

Problem 2.17

Given signal :

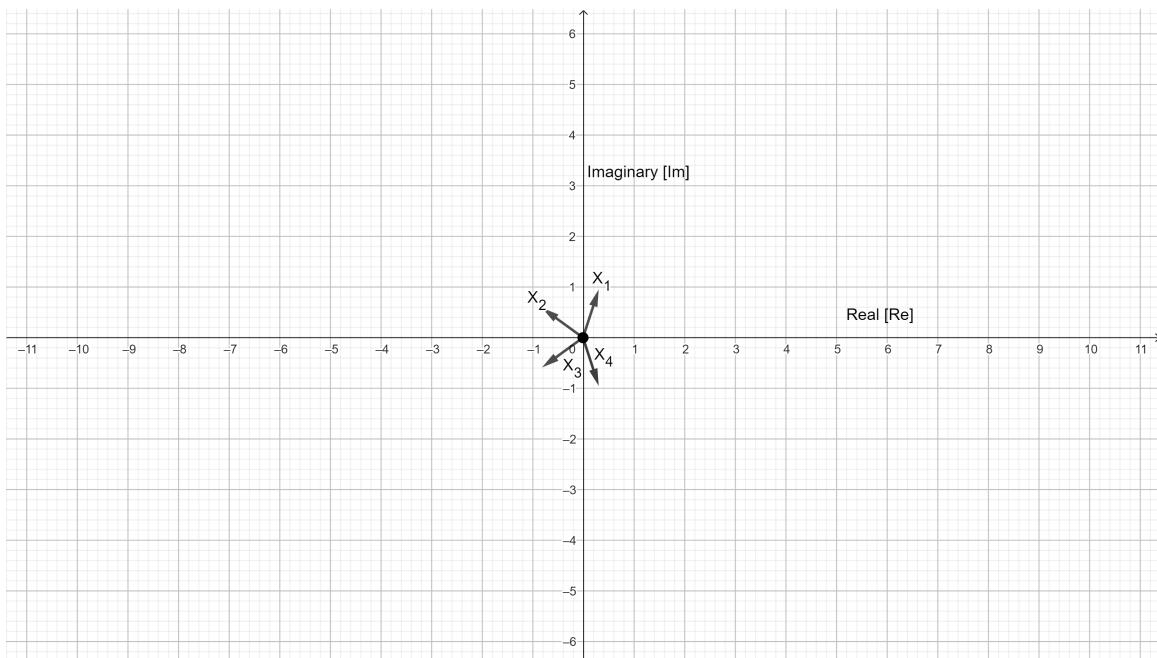
$$\sum_{k=0}^4 \cos(\omega t + \frac{2}{5}\pi k)$$

(a) Representation in terms of Phasors:

$$\sum_{k=0}^4 \cos(\omega t + \frac{2}{5}\pi k) = 1 + e^{j\frac{2}{5}\pi} + e^{j\frac{4}{5}\pi} + e^{j\frac{6}{5}\pi} + e^{j\frac{8}{5}\pi}$$

Phasors in vector plane:

$$X_1 = e^{j\frac{2}{5}\pi}, X_2 = e^{j\frac{4}{5}\pi}, X_3 = e^{j\frac{6}{5}\pi}, X_4 = e^{j\frac{8}{5}\pi}$$



(b) Required to express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$.

$$x(t) = \cos(\omega t) + \cos(\omega t + \frac{2}{5}\pi) + \cos(\omega t + \frac{4}{5}\pi) + \cos(\omega t + \frac{6}{5}\pi) + \cos(\omega t + \frac{8}{5}\pi) \quad - (1)$$

$$\text{where, } \cos(\omega t + \frac{6}{5}\pi) + \cos(\omega t + \frac{8}{5}\pi) = 2\cos(\omega t + \frac{7}{5}\pi)\cos(\frac{2\pi}{5})$$

$$\text{and } \cos(\omega t + \frac{2}{5}\pi) + \cos(\omega t + \frac{4}{5}\pi) = 2\cos(\omega t + \frac{3}{5}\pi)\cos(\frac{2\pi}{5})$$

Using the trigonometric identity,

$\cos A + \cos B = 2\cos((A + B)/2)\cos((A - B)/2)$, (1) can be written as:

$$\begin{aligned} x(t) &= \cos(\omega t) + 2\cos(\frac{2}{5}\pi)[\cos(\omega t + \frac{7}{5}\pi) + \cos(\omega t + \frac{3}{5}\pi)] \\ \implies x(t) &= \cos(\omega t) + 4\cos(\frac{2}{5}\pi) + \cos(\frac{4}{5}\pi) + \cos(\omega t + \pi) \end{aligned}$$

Hence,

$$x(t) = [1 - 4\cos(\frac{2}{5}\pi)\cos(\frac{4}{5}\pi)](\cos(\omega t))$$

$$A = [1 - 4\cos(\frac{2}{5}\pi)\cos(\frac{4}{5}\pi)]$$