Problem 6.8

Given, the frequency response of the LTI filter:

$H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\frac{3\pi}{4}} e^{-j\hat{\omega}})(1 - e^{-j\frac{3\pi}{4}} e^{-j\hat{\omega}})$

(a) Required to find Difference Equation:

$H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})(1 + \sqrt{2}e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$

$= 1 + (\sqrt{2} - 1)e^{-j\hat{\omega}} - (\sqrt{2} - 1)e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$

$= 1 + 0.41e^{-j\hat{\omega}} - 0.41e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$

The difference equation can be expressed as:

$y[n] = x[n] + 0.41x[n - 1] - 0.41x[n - 2] - x[n - 3]$

(b) Step response of the filter when input $x[n] = u[n]$:

$y[n] = u[n] + 0.41u[n - 1] - 0.41u[n - 2] - u[n - 3]$

Hence, $y[n] = 0$ for $n < 0, 1$ for $n = 0, 1.41$ for $n = 1$, 1 for $n = 24$ and 0 for $n \geq 3$.

(c) To find values of $-\pi < \hat{\omega} < \pi$ for which $y[n] = 0$.

For $y[n] = 0$, $H(e^{j\hat{\omega}}) = 0$.

Hence, $(1 - e^{-j\hat{\omega}}) = 0$, $((1 - e^{j\frac{3\pi}{4}} e^{-j\hat{\omega}}) = 0$ and $(1 - e^{-j\frac{3\pi}{4}} e^{-j\hat{\omega}})$.

Thus, $\hat{\omega} = 0$ or $\frac{3\pi}{4}$ or $\frac{-3\pi}{4}$ radians.