

Problem 7.8

Given frequency response of LTI system is:

$$H(e^{j\hat{\omega}}) = \cos(3\hat{\omega})$$

(a) Output of the system for the input signal, $x_1[n] = \delta[n - 3]$:

Taking DTFT, $X(e^{j\hat{\omega}}) = e^{-j\hat{\omega}3}$. Thus, $Y(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}})X(e^{j\hat{\omega}})$.

Hence, $Y(e^{j\hat{\omega}}) = \cos(3\hat{\omega})e^{-j\hat{\omega}3}$.

(b) Output of the system for the input signal, $x_2[n] = 3 \cos(0.25\pi n)$:

$$\begin{aligned}y_2[n] &= 3|H(e^{j0.25\pi})| \cos(0.25\pi n) = \frac{-3\sqrt{2}}{2} \cos(0.25\pi n) \\&= -1.5\sqrt{2} \cos(0.25\pi n)\end{aligned}$$

(c) Output of the system for the input signal, $x_3[n] = u[n] - u[n - 9]$:

$$H(e^{j\hat{\omega}}) = 0.5(e^{j\hat{\omega}3} + e^{-j\hat{\omega}3}). \quad h[n] = 0.5(\delta[n + 3] + \delta[n - 3])$$

Thus, $y[n] = x_3[n] * h[n] = (u[n] - u[n - 9]) * (0.5\delta[n + 3] + 0.5\delta[n - 3])$

$$= 0.5(u[n + 3] + u[n - 3] - u[n - 6] - u[n - 12])$$