Problem 3.22

(a) Given $f_1 = 4800$ Hz , $f_2 = 800$ Hz and $t_1 = 0$ s to $t_2 = 2$ s, Mathematical formula for instantaneous frequency,

$f_i(t) = 2\mu t + f_0$ -(A)

Then $f_1(t_1) = 2\mu t_1 + f_0$, since $t_1 = 0$, $f_1 = f_0$

Similarly, $f_2(t_2) = 2\mu t_2 + f_1 = 4\mu + f_1$

Hence , this implies $\mu = \frac{(f_2 - f_1)}{4}$.

Substitute this in (A),

$f_i(t) = \frac{(f_2 - f_1)}{2} t + f_1$ is the mathematical formula.

$\psi(t) = 2\pi\mu t^2 + 2\pi f_0 t + \phi = -2000\pi t^2 + 9600\pi t + \phi$

Hence, the chirp signal is :

$x(t) = cos(-2000\pi t^2 + 9600\pi t + \phi)$

(b) $y(t) = cos(40\pi t^2 + 500\pi t - \frac{\pi}{4}) - (1)$, $t = [0, 3]s$

$f_i(t) = \frac{1}{2\pi} \frac{d\psi(t)}{dt}$

The quadratic angle function, $\psi(t)$, of (1) is

$400\pi t^2 + 500\pi t - \frac{\pi}{4}$. Then the instantaneous frequency is from (A) , $f_i(t) = 400t + 250$

Starting instantaneous frequency, at t = 0 s,

$f_i(0) = 400(0) + 250 = 250$ Hz

Ending instantaneous frequency, at t = 3 s,

$$f_i(3) = 400(3) + 250 = 1450 \text{ Hz}$$