## Problem 4.23

Given, $-f_m = 13$ rev/s and $f_s = 15$ flash/s.

Position of spot at the $n^{th}$ flash can be denoted as:

$p[n] = re^{j[2\pi(\frac{-f_m}{f_s})n]}e^{j\phi}$

$\implies p[n] = re^{j[2\pi(\frac{13}{15})n]}e^{j\phi}$

(a) The spot is initially at $t = 0$ on the y-axis (imaginary) which corresponds to $n = 0$:

$p[0] = re^0e^{j\phi}$.(Equation 1)

$p(t)$ can also be represented as a complex number:

$p(t) = x(t) + jy(t)$. The coordinates of the spot at $n = 0$ are $(0, r)$.

Thus, $p[0] = jr$. (Equation 2)

Equating Equations 1 and 2, yields, $e^{j\phi} = j$ which gives, $\phi = \frac{\pi}{2}$ radians.

Calculating the movement of the spot,

$\Delta\theta = -360° \times -\frac{780}{900} = 312° = -48°$.

Spot moves 48° clockwise or by $\frac{4\pi}{15}$ radians in clockwise direction because of negative sign.

$p[n]$ is: $p[n] = re^{j[2\pi(\frac{13}{15})n]}e^{j\frac{\pi}{2}}$ which can also be written as: $p[n] = re^{j[-\frac{4\pi}{15})n]}e^{j\frac{\pi}{2}}$

(b) The position of spot in continuous-time can be denoted

as:

$p(t) = p[n]\Big|_{n=15t} = re^{j(-4\pi t+\frac{\pi}{2})}$

Spot rotates by an angle of $-4\pi$ every second.

As 1 rev $= 2\pi$ radians, the apparent speed is 2 rev/s in the clockwise direction because of negative sign.

(c)Given : $f_m$ is unknown. $f_s = 13$ flashes/s and the spot moves counterclockwise by $x = 15°$ with each flash.

$f_m$ is given by:

$f_m = 13(l - \frac{1}{24})$, l is an integer.