Problem 6.12

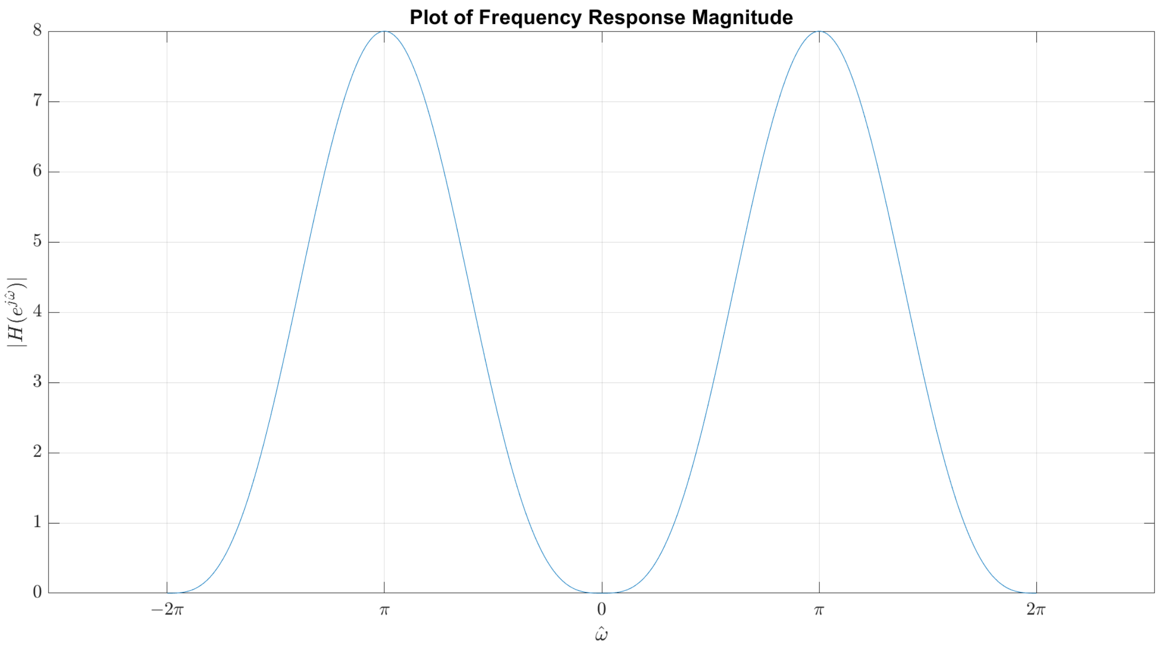
Given, the LTI filter is described by the difference equation, $y[n] = x[n] - 3x[n-1] + 3x[n-2] - x[n-3]$

(a) The frequency response of the system can be expressed as: $H(e^{j\hat{\omega}}) = 1 - 3e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} = (1 - e^{-j\hat{\omega}})^3$ using the identity $(1-a)^3 = 1 - 3a + 3a^2 - a^3$.

$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}\frac{3}{2}}[e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}]^3$

Multiplying Numerator and Denominator by $(2j)^3$ and as $j = e^{j\frac{\pi}{2}}$. Hence, $H(e^{j\hat{\omega}}) = 8\sin^3(\frac{\hat{\omega}}{2})e^{j(-\frac{\pi}{2}-\frac{3\hat{\omega}}{2})}$.

(b) Frequency Response Magnitude as a function of $\hat{\omega}$:



(c) Output when $x[n] = 5 + 6\cos(0.5\pi(n-2))$:
$y[n] = 5|H(e^{j0})| + 6|H(e^{j0.5\pi})|cos(0.5\pi n - \pi + \angle H(e^{j0.5\pi}))$
where, $H(e^{j0}) = 0$ and $H(e^{j0.5\pi}) = 2\sqrt{2}e^{-j\frac{7\pi}{4}}$.
Hence, $y[n] = 12\sqrt{2}\cos(0.5\pi n - \pi - \frac{7\pi}{4})$.

(d) The impulse response of FIR filter:
$h[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$

(e) Output when input $x[n] = 5 + 6\cos(0.5\pi(n)) + 9\delta[n-3]$:
$y[n] = 12\sqrt{2}\cos(0.5\pi n - \frac{7\pi}{4}) + 9h[n-3]$. (Using results found in (c) and (d)).