Problem 9.12

Given system function: $H(z) = (1 - z^{-1})(1 + z^{-2})(1 + z^{-1})$

(a) Time-domain description of system in form of difference equation:

After, multiplying the terms of $H(z)$, it can be written as: $H(z) = 1 - z^{-4}$

Hence, $y[n] = x[n] - x[n-4]$

(b) The frequency response of the system:

$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}4}$

(c) $H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}4} = e^{-j\hat{\omega}2}(e^{(j\hat{\omega}2} - e^{(-j\hat{\omega}2)}) = 2je^{-j\hat{\omega}2}\sin(2\hat{\omega})$

$= 2\sin(2\hat{\omega})e^{j(\frac{\pi}{2}-2\hat{\omega})}$.

Magnitude: $2\sin(2\hat{\omega})$ and Phase:$\frac{\pi}{2} - 2\hat{\omega}$

(d) The input frequencies $\hat{\omega_0}$ are obtained when $H(e^{j\hat{\omega_0}}) = 0$.

This occurs when $2\sin(2\hat{\omega_0}) = 0$. Hence $\hat{\omega_0} = \frac{k\pi}{2}$, k is a integer.

(e) Input is $x[n] = \cos(\frac{\pi n}{3})$

Thus, $y[n] = |H(e^{j\frac{\pi}{3}})|\cos(\frac{\pi n}{3} + \angle H(e^{j\frac{\pi}{3}}))$

$= \sqrt{3}\cos(\frac{\pi n}{3} - \frac{\pi}{6})$