Problem 4.1

Given in the question, $x(t) = 11\cos(7\pi t - \frac{\pi}{3})$.

The discrete-time signal $x[n]$ is obtained by sampling x(t) at a rate f_s samples/s, and the resultant $x[n]$ can be written as :

$x[n] = A\cos(\hat{\omega}_1 n + \phi)$

From the equation of $x(t)$, A $=$ 11, $\omega = 7\pi$ rad/s, $f = 3.5$ Hz and $\phi = -\frac{\pi}{3}$ radians.

(a) Sampling frequency is $f_s = 9$ samples/s

The signal is over-sampled as $f_s > 2f$.

. A $=$ 11, $\hat{\omega}_1 = \frac{\omega}{f_s} = \frac{7\pi}{9}$ radians, $\phi = -\frac{\pi}{3}$ radians.

(b) Sampling frequency is $f_s = 6$ samples/s

The signal is under-sampled as $f_s < 2f$.

A $=$ 11, $\hat{\omega}_1 = \frac{\omega}{f_s} = \frac{7\pi}{6}$ radians, which is greater than π radians, which does not satisfy the constraint of $0 \leq \hat{\omega}_1 \leq \pi$. Hence computing the alias frequency, $2\pi - \hat{\omega}_1 = \frac{5\pi}{6}$ radians, will satisfy the constraint.

$\phi = \frac{\pi}{3}$ radians, because the algebraic sign of the phase angle of the folded alias must be opposite to sign of phase angle of principal alias.

(c) <u>Sampling frequency is $f_s = 3$ samples/s</u>

The signal is under-sampled as $f_s < 2f$.

A = 11, $\hat{\omega_1} = \frac{\omega}{f_s} = \frac{7\pi}{3} > \pi$ radians. Hence computing the alias frequency, $2\pi - \hat{\omega_1} = -\frac{\pi}{3}$ radians. Phase will then be, $\phi = \frac{\pi}{3}$. But, $-\frac{\pi}{3}$ is less than zero radians.

Hence, $x[n] = 11\cos\left(-\frac{\pi}{3}n + \frac{\pi}{3}\right)$ can be expressed as $x[n] = 11\cos\left(\frac{\pi}{3}n - \frac{\pi}{3}\right)$ because of the property $\cos(-\theta) = \cos(\theta)$.

Thus,

A = 11, $\hat{\omega_1} = \frac{\pi}{3}$ radians and $\phi = -\frac{\pi}{3}$ radians.