

### Problem 8.11

Given, periodic signal  $x[n]$  has DFS:

$x[n] = \sum_{k=-9}^9 (1 + k^2)e^{j(0.08\pi k)n}$ . Required to use DTFT to evaluate the convolution:  $y[n] = \frac{(\sin(0.15\pi n))}{5\pi n} * x[n] = h[n] * x[n]$ .

Taking DTFT of  $h[n]$  yields:

$$H(\omega) = \begin{cases} \frac{1}{5} & |\hat{\omega}| \leq 0.15\pi \\ 1 & 0.15\pi < |\hat{\omega}| \leq \pi \end{cases} \quad (5)$$

$$Y(\omega) = H(\omega)X(\omega)$$

From the equation of  $x[n]$   $\hat{\omega} = 0.8\pi k$ , when  $k = 0, \hat{\omega} = 0$ , when  $k = 1, \hat{\omega}_1 = 0.08\pi$  and when  $k = -1, \hat{\omega}_{-1} = -0.08\pi$ . When  $k = 2, \hat{\omega}_2 = 0.16\pi$  and  $k = -2, \hat{\omega}_{-2} = -0.16\pi$ .

$\hat{\omega}_1$  and  $\hat{\omega}_{-1}$  are below the cutoff frequency  $0.15\pi$  and will be considered for the convolution.

Hence,  $y[n] = h(n) * x(n) = \frac{1}{5}(1 + 2e^{j(0.08\pi)} + 2e^{-j(0.08\pi)}) = 0.2 + 0.8 \cos(0.08\pi n)$