Problem 9.8

Given, an LTI system is described by the difference equation:

$y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4] = \sum_{k=0}^{4}(-1)^k x[n-k]$.

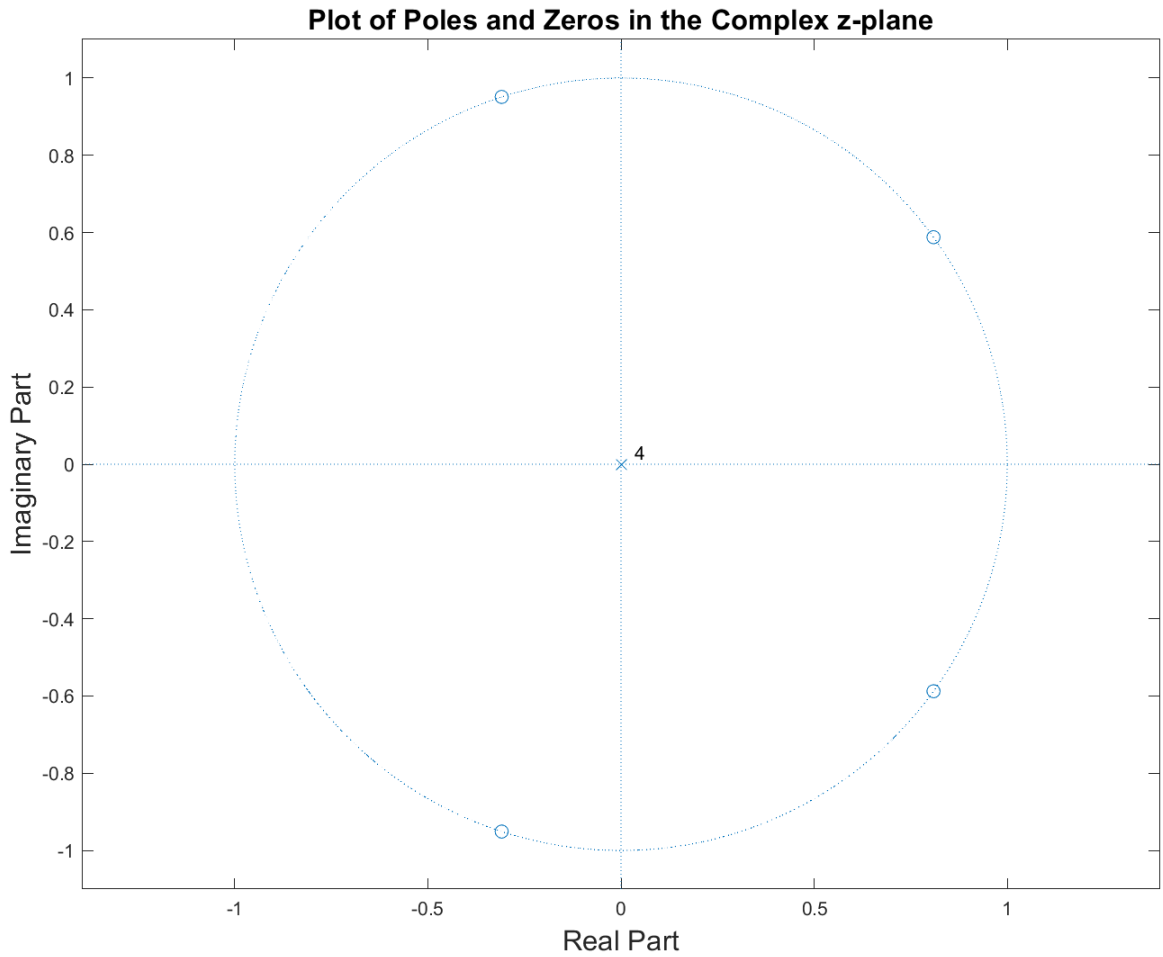
(a) The impulse response of the system is:

$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \delta[n-4] = \sum_{k=0}^{4}(-1)^k \delta[n-k]$

(b) The system function $H(z)$ for the system is:

$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}$

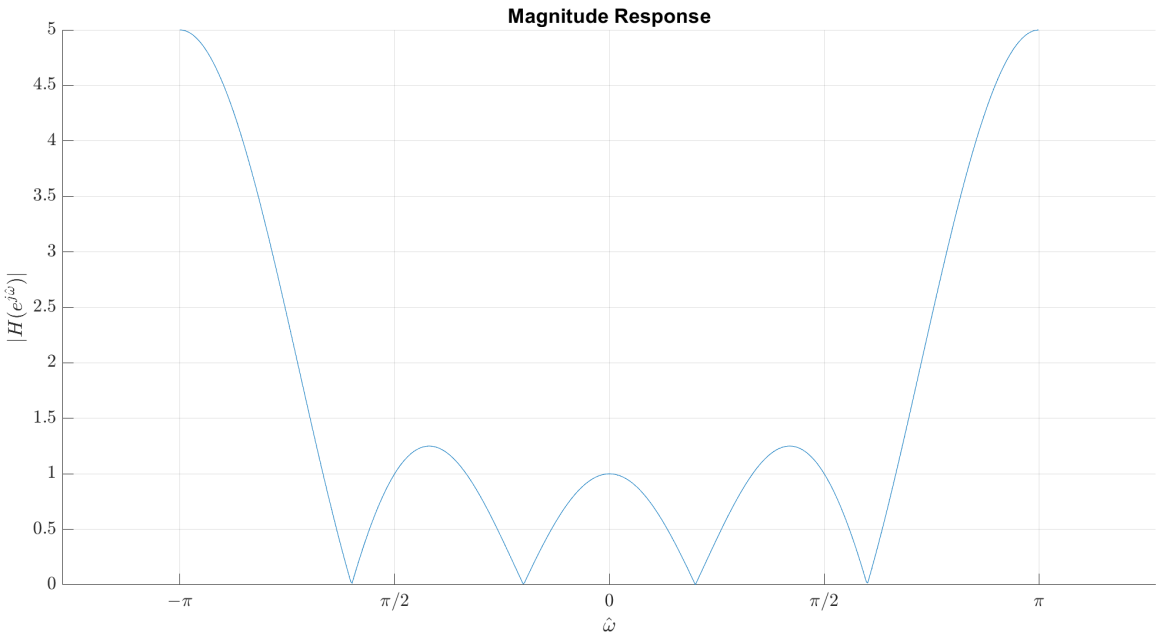
(c) Plot of poles and zeros of $H(z)$ in the complex plane:

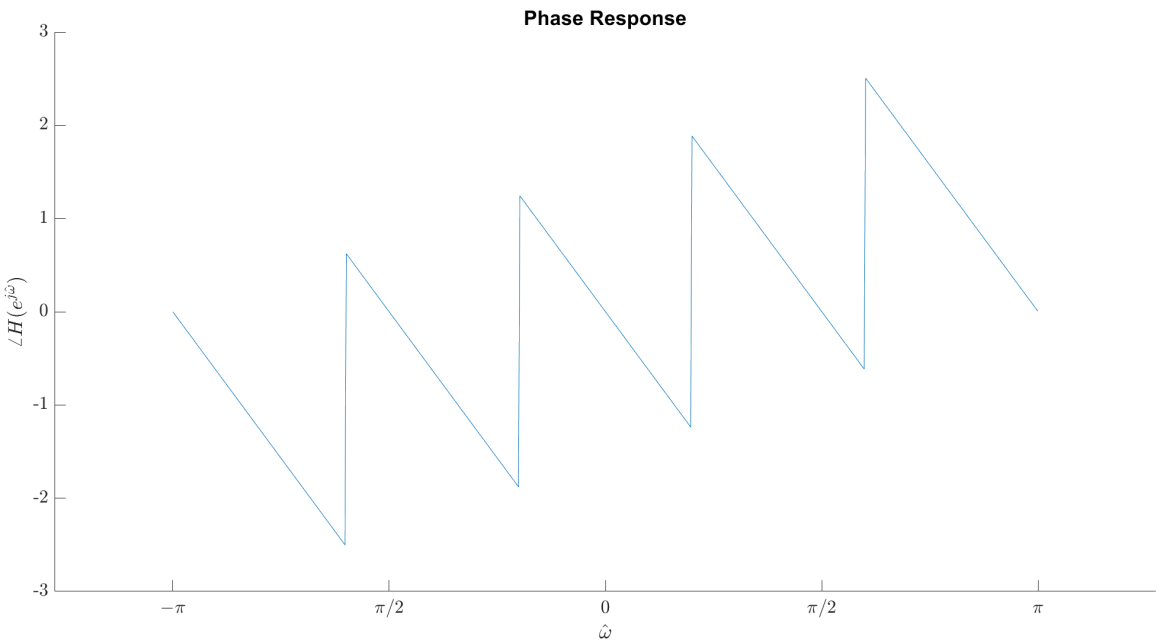


(d) Frequency response of the system:

$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$

(e) Frequency Response (Magnitude and Phase):





(f) Given input is $x[n] = 5 + 4\cos(0.5\pi n) + 3\cos(0.6\pi n + \frac{\pi}{4})$
Then, output $y[n] = 5|H(e^{j0})| + 4|H(e^{j0.5\pi})|\cos(0.5\pi n + \angle H(e^{j0.5\pi})) + 3|H(e^{j0.6\pi})|\cos(0.6\pi n + \angle H(e^{j0.6\pi}) + \frac{\pi}{4}))$
Hence, $y[n] = 5 + 4\cos(0.5\pi n)$.