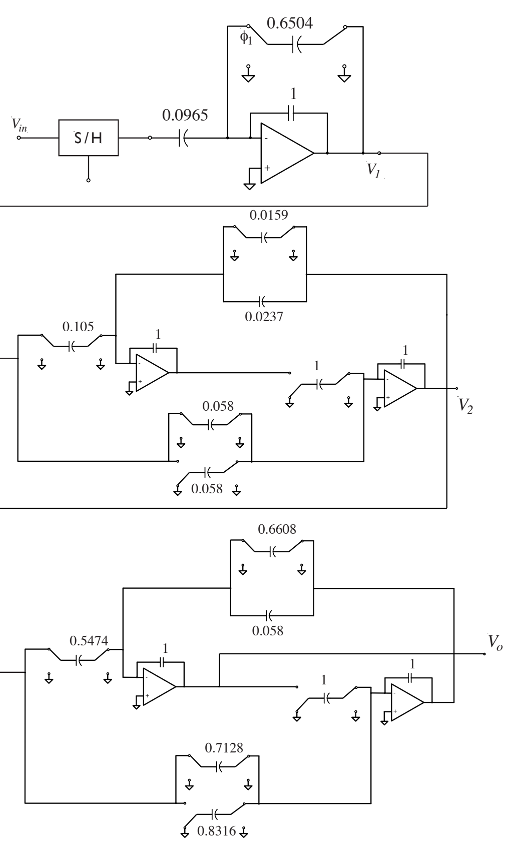
$$\gamma_4(\gamma_9+\gamma_{10}) - \gamma_2(1+\gamma_{15}) = 0.058 \times 0.8388 = 0.0487$$
$$\gamma_2 + \gamma_3(1+\gamma_{15}) - \gamma_6\gamma_{10} - \gamma_9(\gamma_4+\gamma_6) = -0.058 \times 1.5446 = -0.08959$$
$$\gamma_6\gamma_9 - \gamma_3 = 0.058 \times 0.8388 = 0.04865$$
$$1 + \gamma_{15} = 1$$
$$-\gamma_{14}(\gamma_9+\gamma_{10}) + \gamma_{15} + 2 = 1.2807$$
$$1 - \gamma_9\gamma_{14} = 0.9415$$

There are 6 equations and 8 unknowns. We may therefore set $\gamma_3 = 0$ and $\gamma_{14} = 1$. Solving yields

$\gamma_4 = 0.7128, \gamma_6 = 0.8316, \gamma_2 = 0.5474, \gamma_9 = 0.058, \gamma_{10} = 0.6608$ and $\gamma_{15} = 0$



# Final Realization

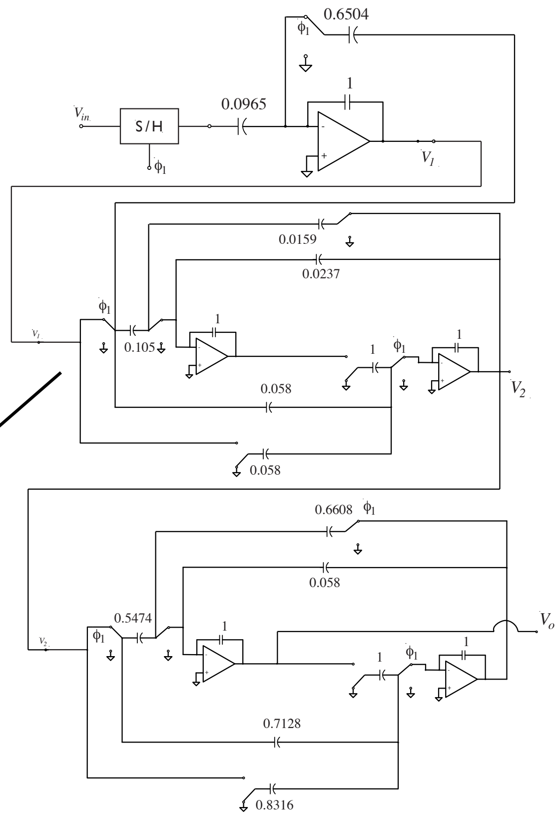



1st Order Stage

Biquad #1

Biquad #2

# Switch Sharing




Switches can be shared to reduce hardware



# Design of SC Ladder Filters

- Procedure very similar to that of analog filters
- We have to employ a transformation because the analog prototype filter operates in s and the corresponding SC filter operates in terms of z.
- Most used s-z transformations are the LDI and the bilinear transformation. Note the bilinear transformation has zero phase error.

LDI (Approximate)

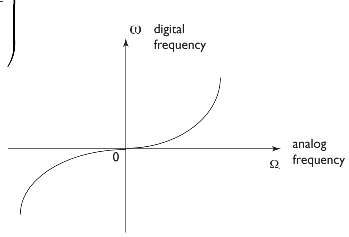
$$s = \frac{1}{T}\left(z^{\frac{1}{2}} - z^{-\frac{1}{2}}\right)$$

$$\Downarrow$$

$$j\Omega = \frac{1}{T}\left(e^{\frac{j\omega T}{2}} - e^{-\frac{j\omega T}{2}}\right)$$

$$\Omega = \frac{2}{T}\sin\left(\frac{\omega T}{2}\right)$$

$$\therefore \frac{\omega T}{2} = \sin^{-1}\left(\frac{\Omega T}{2}\right)$$

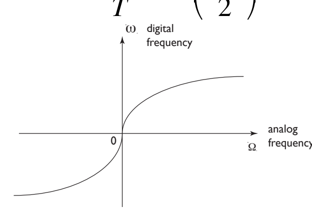



BDI (Exact)

$$s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\Downarrow$$

$$\therefore \omega = \frac{2}{T}\tan^{-1}\left(\frac{\Omega T}{2}\right)$$



# General procedure for the Approximate Design of SC Ladder filters

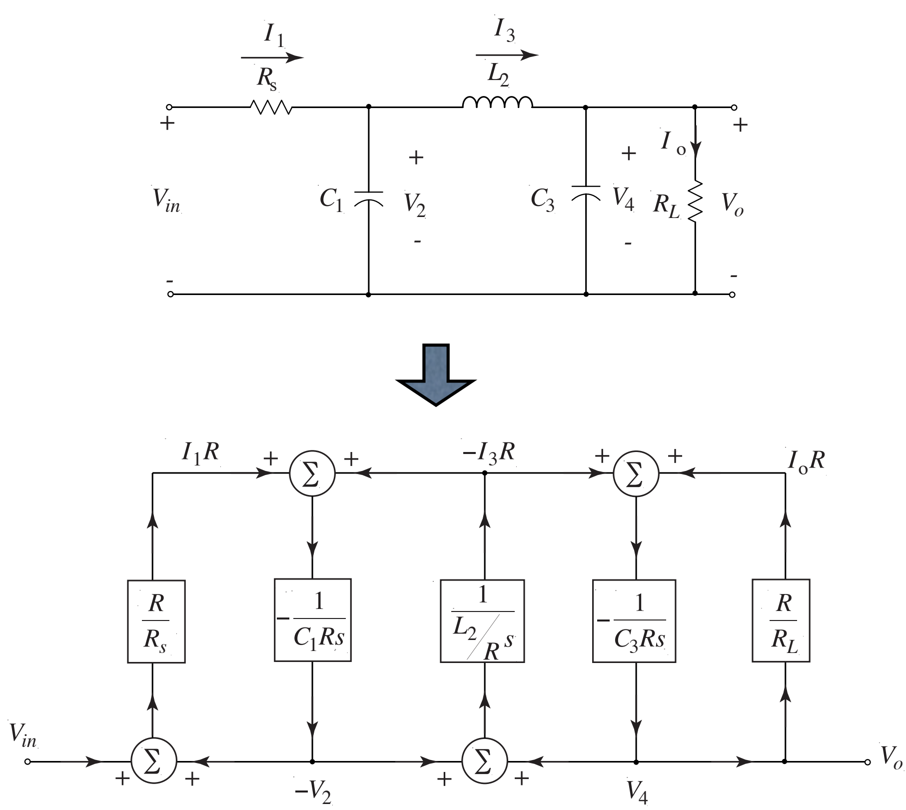
- A doubly terminated LC two port is designed from the specifications. The specifications are prewarped using the relationship

$$\Omega = \frac{2}{T}\sin\left(\frac{\omega T}{2}\right)$$

- Find the state equations of the resulting RLC circuit. The signs of the voltage and current variables must be choosen such that inverting and noninverting integrators alternate in the implementation.
- Construct the block diagram or SFG from the state equations. Transform it directly or via the active RC circuit into the SC filter.
- If necessary, additional circuit transformations can be performed to improve the response of the SCF. These include modifications of the termination sections which improves the passband response or eliminating unnecessary capacitive coupling between opamps in circuits with high clock rates.

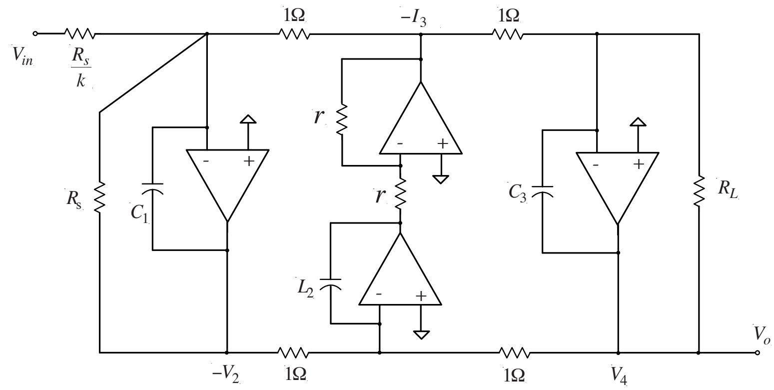


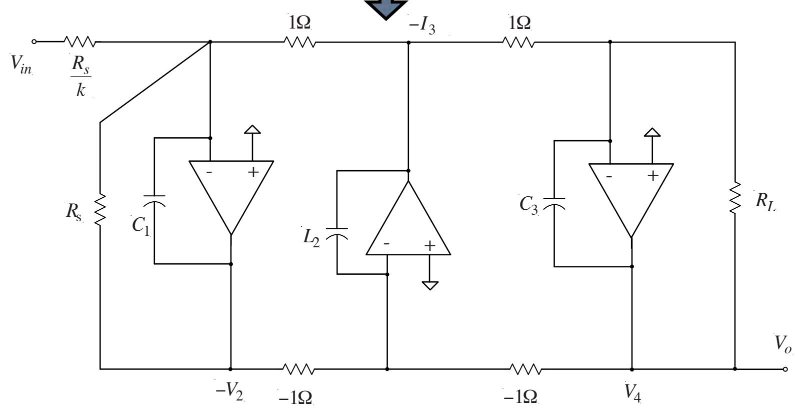
# Design Example - 3rd Order LPF



# Design Example - 3rd Order LPF (Cont'd)

Equal-R
(R=1 Ohm)

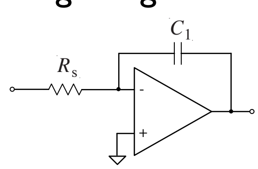


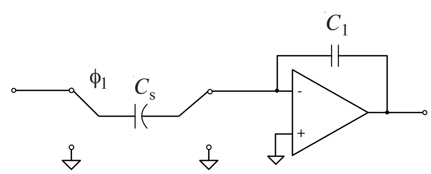




Elimination of 1 opamp by using negative resistors



- Inverting Integrator

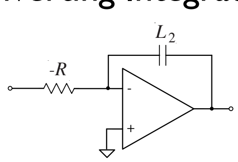


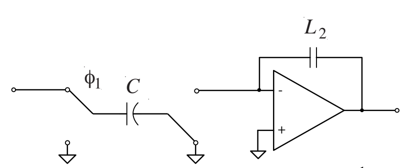




$$H(s) = -\frac{1}{R_s C_1 s}$$

$$H_{inv}(z) = -\frac{C_s}{C_1}\frac{1}{\left(1-z^{-1}\right)}$$

- Noninverting Integrator




$$H(s) = \frac{1}{RL_2 s}$$

$$H_{noninv}(z) = \frac{C}{L_2}\frac{z^{-1}}{\left(1-z^{-1}\right)}$$

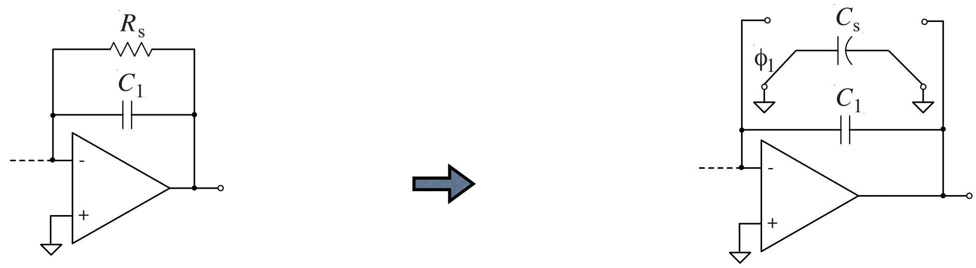
- Note $H(z) = H_{inv}(z)\cdot H_{noninv}(z) = -\frac{C_s}{C_1}\cdot\frac{C}{L_2}\cdot\frac{z^{-1}}{\left(1-z^{-1}\right)^2}$

$$= -\frac{K}{\left(z^{\frac{1}{2}} - z^{-\frac{1}{2}}\right)^2}$$

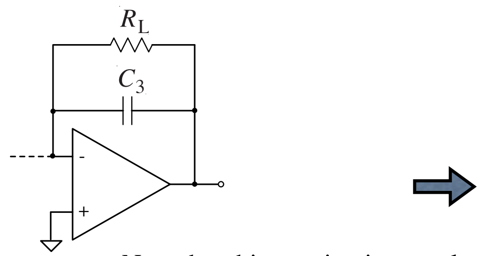
and

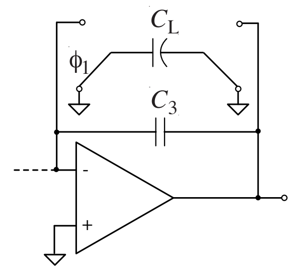
$H\left(e^{j\omega T}\right) = \frac{K}{4\sin^2\left(\frac{\omega T}{2}\right)}$ which is purely real (Zero phase error)

# Termination




$$C_s \cong \frac{T}{R_s}$$




$$C_L \cong \frac{T}{R_L}$$

Note that this termination results in a phase error.

$$\left.\frac{V_2}{V_{in}}(s)\right|_{-I_3=0} = -\frac{1}{R_s}\cdot\frac{R_s}{(R_sC_1s+1)}$$

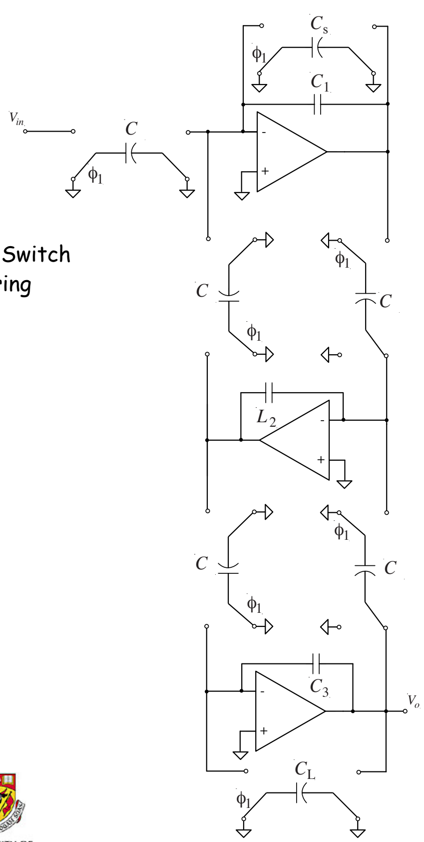
$$\frac{V_2}{V_{in}}(j\omega) = -\frac{C_s}{T}\cdot\frac{T/C_s}{\left(\frac{TC_1}{C_s}j\omega+1\right)} = -\frac{C_s}{T}\cdot\frac{T/C_s}{(Qj+1)}$$

$Q = \omega\frac{TC_1}{C_s}$, represents an approximation error

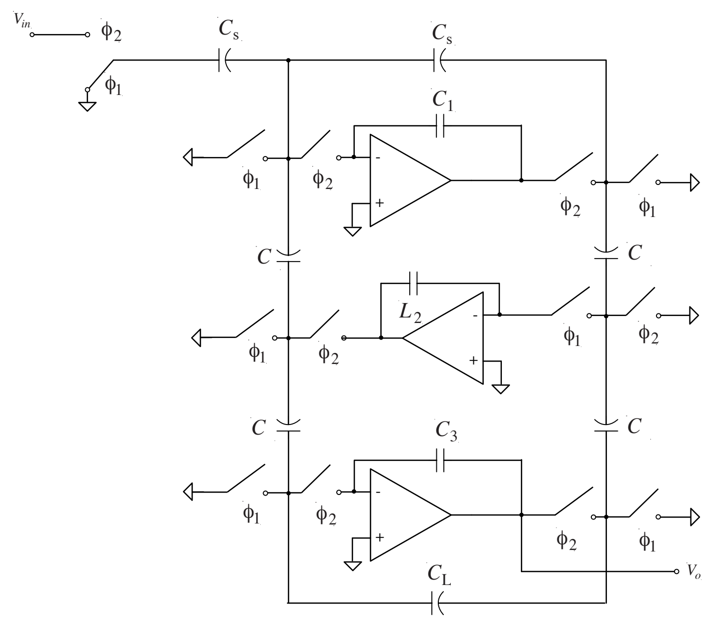


# Final Realization

No Switch sharing

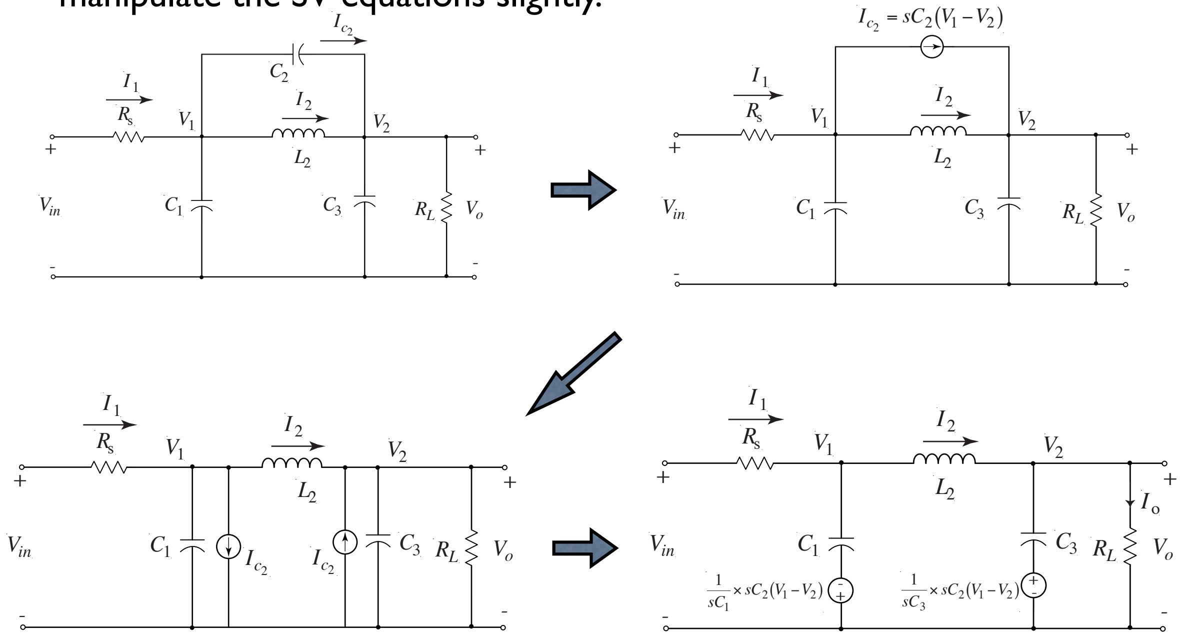



With Switch sharing



# Ladders with Finite Transmission Zeroes

- Procedure is the same as before except that we have to manipulate the SV equations slightly.





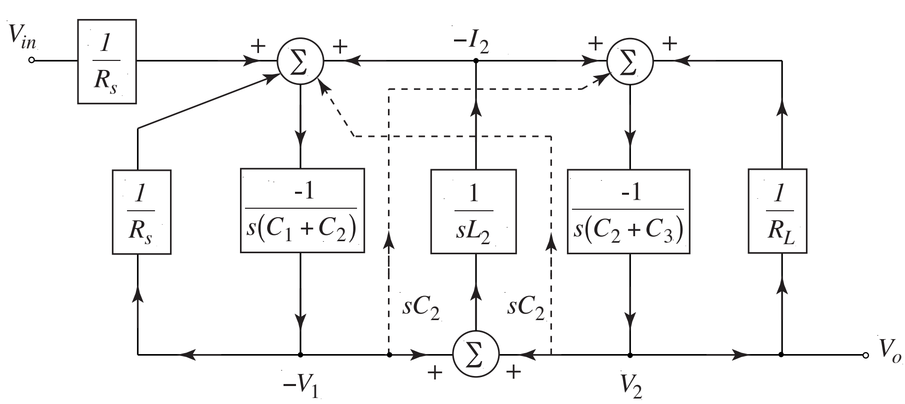

- SV Equations are

$$I_1 = \frac{V_s - V_1}{R_s}$$

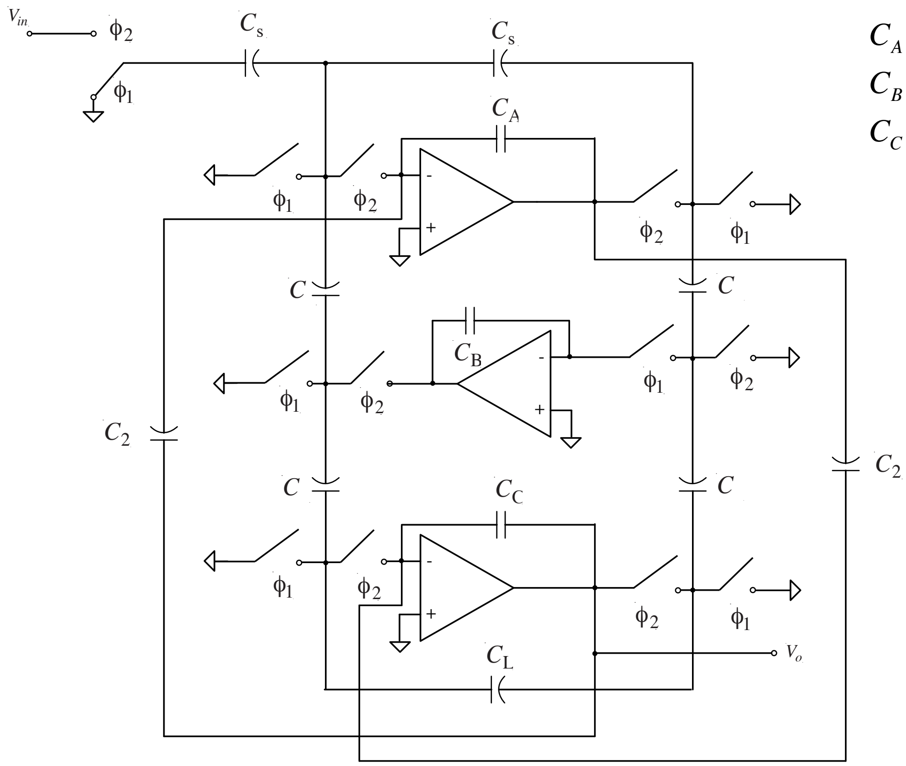
$$V_1 = \frac{1}{s(C_1 + C_2)}(I_1 - I_2) + \frac{sC_2}{s(C_1 + C_2)}V_2$$

$$V_2 = \frac{1}{s(C_2 + C_3)}(I_2 - I_0) + \frac{sC_2}{s(C_2 + C_3)}V_1$$

$$V_o = I_0 R_L = V_2$$



# Final Realization (switch sharing)



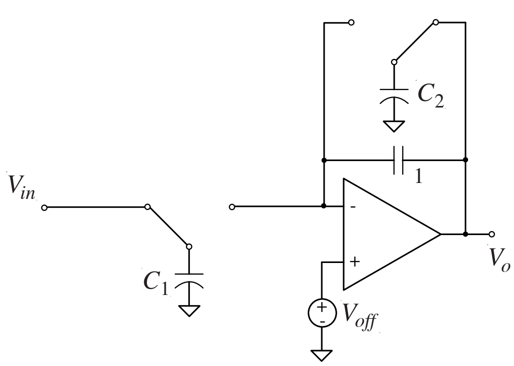

$$C_A = C_1 + C_2$$
$$C_B = L_2$$
$$C_C = C_2 + C_3$$

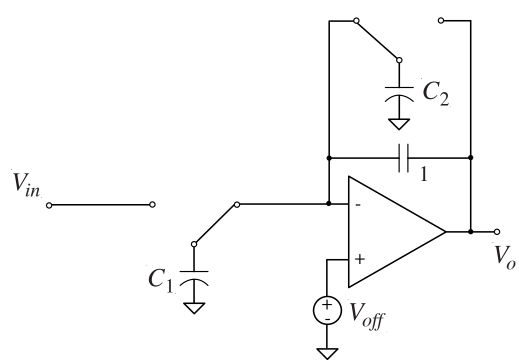


# Non-Ideal Opamp Effects

Non-ideal opamp effects such as DC offset voltage, finite DC gain, finite opamp bandwidth, slew rate, and charge injection are important considerations in SC filters.

- DC Offset voltage effect - consider the simple lossy integrator with opamp offset voltage $V_{off}$.




$n$

$n - \frac{1}{2}$

# Non-Ideal Opamp Effects (Cont'd)-DC Offset

$n$

$$v_o(n)-v_{off}(n)=v_o\left(n-\frac{1}{2}\right)-v_{off}\left(n-\frac{1}{2}\right)+0$$

$n-\frac{1}{2}$

$$v_o\left(n-\frac{1}{2}\right)-v_{off}\left(n-\frac{1}{2}\right)=v_o(n-1)-v_{off}(n-1)+C_1\left[v_{c1}\left(n-\frac{1}{2}\right)-v_{c1}(n-1)\right]$$

$$+\ C_2\left[v_{c2}\left(n-\frac{1}{2}\right)-v_{c2}(n-1)\right]$$

but $v_{c1}\left(n-\frac{1}{2}\right)=v_{off}\left(n-\frac{1}{2}\right), v_{c2}\left(n-\frac{1}{2}\right)=v_{off}\left(n-\frac{1}{2}\right), v_{c1}(n-1)=v_{in}(n-1)$

$v_{c2}(n-1)=v_o(n-1)$ and $v_{off}(n)=v_{off}\left(n-\frac{1}{2}\right)=v_{off}(n-1)$

Substitution and taking z - transforms yields,

$$V_o(z)=-\frac{C_1 z^{-1}}{1-z^{-1}+C_2 z^{-1}}V_{in}(z)+\frac{(C_1+C_2)}{1-z^{-1}+C_2 z^{-1}}V_{off}(z)$$

If $V_{in}=0 \Rightarrow V_o(z)=\dfrac{(C_1+C_2)}{1-z^{-1}+C_2 z^{-1}}V_{off}(z)$

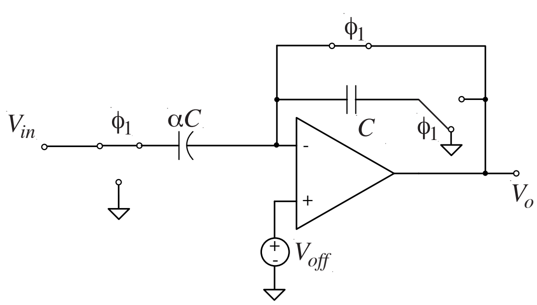
Since $V_{off}$ is constant $s=0 \rightarrow z=1$

$$\therefore V_o=\left(1+\frac{C_1}{C_2}\right)V_{off}$$

- $V_o$ is independent of the feedback capacitor. It depends on the ratio $C_1/C_2$.



# Simple Offset Cancelation scheme (PIS)




By charge conservation we can write

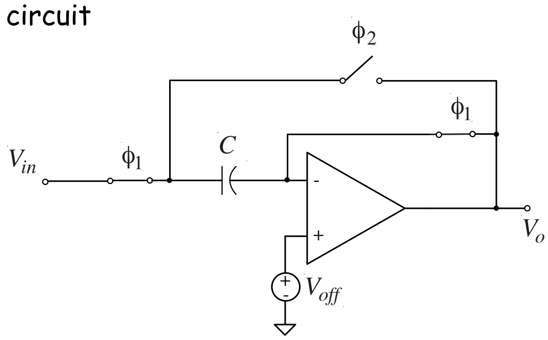
$$\alpha C\left\{v_{off}-\left[v_{off}-v_{in}\left(n-\frac{1}{2}\right)\right]\right\}+C\left\{\left[v_{off}-v_o(n)\right]-v_{off}\right\}=0$$

$\Rightarrow v_o(n)=\alpha v_{in}\left(n-\frac{1}{2}\right)$ and taking z - transforms yields

$$H(z)=\alpha z^{-\frac{1}{2}}$$

If the clock phases at the input terminal are interchanged, then $H(z)=-\alpha$

Another circuit

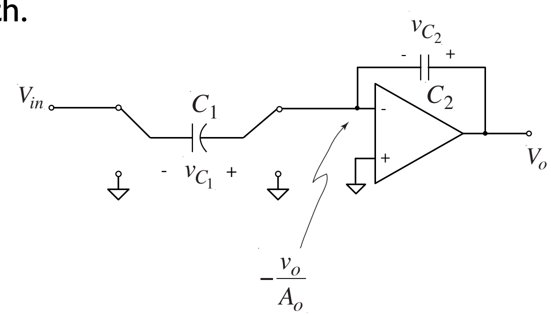



Show that for this circuit $H(z)=z^{-\frac{1}{2}}$

Note the TF only valid for $\phi_2$

# Effects of the finite DC gain

- Here we consider the opamp as having a finite DC gain $A_o$ but an infinite bandwidth.




$n$

$n$: $$C_2\left[v_o(n)-\frac{-v_o(n)}{A_o}\right]=C_2\left[v_o\left(n-\frac{1}{2}\right)-\frac{-v_o\left(n-\frac{1}{2}\right)}{A_o}\right]+C_1\left[v_{C_1}(n)-v_{C_1}\left(n-\frac{1}{2}\right)\right]$$

$n-\frac{1}{2}$: $$C_2\left[v_o\left(n-\frac{1}{2}\right)-\frac{-v_o\left(n-\frac{1}{2}\right)}{A_o}\right]=C_2\left[v_o(n-1)-\frac{-v_o(n-1)}{A_o}\right]$$

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# Effects of the finite DC gain (Cont'd)

but $v_{C_1}(n)=\dfrac{-v_o(n)}{A_o}-v_{in}(n)$ and $v_{C_1}\left(n-\dfrac{1}{2}\right)=0$

Substitution and taking z - transforms yields,

$$H(z)=\frac{V_o}{V_{in}}(z)=-\frac{C_1}{C_2+\dfrac{(C_1+C_2)}{A_o}-C_2\left(1+\dfrac{1}{A_o}\right)z^{-1}}$$

$$=-\frac{(C_1/C_2)\left[1+\dfrac{(1+C_1/C_2)}{A_o}\right]^{-1}z}{z-\left(1+\dfrac{1}{A_o}\right)\Big/\left[1+\dfrac{(1+C_1/C_2)}{A_o}\right]}$$

Compare with the ideal transfer function $H_i(z)=-\dfrac{(C_1/C_2)z}{z-1}=\dfrac{-(C_1/C_2)}{1-z^{-1}}$

Two observations:

- The gain is reduced from $(C_1/C_2)$ to a smaller value
- Pole previously at $z=1$ now has a smaller value

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# Effects of the finite DC gain (Cont'd)

The ideal frequency response obtained from $z = e^{j\omega T}$ is

$$H_i\left(e^{j\omega T}\right) = \frac{-(C_1/C_2)e^{j\omega T}}{2j\sin\left(\frac{\omega T}{2}\right)}$$ and therefore

$$H\left(e^{j\omega T}\right) = \frac{1}{1-m(\omega)-j\theta(\omega)} \cdot H_i\left(e^{j\omega T}\right)$$ where

$$m(\omega) = -\frac{1}{A_o}\left(1+\frac{C_1}{2C_2}\right) \text{ and } \theta(\omega) = \frac{(C_1/C_2)}{2A_o\tan\left(\frac{\omega T}{2}\right)} \cong \frac{(C_1/C_2)}{A_o\omega T} \text{ for } \omega T << 1$$

- For $A_o > 1000$ and typical values of $C_1/C_2$ and $\omega T$, $|m| << 1$ and $|\theta| << 1$ (Except near $\omega = 0$)
- For high sampling rates $(|sT| << 1)$, $z = e^{sT} \cong 1 + sT$

$$H(s) = H(z)\big|_{z\cong 1+sT} = \frac{-(C_1/C_2)\left[1+(1+C_1/C_2)/A_o\right]^{-1}(1+sT)}{sT + (C_1/C_2\,A_o)/\left[1+(1+C_1/C_2)/A_o\right]}$$

$$H(s) \cong \frac{-(C_1/C_2)T}{s+(C_1/C_2A_oT)}$$

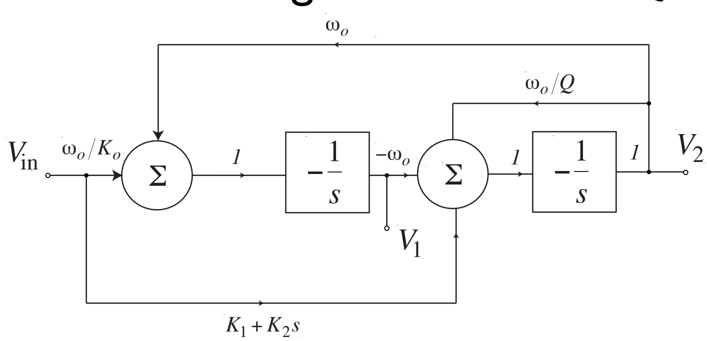
$\therefore$ The s-domain pole of the integrator is $\sigma_i = \dfrac{(C_1/C_2)}{A_oT}$ instead of $s=0$

$$H(s) \cong \frac{-A_o\sigma_i}{s+\sigma_i}$$



# Effects of finite DC gain on the Low Q Biquad

Low Q Biquad




- If each ideal integrator is replaced by its lossy counterpart

$$\frac{1}{Q_{new}} = \frac{1}{Q} + \frac{1}{A_o\omega T}\left[\frac{C_1}{C_F}+\frac{C_2}{C_F}\right],\ C_F \text{ is the feedback capacitor}$$

$$\omega_{o(new)}^2 = \omega_o^2 + \frac{\omega_o\sigma_{i1}}{Q} + \sigma_{i1}\cdot\sigma_{i2}$$

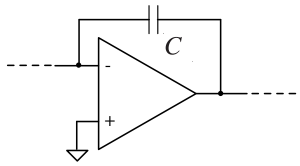
- As an example if $A_o = 1000$, $Q = 15$, and $\dfrac{C_i}{C_F} \cong \omega T\,(i=1,2) \rightarrow Q_{new} \cong 14.56$

  Note the effect is worse for high-Q biquads.
- The change in $\omega_o$ is less dramatic.

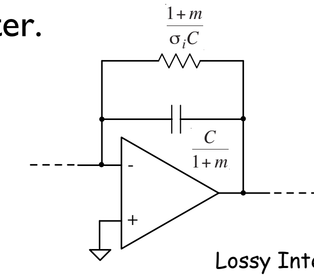
# Effects of finite DC gain on Ladder Filters

- Recall in a ladder filter each integrator simulates a reactive ($L$ or $C$) element of the passive prototype filter.

If the element is C




Ideal Integrator




Lossy Integrator

The branch simulated has the impedance $Z(s)=\dfrac{1}{sC/(1+m)+\sigma_i\, C/(1+m)}$.

The quality factor of the capacitor is therefore,

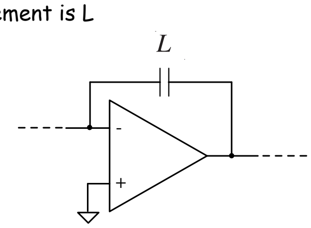
$$Q_c=\omega C R_c=\frac{\omega}{\sigma_i}=\frac{A_o\omega T}{(C_1/C_2)}=\frac{1}{\theta(\omega)}$$

- The change in element value $C\to C/(1+m)$ is usually neglible since the doubly terminated ladder is insensitive to such variations.
- The finite $Q_c$ can result in significant changes in the gain response, especially for narrow-band filters with high-$Q$ poles.

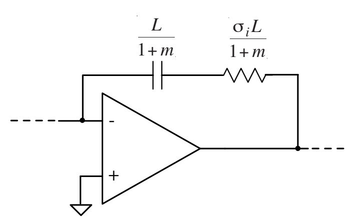


# Effects of finite DC gain on Ladder Filters (Cont'd)

If the element is L




Ideal Integrator




Lossy Integrator

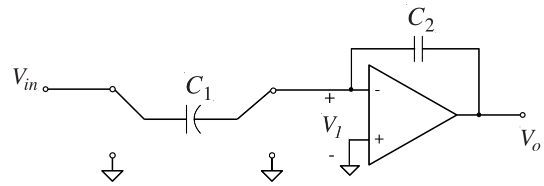
$$Q_L=\frac{\omega L}{R_L}=\frac{\omega}{\sigma_i}=\frac{A_o\omega T}{(C_1/C_2)}=\frac{1}{\theta(\omega)}$$

and

$$Y(s)=\frac{1}{sL/(1+m)+\sigma_i\, L/(1+m)}$$

# Effects of Finite Opamp Bandwidth

Consider the BEDI




- For the opamp $\frac{V_o}{V_1}(s) = -\frac{\omega_o}{s+\sigma} = -\frac{1}{\frac{1}{A_o}+\frac{s}{\omega_o}}$ where $\omega_o = A_o\sigma$ is the unity gain bandwidth product of the opamp.

  The above equation can be written in the time domain as

$$\frac{1}{A_o}v_o(t) + \frac{1}{\omega_o}\frac{dv_o(t)}{dt} = -v_i(t)$$

  Using the opamp time domain behaviour the BEDI can be analyzed for samples $v_o(nT), v_o(nT+T)$, etc and the actual transfer function $H(z) = V_o(z)/V_{in}(z)$ calculated and evaluated for $z = e^{j\omega T}$.

- For the BEDI

$$m(\omega) = -e^{-k_1}\left[1 - k\cos(\omega T)\right]$$



# Effects of Finite Opamp Bandwidth (Cont'd)

and

$$\theta(\omega) = -e^{-k_1}k\sin(\omega T)$$

where

$$k = \frac{C_2}{C_1 + C_2} \text{ and } k_1 = k\frac{\omega_o T}{2}$$

If we define a unity gain frequency $\omega_i$ of the integrator as $\left|H_i\left(e^{j\omega_i T}\right)\right| = 1$ then

$$m(\omega) \cong \theta(\omega) \cong -\omega_i T e^{-\frac{\omega_o T}{2}} \text{ where } \omega_i T << 1 \text{ is assumed.}$$

Thus if $\frac{\omega_o T}{2} = \frac{\pi\omega_o}{\omega_c} >> 1$ ($\omega_c$ – clock frequency), then both $m(\omega)$ and $\theta(\omega)$ become negligible. In general $\omega_o \cong 5\omega_c$ is usually adequate.

– The unity gain bandwidth of the opamp should be at least five times as large as the clock frequency $\omega_c$.

# Effects of Finite Opamp Bandwidth (Cont'd)

- For the FEDI

$$m(\omega) = -(1-k)e^{-k_1}$$

and

$$\theta(\omega) \cong 0$$

For $\omega_i << \omega_c$,

$$m(\omega_i) \cong -\omega_i T e^{-\frac{\omega_o T}{2}}$$

Hence $\omega_o \geq 5\omega_c$ applies for the FEDI as well.

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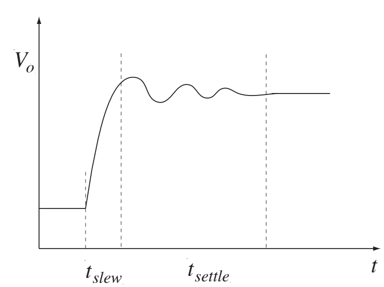
# Effects of Opamp Slew Rate

- All opamps exhibit finite slew rates that are dependent on currents and capacitances in the output stage of the opamp. A typical output in response to an input step function might therefore look like;




- Slew rate is defined as the maximum rate of change $dv_o/dt$ . Thus

$$S_r = \left|\frac{dv_o}{dt}\right| = \frac{I_L}{C_L}$$

where $I_L$ is assumed to be a load current and $C_L$ the load capacitor.

- Note slew rate is not directly related to the frequency response.

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## Effects of Opamp Slew Rate (Cont'd)

- For SC circuits based on the assumption of a sine wave input whose highest passband frequency is $\omega_B$ and amplitude is $V_{\max}$

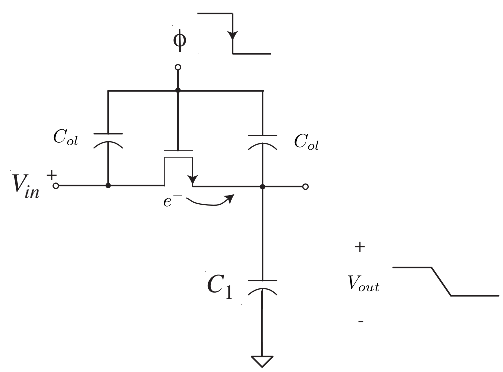
$$S_r \geq \frac{2\omega_B V_{\max}}{\left(t_{slew}/T_2\right)}$$

with $T_1 = T_2 \cong \dfrac{T}{2} = \dfrac{1}{2f_c}$ and $t_{slew} + t_{settle} < T_2$

- Note positive and negative slew rates exist.



## Charge Injection




- Due to overlap capacitance $C_{ol} \rightarrow$ Signal Independent

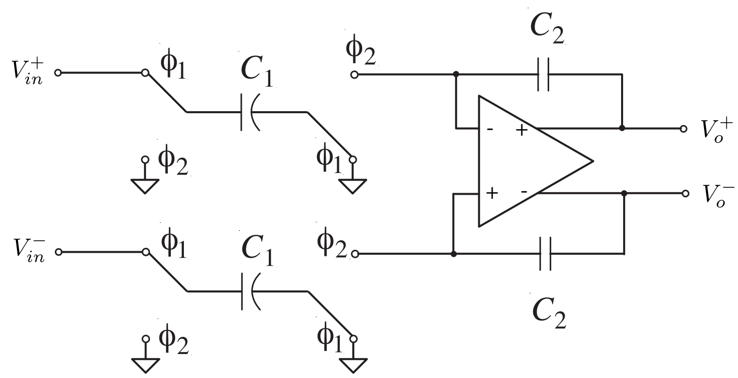
$$\Delta V_{ol} = -\frac{C_{ol}}{C_1 + C_{ol}} \cdot \left(V_{DD} - V_{SS}\right)$$

- Due to channel charge $\Delta Q_{ch} \rightarrow$ Signal Dependent

$$\Delta V_{ch} = \frac{\Delta Q_{ch}/2}{C_1} = -\frac{\mu C_{ox} W/L\left(V_{GS} - V_{TH}\right)}{2C_1} = \frac{\mu C_{ox} W/L\left(V_{DD} - V_S - V_{TH}\right)}{2C_1}$$

# Solution - $\Delta V_{ol}$

- Half-Size Dummy Switch driven by Complementary clocks.
- Transmission Gate
- Fully Differential Approach

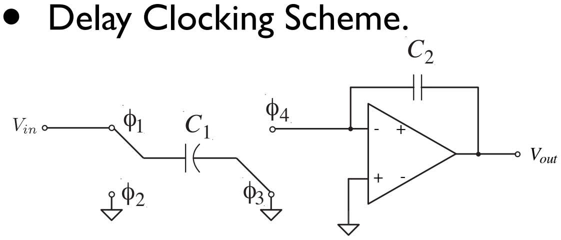


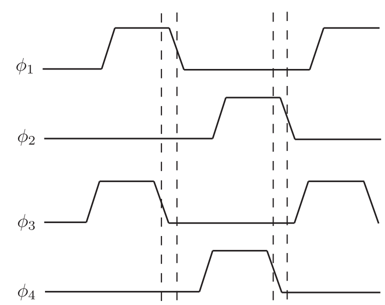

$$\Rightarrow \Delta V_{ol}(+) \ \& \ \Delta V_{ol}(-) \text{ Cancel each other}$$



# Solution - $\Delta V_{ol}$

- Delay Clocking Scheme.




  - Due to $\phi_1$ : Channel Charge Dependent on $V_{in}$
  - Due to $\phi_3$ & $\phi_4$ : Channel Charge Dependent on $GND \rightarrow$ Can be removed
  - Turning off $\phi_3$ & $\phi_4$ Earlier $\rightarrow$ Channel Charge of $\phi_1$ isolated from $C_2$
- Use a Compensation Capacitor

$\Rightarrow$ Insert (-) $\Delta Q_{ch}$ which is stored on the compensation capacitor