

Problem 3.27

(a) $x(t) = \cos(-250\pi t^2)$ $\psi(t) = -250\pi t^2$. To be able to find spectrogram, compute the instantaneous frequency defined as,
 $f_i(t) = \frac{1}{2\pi} \frac{d\psi(t)}{dt}$ -(A).

For (a) the instantaneous frequency is 250t Hz.

Hence, (a) corresponds to (2).

(b) $x(t) = \cos(100\pi t - \frac{\pi}{4}) + \cos(400\pi t)$

Here,

$\psi_1(t) = (100\pi t - \frac{\pi}{4})$ and $\psi_2(t) = (400\pi t)$

The instantaneous frequencies are then ,

$f_1(t) = 50$ Hz and $f_2(t) = 200$ Hz

Hence, (b) corresponds to (5).

(c) $x(t) = \cos(100\pi t)\cos(400\pi t)$

$\implies x(t) = 0.5[\cos(500\pi t) + \cos(300\pi t)]$

Here,

$\psi_1(t) = (500\pi t)$ and $\psi_2(t) = (300\pi t)$

The instantaneous frequencies are then ,

$f_1(t) = 250$ Hz and $f_2(t) = 150$ Hz

Hence, (c) corresponds to (1).

(d) $x(t) = \cos(200\pi t^2)$

$\psi(t) = 200\pi t^2$. To be able to find spectrogram, compute the instantaneous frequency defined as, $f_i(t) = \frac{1}{2\pi} \frac{d\psi(t)}{dt}$

For (d) the instantaneous frequency is $200t$ Hz.

Hence, (d) corresponds to (6).

(e) $x(t) = \cos(30e^{2t})$

$\psi(t) = 30e^{2t}$. The instantaneous frequency is then $f(t) = \frac{30e^{2t}}{\pi}$ Hz. Hence, (e) corresponds to (3)