Problem 4.8

Given, sampling rate of C-D and D-C converters is $f_s = 700$ samples/s.

(a) Maximum frequency $f_{max} = 500$ Hz, from spectrum of $x(t)$. Hence, Nyquist rate is $2 \times f_{max} = 1000$ Hz.

As $f_s < 1000$ Hz, (under-sampling) aliasing occurs.

(b) $x(t) = 7e^{-j(0.3\pi)}[2\cos(2\pi(500)t)] + 13\cos[2\pi(200)t + 0.7\pi]$
$\implies x(t) = 14[\cos(0.3\pi)][\cos(2\pi(500)t)] + 26\cos[2\pi(200)t + 0.7\pi]$

Sampling frequency $= f_s = 700$ Hz.

Then,

$x[n] = 14[\cos(0.3\pi)][\cos\frac{10\pi}{7}n] + 26\cos[\frac{4\pi}{7}n + 0.7\pi]$

There will be aliasing as $\hat{\omega}_1 = \frac{10\pi}{7}$ will alias to $\hat{\omega}_2 = -\frac{4\pi}{7}$, called folded alias, by the property $\hat{\omega} = \frac{\omega_1}{f_s} - 2\pi l$.

Hence, $x[n]$ can be written as:

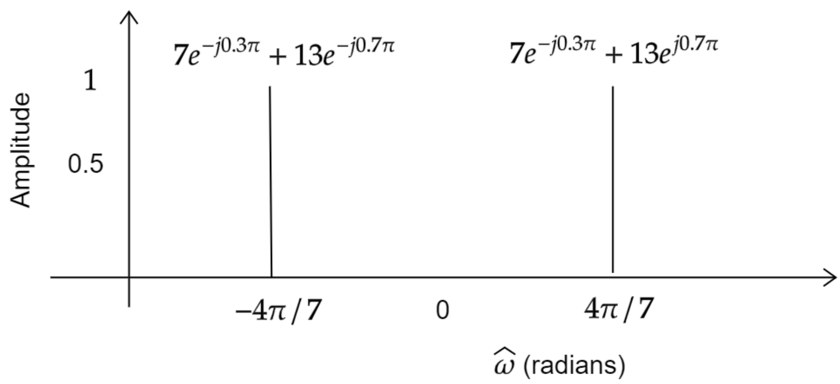
$x[n] = 14[\cos(0.3\pi)][\cos(-\frac{4\pi}{7}n)] + 26\cos[\frac{4\pi}{7}n + 0.7\pi]$

$\implies x[n] = 14[\cos(0.3\pi)][\cos(\frac{4\pi}{7}n)] + 26\cos[\frac{4\pi}{7}n + 0.7\pi]$

By Inverse Euler formula,

$x[n] = e^{j\frac{4\pi}{7}}[7e^{-j0.3\pi} + 13e^{j0.7\pi}] + e^{-j\frac{4\pi}{7}}[7e^{-j0.3\pi} + 13e^{-j0.7\pi}]$

Spectrum:



(c) When $f_s = 700$ Hz , $\omega = \frac{\hat{\omega}}{f_s} = 400\pi$ rad/s.
$y(t) = 14\cos(0.3\pi)\cos(400\pi t) + 26[cos(400\pi t + 0.7\pi)]$