Problem 6.20

(a) Required to find the FIR filter that nulls out the signal $x[n] = 3 + 7\cos(0.2\pi n - 0.4\pi)$.

$\cos(0.2\pi n) = (e^{j0.2\pi n} + e^{-j0.2\pi n})/2$. Hence, the input signals that need be nulled are of the form, $e^{j0.2\pi n}$, $e^{-j0.2\pi n}$, e^{j0n}.

Thus, the frequency response can be represented as:

$H(e^{j\hat{\omega}}) = (1 - e^{j0.2\pi}e^{-j\hat{\omega}})(1 - e^{-j0.2\pi}e^{-j\hat{\omega}})(1 - e^{-j\hat{\omega}})$

$= (1 - 2\cos(0.2\pi)e^{-j\hat{\omega}} + e^{-j\hat{\omega}2})(1 - e^{-j\hat{\omega}})$

$= 1 - 2.61e^{-j\hat{\omega}} + 2.61e^{-j\hat{\omega}2} - e^{-j\hat{\omega}3}$

The difference equation filter coefficients are: {1,-2.61,2.61,-1}

(b) To design the minimum order FIR filter that nulls out the signal:

$$x[n] = \sum_{k=0}^{5}(k^2 + 9)\cos(0.2\pi kn)$$

Nulling of the signal will happen at the frequencies $\hat{\omega}_k = \pm 0.2\pi k$, $k = 1, 2, 3, 4, 5$.

Required to find frequency response $H(e^{j\hat{\omega}}) = 0$ at the above frequencies. This is possible when $H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}_k 10}$ where $e^{-j\hat{\omega}_k 10} = e^{-j(0.2\pi k)10} = e^{-j2\pi k} = 1$.

The filter coefficients of the difference equations for the nulling filter are: $b_0 = 1$ and $b_{10} = -1$.