Problem 7.3

To determine the forward or inverse DTFT: (a)

$$V_1(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| \le 0.3\pi \\ 0 & 0.3\pi < |\hat{\omega}| \le \pi \end{cases}$$
(1)

The IDFT needs to be computed. Using the table of DTFT pairs, it is given by:

 $v_1[n] = \frac{\sin(0.3\pi n)}{\pi n}.$

(b)

$$v_2[n] = \begin{cases} 0 & n < 0\\ 1 & n = 0, 1, 2, ..9\\ 0 & n > 9 \end{cases}$$
(2)

The DTFT needs to be computed. It is given by:

$$V_2(e^{j\hat{\omega}}) = \sum_{n=0}^{9} e^{-j\hat{\omega}n} = \frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}4.5}.$$

(c)

$$V_{3}(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| \le 0.3\pi \\ 1 & 0.3\pi < |\hat{\omega}| \le \pi \end{cases}$$
(3)

The IDFT must be computed.

Additionally, it is seen that $V_3(e^{j\hat{\omega}})$ can be related to $V_1(e^{j\hat{\omega}})$

as
$$V_3(e^{j\hat{\omega}}) = 1 - V_1(e^{j\hat{\omega}}).$$

Then, $v_3[n] = \delta[n] - v_1[n]$, where $v_1[n] = \frac{\sin(0.3\pi n)}{\pi n}.$

(d)

$$v_4[n] = \begin{cases} 0 & n < 0\\ (-1)^n & n = 0, 1, 2, ..9\\ 0 & n > 9 \end{cases}$$
(4)

The DTFT must be computed. This is given by:

$$V_4(e^{j\hat{\omega}}) = \sum_{n=0}^9 (-1)^n e^{-j\hat{\omega}n} = e^{j\pi n} e^{-j\hat{\omega}n} = \frac{\sin(5(\hat{\omega}-\pi))}{\sin(\hat{\omega}-\pi)/2} e^{-j(\hat{\omega}-\pi)4.5}.$$