Problem 5.6

(a)Given system defined as:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

(a) Given input x[n] is nonzero for $0 \leq n \leq N-1$. To show $y[n]$ is non-zero from $0 \leq n \leq P-1$ and find P in terms of M and N.

Since $x[n]$ is non-zero for $0 \leq n \leq N-1$, it is zero when $n > 0$ and $n > N-1$.

$y[n]$ can be expressed as:

$y[n] = b_m x[n-M] + b_{m-1} x[n-M+1] + b_{m-2} x[n-M+2] + b_{m-3} x[n-M+3] + ...$

In order to find, largest n value for which $y[n]$ is non-zero, analyzing the term $x[n-M]$, $n-M < N$ for $x[n-M]$ to be non-zero, which implies $n < M+N$ for $y[n]$ to be non-zero. Hence, the support of $y[n]$, which is P, can be expressed as $P = M+N$.

(b) Given, $x[n]$ is non-zero for $N_1 \leq n \leq N_2$.

Length of $x[n]$ is $N_2 - N_1 + 1$.

To show that $y[n]$ is non-zero over interval $N_3 \leq n \leq N_4$.

The output $y[n]$ will start at $n = N_1$ and end at $n = N_2 + M$.

Hence, $N_3 = N_1$ and $N_4 = N_2 + M$.

The length of $y[n] = N_4 - N_3 + 1 = N_2 + M - N_1 + 1$.