Problem 2.15

Given, $x(t) = 2cos(\omega t+5)+8cos(\omega t+9)+4cos(\omega t)$. Required to express x(t) in the form $x(t) = Acos(\omega t + \phi)$.

Let, $x_1(t) = 2cos(\omega t + 5)$

$x_2(t) = 8cos(\omega t + 9)$

$x_3(t) = 4cos(\omega t)$

Step 1: Represent $x_1(t), x_2(t)$ and $x_3(t)$ by the phasors:

$X_1 = 2e^{j5}$, $X_2 = 8e^{j9}$, $X_3 = 4e^{j10}$.

Step 2 : Convert X_1,X_2,X_3 to rectangular form:

Method :

$X_1 = 2e^{j5}$ can be converted to form a + ib

where $a = Acos\phi$ and $b = Asin\phi$; A is amplitude of the signal, for X_1, $A = 2$ and $\phi = 5$.

Hence,

$X_1 = 0.567 + j(-1.9178)$

$X_2 = -7.2890 + j(3.29)$

$X_3 = 4 + j0$

Step 3 : Add the phasors

$X_4 = X_1 + X_2 + X_3 = -2.722 + j(1.3722)$

Step 4: Convert back to Phasor Form

$X_4 = 3.048e^{j2.674}$

Step 5: Expressing the Complex Exponential in the form $x(t) = Acos(\omega t + \phi)$:

$3.048e^{j2.674} = 3.048cos(\omega t + 2.674)$

Phasor diagram: