Problem 6.9

Given the frequency response of the LTI filter is:

$H(e^{j\hat{\omega}}) = (1 + e^{-j2\hat{\omega}})(1 - \frac{1}{2}e^{-j\hat{\omega}} + \frac{1}{4}e^{-j2\hat{\omega}})$

(a) Required to find Difference Equation:

After multiplying the two factors of $H(e^{j\hat{\omega}})$, then

$H(e^{j\hat{\omega}}) = 1 - \frac{1}{2}e^{-j\hat{\omega}} + \frac{5}{4}e^{-j2\hat{\omega}} - \frac{1}{2}e^{-j3\hat{\omega}} + \frac{1}{4}e^{-j4\hat{\omega}}$

Hence, the difference equation is :

$y[n] = x[n] - \frac{1}{2}x[n-1] + \frac{5}{4}x[n-2] - \frac{1}{2}x[n-3] + \frac{1}{4}x[n-4]$

(b) The impulse response of the LTI filter is:

$h[n] = \delta[n] - \frac{1}{2}\delta[n-1] + \frac{5}{4}\delta[n-2] - \frac{1}{2}\delta[n-3] + \frac{1}{4}\delta[n-4]$

(c) To find values of $-\pi < \hat{\omega} < \pi$ for which $y[n] = 0$.

This can be done by setting $H(e^{j\hat{\omega}}) = 0$.

Thus, $(1 + e^{-j2\hat{\omega}}) = 0$ and $(1 - \frac{1}{2}e^{-j\hat{\omega}} + \frac{1}{4}e^{-j2\hat{\omega}}) = 0$

For the first term , the values of $\hat{\omega}$ are $\hat{\omega} = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ radians.

The second term yields a quadratic equation which will have no real solution, hence, no values of $\hat{\omega}$.