Problem 6.6

Given, LTI system is described by difference equation :

$y[n] = x[n] - x[n-4]$

(a)The frequency response is given by:

$H(e^{j\hat{\omega}}) = \sum_{k=0}^{4} b_k e^{-j\hat{\omega}k} = 1 - e^{-j\hat{\omega}4} = e^{-j\hat{\omega}2}\{e^{j\hat{\omega}2} - e^{-j\hat{\omega}2}\} = 2j\sin(2\hat{\omega})e^{-j\hat{\omega}2} = 2\sin(2\hat{\omega})e^{j\frac{\pi}{2}}e^{-j\hat{\omega}2}$

(b) Input is $x[n] = 4 + cos(0.25\pi n - \frac{\pi}{4})$.

Output signal can be represented as:

$y[n] = 4|H(e^{j0})| + |H(e^{j0.25\pi})|cos(0.25\pi n - \frac{\pi}{4} + \angle H(e^{j0.25\pi}))$

$|H(e^{j0})| = 0$ and $|H(e^{j0.25\pi})| = 2sin(\frac{\pi}{2}) = 2.$

Therefore, $y[n] = 2\cos(0.25\pi n - \frac{\pi}{4})$

(c) The output when the signal is $x_1[n] = (4 + cos(0.25\pi n - \frac{\pi}{4}))u[n]$ will be equal to the output in (b) when $n \geq 4$ as the value of the order of filter M is 4.