Problem 8.9

Given DTFT of length-32 real sinusoid signal sequence consists of 2 Dirchlet peaks given by:

$$Q(e^{j\hat{\omega}}) = \frac{1}{2}A\frac{\sin((\frac{1}{2})(32)(\hat{\omega}-\hat{\omega_0}))}{\sin((\frac{1}{2})(\hat{\omega}-\hat{\omega_0}))}e^{-j\frac{31}{2}(\hat{\omega}+\hat{\omega_0})}+\frac{1}{2}A\frac{\sin((\frac{1}{2})(32)(\hat{\omega}+\hat{\omega_0}))}{\sin((\frac{1}{2})(\hat{\omega}-\hat{\omega_0}))}e^{-j\frac{31}{2}(\hat{\omega}+\hat{\omega_0})}$$

(a) Given signal is :

$s[n] = 0.1 + A\cos(\hat{\omega}n)$. After taking DFT of the signal the result is $S[k]$. Value of $S[k] = 0$ for 29 of 30 coefficients and maximum at $k_1 = 10$.

$\hat{\omega} = \frac{2\pi k}{N} = \frac{2\pi k}{32}, N = 32.$

Peak frequency occurs at $\hat{\omega_{k_1}} = \frac{2\pi k_1}{32}$. As $k_1 = 10$, the frequency of sinusoid is given as $\hat{\omega_0} = \hat{\omega_{k_1}} = \frac{2\pi k_1}{31} = \frac{20\pi}{32} = \frac{10\pi}{16}$ radians.

(b) Required to find k_2 and k_3 for which $S[k_2] \neq 0$ and $S[k_3] \neq 0$.

The first peak occurs at $k_1 = 10$, then second peak occurs at k_2. Hence, $\hat{\omega_{k_2}} = \hat{\omega_{N-k_1}} = \frac{2\pi(32-k_1)}{32} = \frac{2\pi(32-k_1)}{32} = \frac{44\pi}{32} = \frac{11\pi}{8}$ radians. Thus, $k_2 = 22$.

The third peak occurs when $k_3 = 0$.

(c) Given, $k_1 = 10$ and $|S[k_1]| = 50$. Amplitude of signal can be determined using the equation of $Q(e^{j\hat{\omega}})$.

Hence, $|Q(e^{j\hat{\omega}})| = \lim_{\hat{\omega}\to\hat{\omega_0}} \frac{1}{2}A\frac{\sin((\frac{1}{2})(32)(\hat{\omega}-\hat{\omega_0}))}{\sin((\frac{1}{2})(\hat{\omega}-\hat{\omega_0}))} = \frac{1}{2}A\frac{1/2(32)}{1/2}$.

Since, $|Q(e^{j\hat{\omega}})| = 50$, then $A = 100/32 = 50/16 = 3.125$.