Problem 2.20

(a) The two equations:

$cos(7t) = A_1 cos(7t + \phi_1) + A_2 cos(7t + \phi_2)$ $-(1)$

$sin(7t) = 2A_1 cos(7t + \phi_1) + A_2 cos(7t + \phi_2)$ $-(2)$

can be represented in phasor form as:

$1 = A_1 e^{j(\phi_1)} + A_2 e^{j(\phi_2)}$ $-(3)$

$e^{-j\frac{\pi}{2}} = A_1 e^{(j\phi_1)} + A_2 e^{(j\phi_2)}$ $-(4)$

This is the representation because sin function and cos function are $\frac{\pi}{2}$ phase apart.

(b) Let $z_1 = A_1 e^{(j\phi_1)}$ and $z_2 = A_2 e^{(j\phi_2)}$

Then equations (3) and (4) can be written as:

$1 = z_1 + z_2$ $-(5)$

$-j = 2z_1 + z_2$ $-(6)$

(c) Solving simultaneous pair of linear equations (5) and (6) gives:

$z_2 = 2 + j$ and $z_1 = 1 - z_2 = -1 - j$.

Representing in polar form gives, $z_1 = \sqrt{2}e^{-j\frac{3\pi}{4}}$ or $z_1 = \sqrt{2}e^{j\frac{5\pi}{4}}$.

$z_2 = \sqrt{5}e^{j0.4637}$

(d) Hence, $A_1 = \sqrt{2}$ and $\phi_1 = -\frac{3\pi}{4}$ or $\frac{5\pi}{4}$ radians and $A_2 = \sqrt{5}$ and $\phi_2 = 0.4637$ radians