

## Problem 2.5

(a)  $\cos(\theta_1 + \theta_2)$  is expressed in terms of the real part of the complex exponential, which can be derived from Euler's formula,  $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$$\text{Hence, } \cos(\theta_1 + \theta_2) = \Re\{e^{j(\theta_1+\theta_2)}\} = \Re\{e^{j(\theta_1)}e^{j(\theta_2)}\}$$

$$\begin{aligned}\Re\{e^{j(\theta_1)}e^{j(\theta_2)}\} &= \Re\{(\cos(\theta_1) + j\sin(\theta_1))(\cos(\theta_2) + j\sin(\theta_2))\} \\ &= \Re\{\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) + j(\sin(\theta_1)\cos(\theta_2) + \sin(\theta_2)\cos(\theta_1))\} \\ &= \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) \quad (\text{proved})\end{aligned}$$

$$(b) \cos(\theta_1 - \theta_2) = \Re\{e^{j(\theta_1-\theta_2)}\} = \Re\{e^{j(\theta_1)}e^{-j(\theta_2)}\}$$

$$\begin{aligned}\Re\{e^{j(\theta_1)}e^{-j(\theta_2)}\} &= \Re\{(\cos(\theta_1) + j\sin(\theta_1))(\cos(\theta_2) - j\sin(\theta_2))\} \\ &= \Re\{\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2) + j(\sin(\theta_1)\cos(\theta_2) - \sin(\theta_2)\cos(\theta_1))\} \\ &= \cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2) \quad (\text{proved})\end{aligned}$$