Problem 9.4

Given, LTI filter is described by the difference equation:

$y[n] = 0.1(x[n] - x[n-1] + x[n-2])$

(a) The system function $H(z)$ for the system:

$H(z) = 0.1(1 - z^{-1} + z^{-2})$

(b) Plot the poles and zeroes of $H(z)$ in the z-plane:

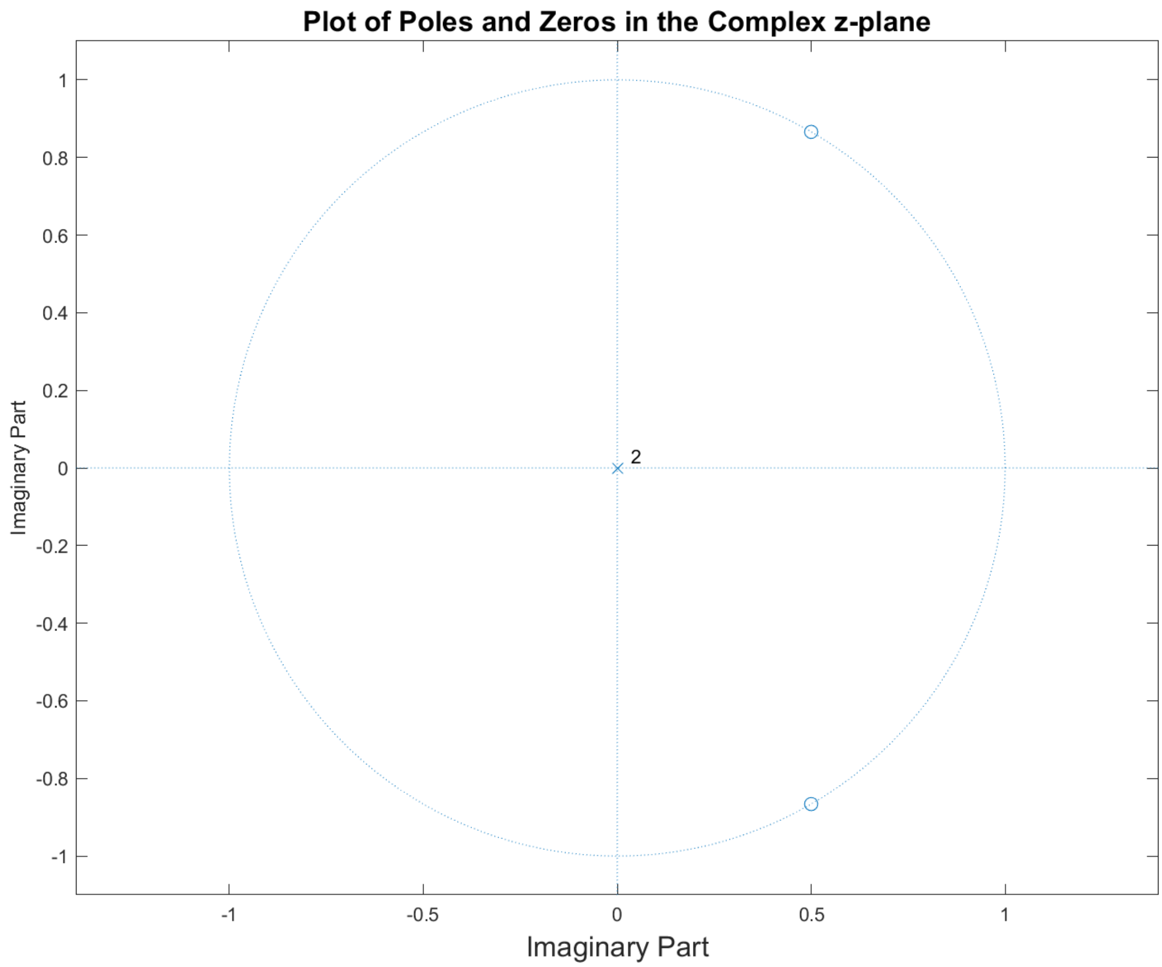
$H(z) = 0.1(1 - z^{-1} + z^{-2}) = \frac{0.1z^2 - 0.1z + 0.1}{z^2}$.

There are 2 poles at $z = 0$ and zeros are found by solving the numerator.

The zeros are $1/2 + \sqrt{3}/2i$.

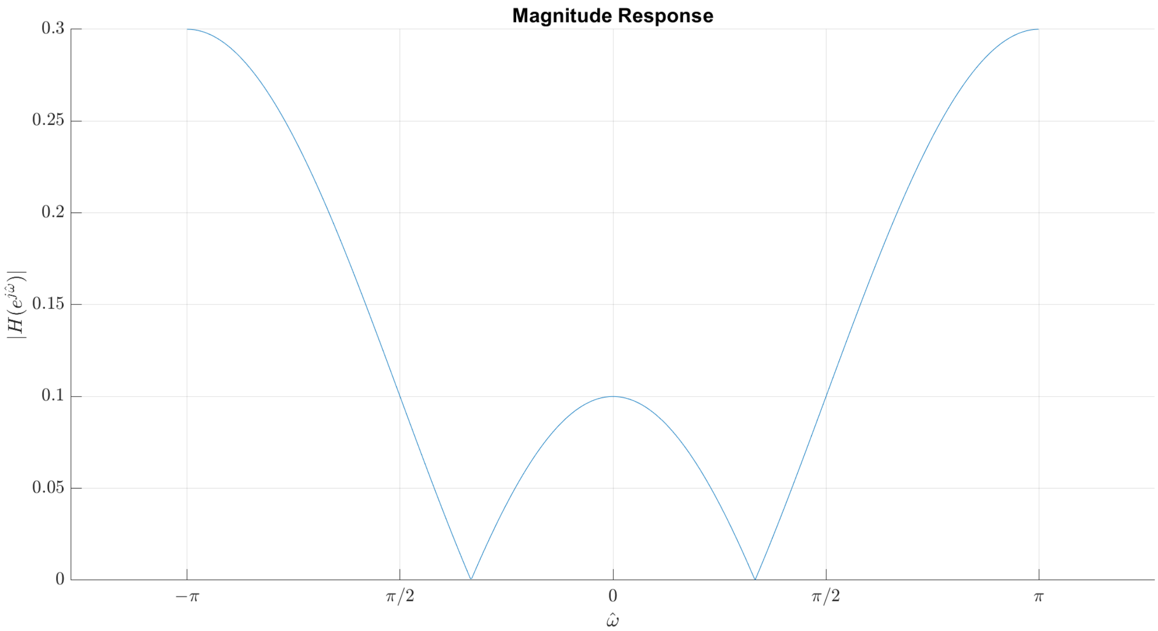
On converting to polar coordinates the zeros can be expressed as: $1e^{j\frac{\pi}{3}}$

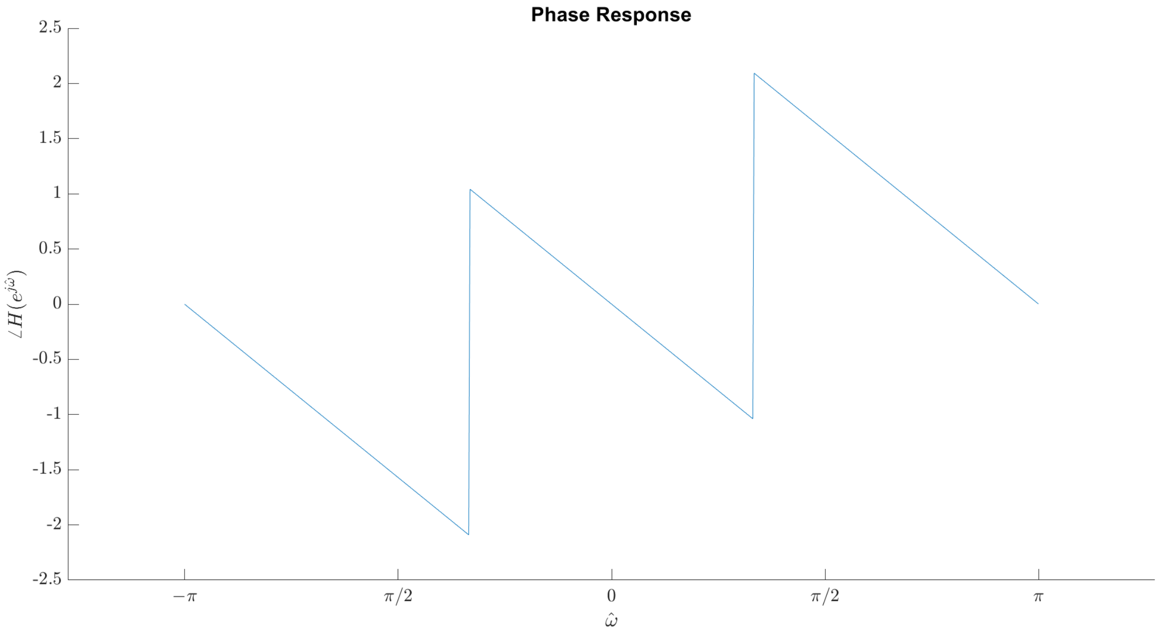
Plot on the z-plane:



(c) From $H(z)$ the expression of $H(e^{j\hat{\omega}})$ can be expressed as: $H(e^{j\hat{\omega}}) = 0.1(1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$ since $z = e^{j\hat{\omega}}$.

(d) Frequency Response (Magnitude and Phase) from $-\pi \leq \hat{\omega} \leq \pi$





(e) The output if the input is $x[n] = 9 - 8\cos[0.25\pi(n-1)] + 7\cos[(\frac{2\pi}{3})n]$ is given by :

$y[n] = 9|H(e^{j0})| - 8|H(e^{j0.25\pi})|\cos(0.25\pi(n-1) + \angle H(e^{j0.25\pi})) + 7|H(e^{j\frac{2\pi}{3}})|\cos(\frac{2\pi}{3}n + \angle H(e^{j\frac{2\pi}{3}\pi}))$.

Thus, $y[n] = 0.9 - 8(0.0414)\cos(0.25\pi(n-1) - \frac{\pi}{4}) + 7(-0.2)\cos(\frac{2\pi}{3}n - \frac{2\pi}{3}))$