

Problem 10.15

Given IIR filter defined by the difference equation :

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$

(a) According to the question, $x[n] = u[n]$. Taking z-transform $X(z) = \frac{1}{1-z^{-1}}$.

Taking z- transform of $y[n]$ gives, $Y(z) = \frac{1}{2}Y(z)z^{-1} + X(z)$

Thus, $\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$.

Hence, $Y(z) = H(z)X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \frac{1}{1-z^{-1}}$.

From partial fractions, $Y(z)$ can be expressed as: $Y(z) = \frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1-z^{-1})}$, where $A = -1$ and $B = 2$.

Hence, $y[n] = 2u[n] - \frac{1}{2}^n u[n]$.