Problem 6.16

Given, input to the C-D converter is:

$x(t) = 10 + 20\cos(\omega_0 t + \frac{\pi}{3})$

(a) The impulse response of LTI system is $h[n] = \delta[n]$ and $\omega_0 = 2\pi(500)$. The values of f_s for which $y(t) = x(t)$ is for $f_s > 2f_{max}$, where $f_{max} = 500Hz$.

(b) Impulse response of LTI system is changed to: $h[n] = \delta[n-10]$, required to find sampling rate and range of values for ω_0 so output of the system is:

$y(t) = x(t-0.001) = 10 + 20\cos(\omega_0(t-0.001) + \frac{\pi}{3})$ [Equation A]

From what's given in the question $y[n]$ can be expressed as:

$y[n] = \delta[n] * x[n] = x[n-10]$

Hence, $y[n] = 10 + 20\cos(\hat{\omega}[n-10] + \frac{\pi}{3})$

$y(t) = y[n]|_{n=f_s t} = 10 + 20\cos(\hat{\omega}(tf_s - 10) + \frac{\pi}{3})$

$= 10 + 20\cos(\omega_0(tf_s - 10)/f_s + \frac{\pi}{3})$ [Equation B]

Both Equation A and Equation B are equivalent so equating the ω_0 terms, $-\omega_0(0.001) = \frac{-10\omega_0}{f_s}$ and solving for f_s.

Hence $f_s = 10000$ Hz.

(c) Given the LTI system is a 5-point moving averager with frequency response:

$H(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{5\sin(\hat{\omega}/2)}e^{-j\hat{\omega 2}}$ and $f_s = 2000$ samples/s.

In order for $y(t)$ to be a constant, the cosine signal must be nullified. Thus can be done by analyzing the null frequencies of $H(e^{j\hat{\omega}})$. The null frequencies of $H(e^{j\hat{\omega}})$ occur at $\hat{\omega} = \pm 2\pi k/L, L = 5$. Hence, the values of ω_0 for which $y(t)$ is constant is: $\omega_0 = \frac{\hat{\omega}}{f_s} = \pm 2\pi(400)k$ radians.

Thus, value of $y(t) = 10 = A$.