Problem 10.5

Given, the impulse response of LTI system is $h[n] = 10(\sqrt{2})^{-n}u[n]$ and input signal is $x[n] = \delta[n] + \beta \delta[n-2] + \delta[n-4]$. Required to find value of β so output y[n] = 0 for $n \ge 4$.

$$\begin{split} y[n] &= x[n] * h[n] = h[n] + \beta h[n-2] + h[n-4] \\ &= 10(\sqrt{2})^{-n} u[n] + \beta 10(\sqrt{2})^{-(n-2)} u[n-2] + 10(\sqrt{2})^{-(n-4)} u[n-4] \\ &\text{For } n \ge 4, u[n] = u[n-2] = u[n-4] = 1. \\ &\text{Then, } y[n] = 10(\sqrt{2})^{-n} [1 + \beta 10(\sqrt{2})^2 + (\sqrt{2})^4] \\ &[1 + \beta 10(\sqrt{2})^2 + (\sqrt{2})^4] \text{ should be zero for } y[n] = 0. \\ &\text{Therefore, } 1 + 2\beta + 4 = 0 \implies \beta = -2.5 \end{split}$$