Problem 5.18

(a) The impulse response for each individual system:

$h_1[n] = \delta[n] + \delta[n-1]$

$h_2[n] = \delta[n] - \delta[n-2]$

$h_3[n] = \delta[n-1] + \delta[n-2]$

(b) The impulse response of the overall system is:

$h[n] = h_1[n] * h_2[n] * h_3[n]$

$h_2[n] * h_3[n] = (\delta[n] - \delta[n-2]) * (\delta[n-1] + \delta[n-2])$

$= \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$

Convolving $h_2[n] * h_3[n]$ with $h_1[n]$:

$= (\delta[n] + \delta[n-1]) * (\delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4])$

$= \delta[n-1] - 2\delta[n-3] + \delta[n-5]$

(c) Difference equation that defines the overall system in terms of $x[n]$ and $y[n]$:

$y[n] = h[n] * x[n] = (\delta[n-1] - 2\delta[n-3] + \delta[n-5]) * x[n]$

$\implies y[n] = x[n-1] - 2x[n-3] + x[n-5]$.