Problem 6.14

Given, LTI filter is a 5-point running sum described by the difference equation:

$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$

(a) Closed-loop form expression for frequency response of the system:

$H(e^{j\hat{\omega}}) = \frac{1-e^{j\hat{\omega}5}}{1-e^{j\hat{\omega}}} = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega 2}}$ where $D_5(e^{j\hat{\omega}}) = \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega 2}}$

(b) Frequencies where frequency response is zero in the range $-\pi \leq \hat{\omega} \leq \pi$:

$\hat{\omega} = 0.4\pi, 0.8\pi, -0.4\pi$ and -0.8π radians as

$D_5(e^{j\hat{\omega}}) = 0$ at frequencies $\hat{\omega} = \frac{2\pi k}{5}$ radians.

(c) Given input is: $x[n] = 7 + 8\cos(0.25\pi n) + 9\cos(0.4\pi n)$

Output $y[n] = 7|H(e^{j0})| + 8|H(e^{j0.25\pi})|\cos(0.25\pi n + \angle H(e^{j0.25\pi}))$

$+ 9|H(e^{j0.4\pi})|\cos(0.4\pi n + \angle H(e^{j0.4\pi}))$

Hence, $y[n] = 35 + 19.28\cos(0.25\pi n - 0.5\pi)$

(d) Given input is: $x_1[n] = [7 + 8\cos(0.25\pi n) + 9\cos(0.4\pi n)]u[n]$

For n greater than and equal to 4 , $y_1[n] = y[n]$.