

Problem 8.3

To determine formula for 12-point DFT.

$$(a) y_0[n] = 3(-1)^n \text{ for } n = 0, 1, 2, 3, \dots, 11$$

Taking DFT, $Y_0(e^{j\hat{\omega}k}) = \sum_{n=0}^{N-1} y_0[n]e^{-j\hat{\omega}_k n}$, where $N = 12$ and

$$\hat{\omega}_k = \frac{2\pi k}{N}, k = 0, 1, 2, \dots, N-1. \text{ Hence, } Y_0(k) = \sum_{n=0}^{N-1} y_0[n]e^{j(2\pi/N)kn}.$$

$$(-1)^n = (e^{j\pi})^n. \text{ Thus, } Y_0(k) = \sum_{n=0}^{11} 3(-1)^n e^{-j\frac{2\pi}{12}kn} = \sum_{n=0}^{11} 3e^{j\frac{2\pi}{12}6n} e^{-j\frac{2\pi}{12}kn}$$

$$e^{j\frac{2\pi}{12}6n} = e^{j\pi n}$$

$$Y_0(k) = 3 \sum_{n=0}^{11} e^{j\frac{2\pi}{12}(k-6)n}$$

$$\text{Let } r = e^{j\frac{2\pi}{12}(k-6)n}$$

By geometric series, $S_n = ar^0 + ar^1 + \dots + ar^{N-1}$ for $r \neq 1$

$$S_n = \frac{1-r^N}{1-r}$$

$$\text{Thus, } Y_0(k) = \frac{1-e^{j2\pi(k-6)}}{1-e^{j\frac{2\pi}{12}(k-6)}}$$

(b)

$$y_1[n] = \begin{cases} 1 & n = 0, 1, 2, 3 \\ 0 & n = 4, 5, \dots, 11 \end{cases} \quad (3)$$

$y_1[n]$ can be represented as $y_1[n] = u[n] - u[n-4]$

$$\begin{aligned} \text{The DFT of } y_1[n] \text{ is } Y_1[k] &= \frac{\sin(\frac{1}{2}4(2\pi k/12))}{\sin(\frac{1}{2}(2\pi k/12))} e^{-j(\frac{2\pi k}{12}(4-1)/2)} \\ &= \frac{\sin(\frac{\pi k}{3})}{\sin(\frac{\pi k}{12})} e^{-j(\frac{\pi k}{4})} \end{aligned}$$

(c)

$$y_2[n] = \begin{cases} 1 & n = 0, 2, 4, 6, 8, 10 \\ 0 & n = 1, 3, 5, 7, 9, 11 \end{cases} \quad (4)$$

$$\begin{aligned} y_2[n] \text{ is expressed as } y_2[n] &= \frac{1}{2}[1 + e^{j\frac{2\pi}{12}(6n)}](u[n] - u[n - 12]) \\ &= \frac{1}{2}(u[n] - u[n - 12]) + e^{j(\frac{2\pi(6n)}{12})}(u[n] - u[n - 12]) \end{aligned}$$

Taking DFT yields:

$$Y_2(k) = \frac{1}{2} \frac{\sin(\pi k)}{\sin(\frac{\pi k}{12})} e^{-j\frac{2\pi k}{12}\frac{(12-1)}{2}} + \frac{1}{2} \frac{\sin(\pi(k-6))}{\sin(\frac{\pi(k-6)}{12})} e^{-j\pi(k-6)11/12}$$

Hence,

$$Y_2(k) = \frac{1}{2} \frac{\sin(\pi k)}{\sin(\frac{\pi k}{12})} e^{-j\pi k 11/12} + \frac{1}{2} \frac{\sin(\pi(k-6))}{\sin(\frac{\pi(k-6)}{12})} e^{-j\pi(k-6)11/12}$$