Problem 2.4

Given in the question,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$$

Then,

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + ...$$

It is known that,

$$j = \sqrt{-1}$$

Hence,

$$e^{j\theta} = 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + ... \qquad (1)$$

$$cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + ...$$

and

$$sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + ...$$

are given in the question. Rearranging terms in equation (1) yields,

$$e^{j\theta} = (1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - ....) + j(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + ...)$$

Therefore, $e^{j\theta} = cos(\theta) + jsin(\theta)$, Euler's formula is verified.