ECE 2040 Lab 2

March 14th, 2019

Introduction

In Lab 2, we built a simple RC circuit and measured several of its parameters using the myDAQ. We then analyzed the data in MATLAB and compared that with the theoretical values we should obtain. To start off, we created a ramp waveform that went from 0V to 5V over the course of 5 seconds and passed that signal into the circuit. We then used the datalogger to record the output and input waveforms as an lvm file. This process was repeated for an input step of 1V lasting for 5 seconds, and a chirp signal going from 0 Hz 600 Hz over the course of 30 seconds. We then switched out the capacitor and resistor values, and repeated the frequency response experiment. The files were then parsed and graphed in MATLAB. The plot of the voltage sweep was given a curve fit, the plot of the input step response was given a curve fit, and both frequency responses were plotted on a logarithmic scale.
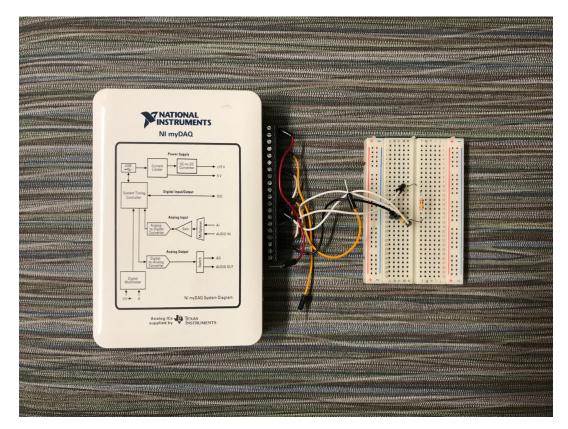

Figure 1. RC circuit connected to MyDAQ analog input and output pins. The same setup with different resistor and capacitor values was used for the second frequency response.

Calculations

After transforming our circuit into the Laplace space, the circuit can be represented with the equation: $H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+s\tau}$ This equation can be transformed into the frequency domain using the equation $s=j\omega$. Thus, the equation to the frequency response becomes $H(j\omega) = \frac{1}{1+j\omega\tau}$. In order to find the magnitude of the frequency response $|H(j\omega)| = \frac{1}{|1+j\omega\tau|} = \frac{1}{\sqrt{1+\omega^2\tau^2}}$.
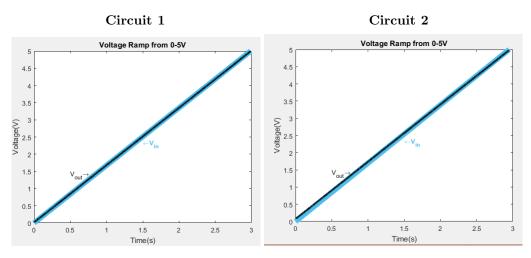
To calculate our time constant, we used the equation: $\tau = RC$. For the first circuit, R = 20kΩ, and C = 0.1μF. So, $\tau = 0.002$ seconds or 2 milliseconds. For the second circuit, R = 40kΩ, and C = 0.22μF. So, $\tau = 0.0088$ seconds or 8.8 milliseconds.

Putting the value of τ into the equation, we have $H(j\omega) = \frac{1}{\sqrt{1+\omega^2(0.002)^2}}$ where ω is frequency in rad/s or $\frac{1}{\sqrt{1+0.000016\pi^2 f^2}}$ where f is the frequency in Hertz for the first equation. We have $H(j\omega) = \frac{1}{\sqrt{1+\omega^2(0.0088)^2}}$ where ω is frequency in rad/s or $\frac{1}{\sqrt{1+0.00030976\pi^2 f^2}}$ where f is the frequency in Hertz for the second equation.
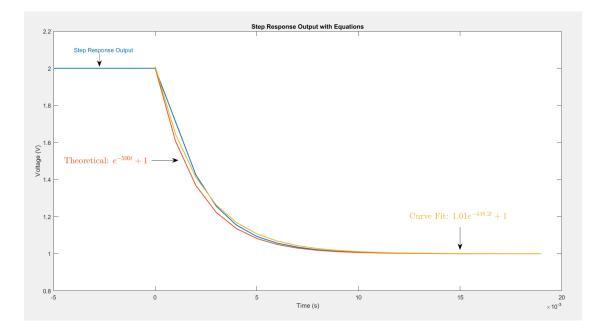
Measurements and Analysis

Ramp

First, we created a voltage sweep input waveform where the voltage increased linearly from 0V to 5V over the course of 5 seconds. We then ran this through the circuit and recorded the output voltage across the capacitor. We measured that the output voltage was almost identical to the input voltage, as expected.
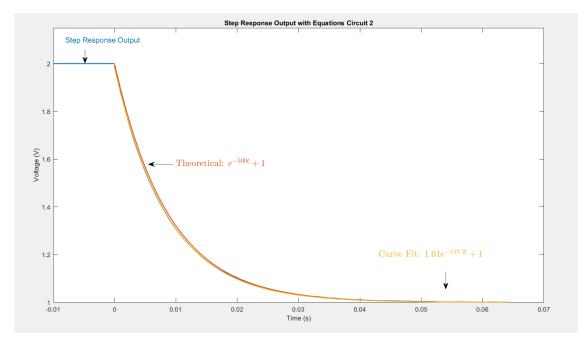
Circuit 1	Circuit 2

Step

Next, we ran a step function through our first circuit and measured the output. This consisted of a DC voltage level of 1V that stepped to 2V for 0.8 seconds, and then stepped back down to 1V. The falling edge of the output is displayed in the graph.
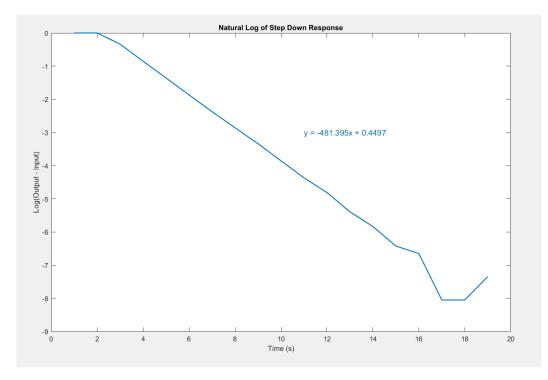
As displayed, the blue line indicates the actual values that we recorded for the step response output. The Orange line is the theoretical equation the data should follow based on our calculated time constant. The yellow line is the equation we obtained when curve fitting an exponential equation to the data.

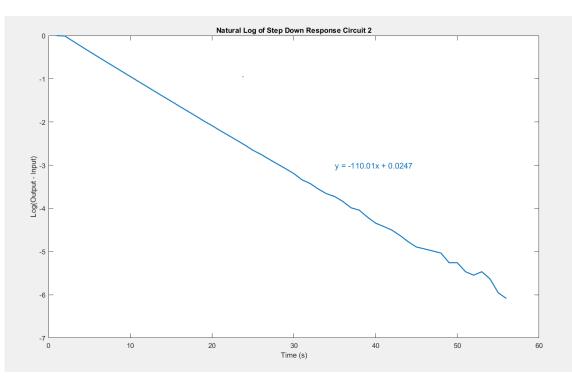We followed the same procedure for the second circuit with a time constant of 0.0088 seconds.

The graph for circuit 2 is very similar to circuit 1, but due to larger time constant values, our curve fit equation more closely resembled the data and the theoretical equation.

We also took the natural logarithm of the output for the first circuit, and ran a linear regression fit on the data.

Natural Log of Step Down Response

This gave us a linear equation with a slope of -481.395. This corresponds to a $\tau = \frac{1}{-481.395} = -0.00208$ seconds. This value is very close to calculated value, with an error of 3.9%.
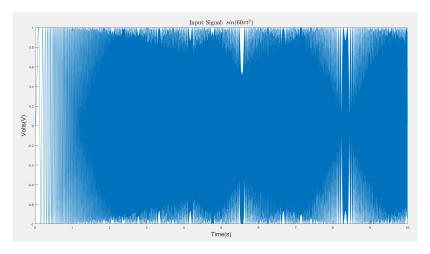
Lastly, we ran the same logarithm procedure on the step down response for circuit 2.
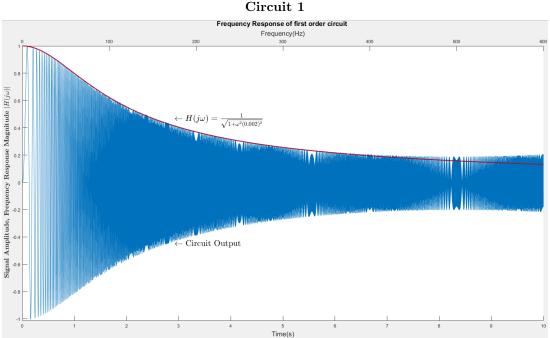

Natural Log of Step Down Response Circuit 2

This gave us a slope of -110.01. This corresponds to a $\tau = \frac{1}{-110.01} = -0.0091$ seconds. This value is also very accurate, with an error of 3.3%.
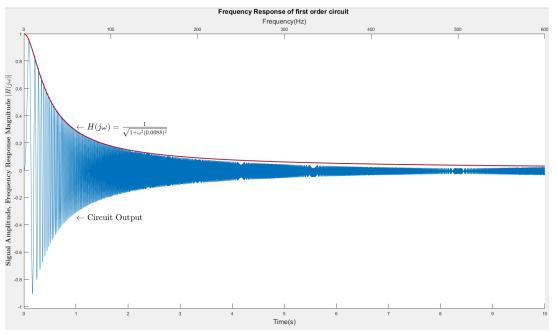
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Frequency Response

In order to get the frequency response of the circuit, we created a chirp signal that would act like a frequency ramp. The equation we used for this was $sin(60\pi t^2)$ which generates a frequency ramp of 60Hz per second. The anomalies in the input graph are due to the fact that our sampling rate is only 1000 samples/second. This creates strange artifacts but does not severely impact our output.

After sending this signal into the circuit, we observed that the output amplitude decreased as our frequency increased. When plotted against our frequency response, we can see that the amplitude roughly mimics the magnitude of the frequency response.

Circuit 1

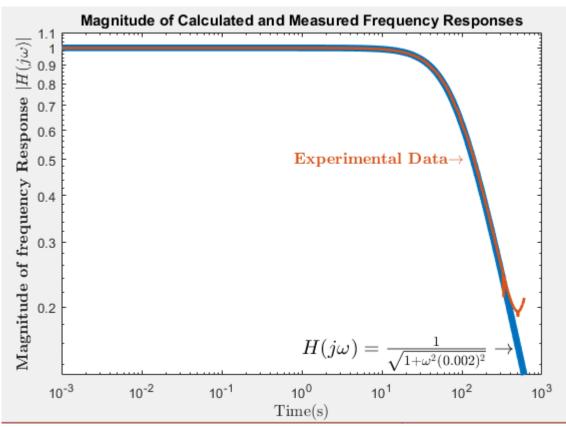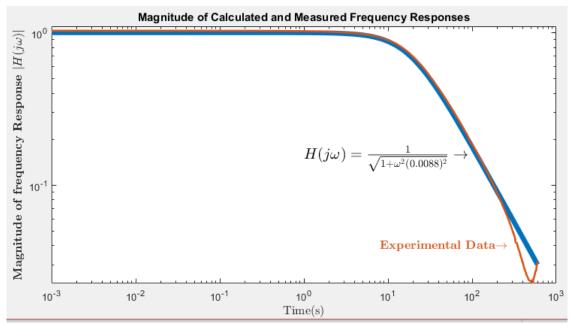**Circuit 2**

Frequency Response of first order circuit

Frequency Response: Analysis

In order to analyze the accuracy of our circuit's frequency response, we ran the MATLAB function envelope function which found the top envelope of our output sinusoid. From this, we graphed the theoretical frequency response vs. the measured frequency response for both circuits.

Circuit 1

Magnitude of Calculated and Measured Frequency Responses

Experimental Data→

$$H(j\omega) = \frac{1}{\sqrt{1+\omega^2(0.002)^2}} \rightarrow$$

x-axis: Time(s)

y-axis: Magnitude of frequency Response $|H(j\omega)|$

Circuit 2

Magnitude of Calculated and Measured Frequency Responses

$$H(j\omega) = \frac{1}{\sqrt{1+\omega^2(0.0088)^2}} \rightarrow$$

Experimental Data→

x-axis: Time(s)

y-axis: Magnitude of frequency Response $|H(j\omega)|$

Both Circuits

Magnitude of Calculated and Measured Frequency Responses
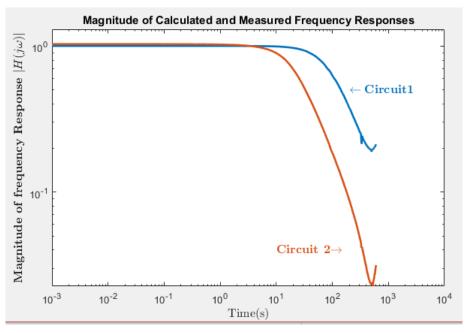
We found that our measured frequency response was very close to the theoretical frequency response except for in the end. This is due to two reasons. The first is that once the frequency reaches 500Hz we run into sampling issues due to the fact that our sampling rate is 1000 Samples/second which is exactly equal to the Nyquist rate. This means that we run into aliasing issues past 500Hz. The second reason that the experimental data does not match our theoretical frequency response at higher frequencies is because the frequency approaches and exceeds our time constant τ. When our frequency exceeds our time constant τ, the signal moves faster than our capacitor can process it. This, however is the nature of a **low pass filter**. This circuit uses the fact that it cannot process high frequencies as a feature which allows it to only pass low frequencies and remove high frequencies.

Remarks on Error

We had many sources of error. The main source of error was our sampling rate which was adequate for the ramp and step functions, but began to present a problem when we reached the frequency response. We would have liked to look at higher frequencies, but the MyDAQ can only read data at a maximum of 1000 Samples/second.

Another large source of error was the fact that for our second circuit, we used 3 resistors in series to achieve the 40kΩ resistance. The tolerance and range of this as well as the tolerance and range of the capacitor likely contributed to the strange frequency response results that our output resulted.