Problem 6.4

Given, LTI system is described by difference equation :

$y[n] = 2x[n] - 2x[n-1] + 2x[n-2]$

(a) The frequency response is given by:

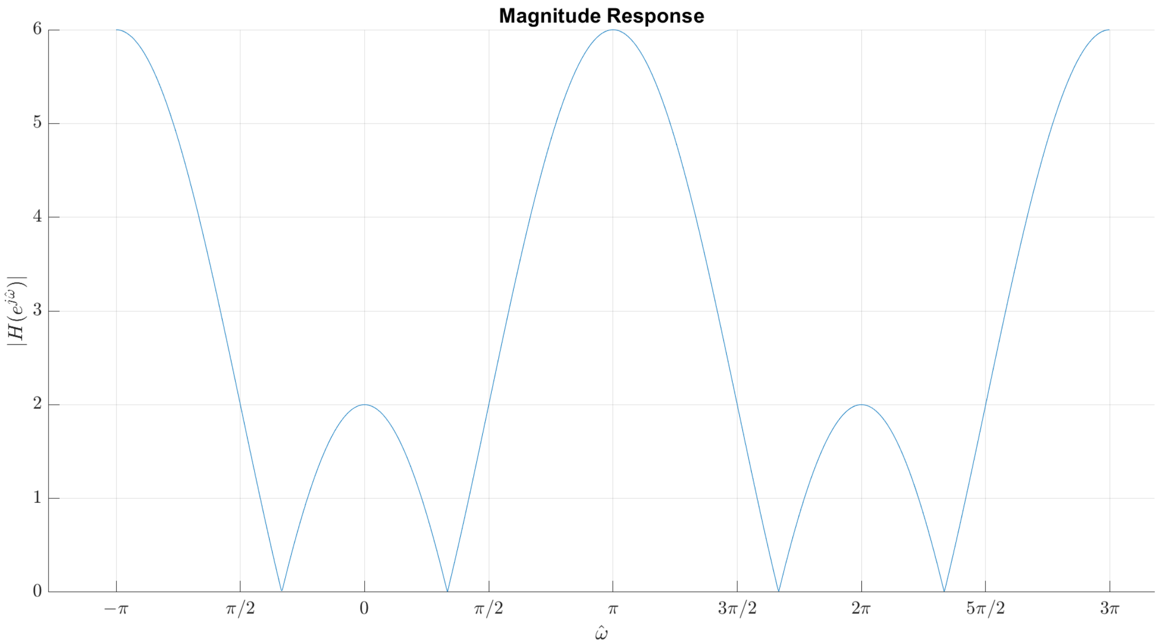
$H(e^{j\hat{\omega}}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 2 - 2e^{-j\hat{\omega}1} + 2e^{-j\hat{\omega}2}$

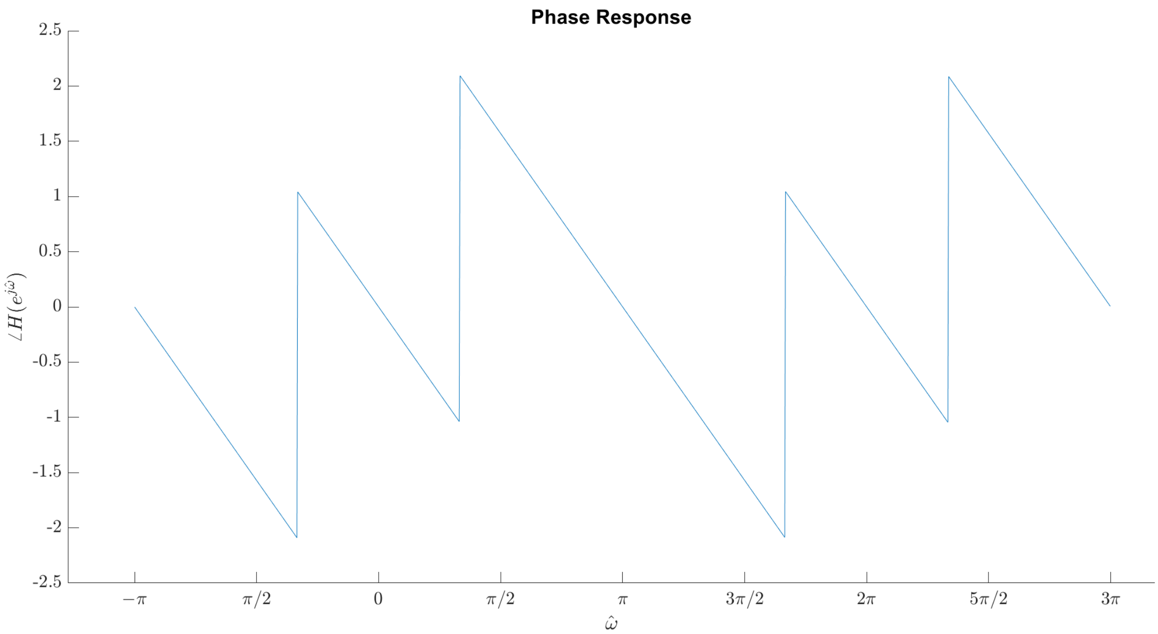
We can simplify the above equation, by factoring out an exponential whose phase is half of the filter order, in this case, M = 2.

$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\{2e^{j\hat{\omega}} - 2 + 2e^{-j\hat{\omega}}\} = (4cos\hat{\omega} - 2)e^{-j\hat{\omega}}$ is the equation for frequency response.

(b) As $cos\hat{\omega}$ has a period 2π, hence, $H(e^{j\hat{\omega}})$ has a period 2π.

(c) Plot of the Magnitude and Phase Response for $-\pi < \hat{\omega} < 3\pi$:





(d) Find the frequency $\hat{\omega}$ in $-\pi < \hat{\omega} < \pi$, for output response to be zero to input $e^{j\hat{\omega}n}$.

The magnitude of frequency response should be 0 for output response to be zero. Hence, $(4\cos\hat{\omega} - 2) = 0 \implies \cos\hat{\omega} = 0.5$ Thus, $\hat{\omega} = \frac{\pi}{3}$ or $-\frac{\pi}{3}$ radians.

(e) Given, input to the system is $x[n] = cos(0.5\pi n)$. $\hat{\omega} = 0.5\pi$

So, the output signal will be of the form :

$y[n] = |H(e^{j0.5\pi})| \cos(0.5\pi + \angle H(e^{j0.5\pi}))$.

Evaluating frequency response at $H(e^{j0.5\pi})$. $H(e^{j0.5\pi}) = 2e^{j0.5\pi}$.

Thus , $y[n] = 2\cos(0.5\pi n + 0.5\pi)$.