

### Problem 9.18

Given,  $H(z) = b_0 + b_1 z^{-1} - b_1 z^{-3} - b_0 z^{-4}$

$$(a) H(z)|_{z=1} = b_0 + b_1 - b_1 - b_0 = 0. \text{ (Proved)}$$

$$(b) H(e^{j\hat{\omega}}) = b_0 + b_1 e^{-j\hat{\omega}} - b_1 e^{-j\hat{\omega}3} - b_0 e^{j\hat{\omega}4} = e^{-j\hat{\omega}2}(b_0 e^{j\hat{\omega}2} + b_1 e^{j\hat{\omega}} - b_1 e^{-j\hat{\omega}} - b_0 e^{-j\hat{\omega}2}) = [2b_0 \sin(2\hat{\omega}) + 2b_1 \sin(\hat{\omega})]e^{(j\frac{\pi}{2} - j\hat{\omega}2)}$$

Note:  $e^{j\frac{\pi}{2}} = j$

(c)  $H(\frac{1}{z}) = b_0 + b_1 z - b_1 z^3 - b_0 z^4$ . Taking  $z^4$  out common:

$$\implies H(\frac{1}{z}) = z^4(-b_0 - b_1 z^{-1} + b_1 z^{-3} + b_0 z^{-4})$$

$$\text{Then, } H(\frac{1}{z}) = -z^4(b_0 + b_1 z^{-1} - b_1 z^{-3} - b_0 z^{-4}) = -z^4 H(z)$$

(d) When M is even:

$$H(e^{j\hat{\omega}}) = 2e^{j(\pi/2 - \hat{\omega}M/2)} \left[ \sum_{k=0}^{M/2} b_{M/2-k} \sin(\hat{\omega}k) \right] + b_{M/2} e^{-j\frac{\hat{\omega}M}{2}}$$

When M is odd:

$$H(e^{j\hat{\omega}}) = 2e^{j(\frac{\pi}{2} - M\hat{\omega}/2)} \sum_{k=1}^{(M+1)/2} b_{[(M+1)/2]-k} \sin\left(\frac{\hat{\omega}}{2}(2k-1)\right)$$