Problem 9.6

LTI 1 described can be expressed by taking z transform of the difference equation $y[n]$ as: $H_1(z) = \frac{1+z^{-1}+z^{-2}}{3}$.

LTI 2 is expressed by system function : $H_2(z) = \frac{1+z^{-1}+z^{-2}+z^{-3}}{4}$.

(a)System function of overall cascade system:

$H(z) = H_1(z)H_2(z) = (\frac{1+z^{-1}+z^{-2}}{3})(\frac{1+z^{-1}+z^{-2}+z^{-3}}{4})$

$= (\frac{1+2z^{-1}+3z^{-2}+3z^{-3}+2z^{-4}+z^{-5}}{12})$

(b) Given, now the LTI 1 is 4-point averager, $H_1(z) = \frac{1+z^{-1}+z^{-2}+z^{-3}}{4}$ and LTI 2 is a 3-point averager, $H_2(z) = \frac{1+z^{-1}+z^{-2}}{3}$.

Thus, $Y_2(z) = H_2(z)X(z) = \frac{1-z^{-3}}{3}$, where $X(z) = 1 - z^{-1}$.

$W(z) = Y_2(z)H_1(z) = (\frac{1+z^{-1}+z^{-2}-z^{-4}-z^{-5}-z^{-6}}{12})$

(c) The system function of the overall cascade system is :

$H(z) = H_1(z)H_2(z) = (\frac{1+2z^{-1}+3z^{-2}+3z^{-3}+2z^{-4}+z^{-5}}{12})$.

Then the difference equation that relates $y[n]$ to $x[n]$:

$y[n] = x[n]+2x[n-1]+3[n-2]+3x[n-3]+2x[n-4]+x[n-5]$

(d) Poles and Zeros of $H(z)$