Problem 5.14

Given for the FIR filter:

$y[n] = x[n] - ax[n-1]$

Find $y[n]$ when:

(a) $x[n] = a^n u[n]$

Then, $y[n] = h[n] * x[n]$, $h[n] = \delta[n] - a\delta[n-1]$

$\implies y[n] = x[n] - ax[n-1] = a^n u[n] - a[a^{n-1}u[n-1]]$

$= a^n u[n] - a^0 a^n u[n-1]$

$u[n] = \delta[n] + u[n-1]$

Thus, $y[n]$ can be expressed as

$y[n] = a^0\delta[n] + a^n u[n-1] - a^n u[n-1] = \delta[n]$ -(A)

(b) $x[n] = a^n(u[n] - u[n-8]) \implies x[n] = a^n u[n] - a^n u[n-8]$

Since, from (a) $y[n]$ is computed for $x[n] = a^n u[n]$, as seen from (A). $y[n]$ can be expressed as, $y[n] = y_1[n] + y_2[n]$, where $y_1[n] = \delta[n]$. Hence, $y_2[n]$ can be computed corresponding to the second term, $-a^n u[n-8]$ and its value can be added to $y_1[n]$.

$y_2[n] = -a^n u[n-8] - aa^{n-1}(u[n-9])$

$= -au[n-8] - a^0 a^n(u[n-9])$

$u[n-8] = \delta[n-8] + u[n-9]$

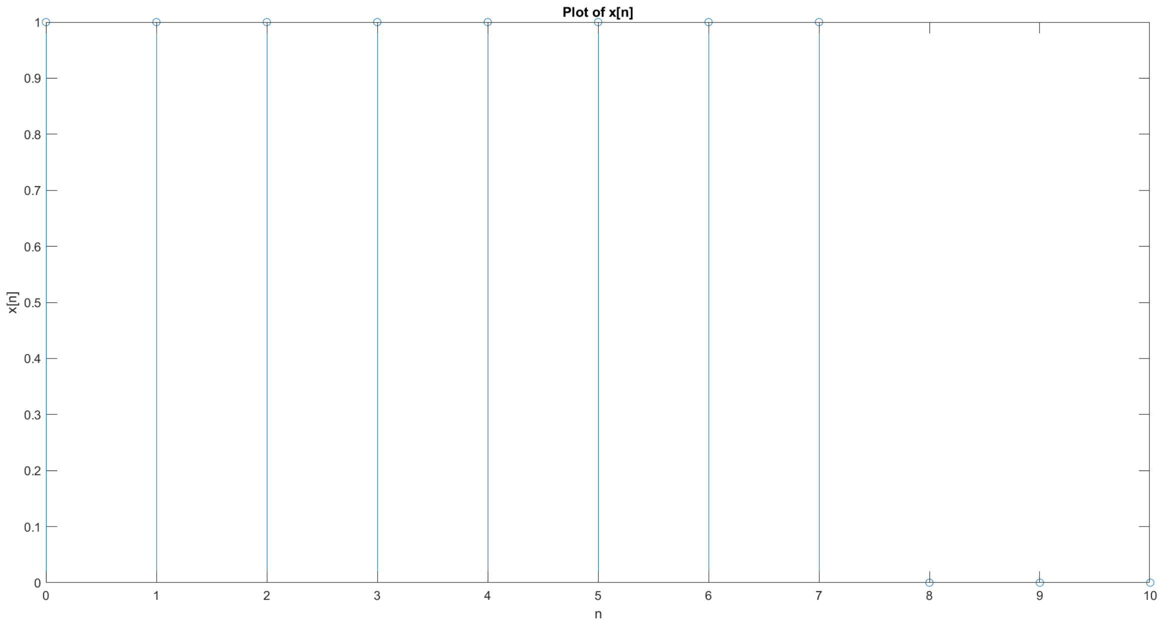
$y_2[n] = -a^8\delta[n-8] - a^n u[n-9] + a^n u[n-9]$

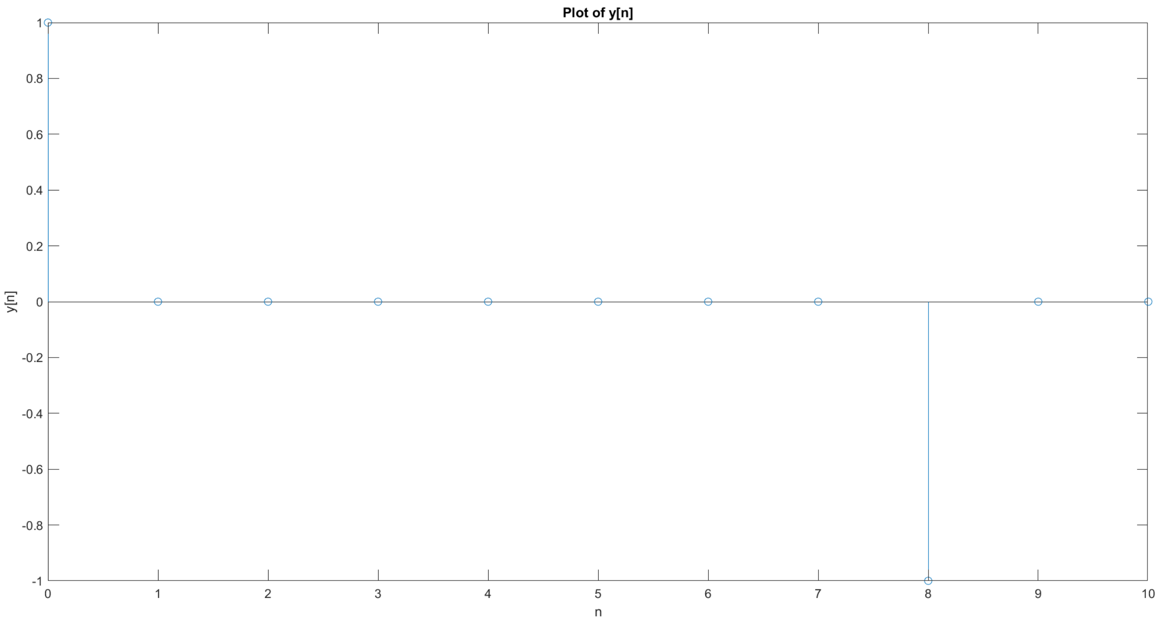
$= -a^8\delta[n-8]$

Thus, $y[n] = \delta[n] - a^8\delta[n-8]$.

(c) Plot of $x[n]$ and $y[n]$ for part (b):

Assuming $a = 1$.




Plot of y[n]
y[n]
n
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10