Problem 7.4

To determine the DTFT of:

(a) $h_1[n] = 5\delta[n] - \frac{\sin(0.25\pi n)}{0.2\pi n}$

$\frac{\sin(0.25\pi n)}{\pi n} = \begin{cases} 1 & |\hat{\omega}| \leq 0.25\pi \\ 0 & 0.25\pi < |\hat{\omega}| \leq \pi \end{cases}$ and $5\delta[n]$ after taking DTFT is 5.

Hence, $h_1[n]$ after taking DTFT is:

$$H_1(e^{j\hat{\omega}}) = 5 - \frac{1}{0.2} \begin{cases} 1 & |\hat{\omega}| \leq 0.25\pi \\ 0 & 0.25\pi < |\hat{\omega}| \leq \pi \end{cases}$$

(b)$h_2[n] = \frac{\sin(0.4\pi n)}{0.1\pi n} - \frac{\sin(0.1\pi n)}{0.1\pi n}$

DTFT of $\frac{\sin(0.4\pi n)}{\pi n}$ is $\begin{cases} 1 & |\hat{\omega}| \leq 0.4\pi \\ 0 & 0.4\pi < |\hat{\omega}| \leq \pi \end{cases}$

DTFT of $\frac{\sin(0.1\pi n)}{\pi n}$ is $\begin{cases} 1 & |\hat{\omega}| \leq 0.1\pi \\ 0 & 0.1\pi < |\hat{\omega}| \leq \pi \end{cases}$

Hence, DTFT of $h_2[n]$ is:

$$H_2(e^{j\hat{\omega}}) = \frac{1}{0.1} \begin{cases} 1 & |\hat{\omega}| \leq 0.4\pi \\ 0 & 0.4\pi < |\hat{\omega}| \leq \pi \end{cases} - \frac{1}{0.1} \begin{cases} 1 & |\hat{\omega}| \leq 0.1\pi \\ 0 & 0.1\pi < |\hat{\omega}| \leq \pi \end{cases}$$

(c)$h_3[n] = \frac{\sin(0.4\pi(n-8)}{0.1\pi(n-8))} - \frac{\sin(0.1\pi(n-8))}{0.1\pi(n-8))}$

$h_3[n]$ is related to $h_2[n]$ as $h_3[n] = 0.1h_2[n-8]$

Hence, $H_3(e^{j\hat{\omega}}) = 0.1H_2(e^{j\hat{\omega}})e^{-j\hat{\omega}8}$, where $H_2(e^{j\hat{\omega}})$ is computed in (b).