Problem 10.13

$(a) X_a(z) = \frac{1-z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$

$X_a(z)$ can also be represented as:

$\frac{z(z-1)}{z^2-\frac{1}{6}z-\frac{1}{6}} = \frac{z(z-1)}{(z-\frac{1}{2})(z+\frac{1}{3})}$

Using partial fractions,

$X_a(z) = \frac{A}{(z-\frac{1}{2})} + \frac{B}{(z+\frac{1}{3})}$

On solving for the constants yields, $B = 1.6$ and $A = -0.6$

Then, $X_a(z) = \frac{-0.6}{(z-\frac{1}{2})} + \frac{1.6}{(z+\frac{1}{3})}$

Hence, $x_a[n] = -(0.6)(\frac{1}{2})^n u[n] + 1.6(\frac{-1}{3})^n u[n]$,

$(b) X_b(z) = \frac{1+z^{-1}}{1+0.9z^{-1}+0.81z^{-2}}$

Let $z^{-1} = x$, then,$X_b(z = x^{-1}) = \frac{1+x^2}{1+0.9x+0.81x^2}$. The roots of the denominator term are: $x_1 = -0.56 + 0.96j$ and $x_2 = -0.56 - 0.96j$. The product of the roots is $x_1 x_2 = \frac{1}{0.81}$.

Converting x_1 and x_2 to polar coordinates yields: $x_1 = 1.11e^{-j\frac{2\pi}{3}}$ and $x_2 = 1.11e^{j\frac{2\pi}{3}}$, then $\frac{1}{x_1} = 0.90e^{j\frac{2\pi}{3}}$ and $\frac{1}{x_2} = 0.90e^{-j\frac{2\pi}{3}}$.

$X_b(z = x^{-1})$ can be expressed as $X_b(z = x^{-1}) = \frac{(1+x^2)}{x_1 x_2(1-x/x_1)(1-x/x_2)}$

Substituting values as found above, $X_b(z) = \frac{0.81(1+z^{-2})}{(1-0.90e^{j1.042})(1-0.90e^{-j1.042})}$

This can be expressed in partial fractions as: $X_b(z) = A + \frac{B}{(1-0.90e^{j1.042})} + \frac{C}{(1-0.90e^{-j1.042})}$

Hence, the equations formed in terms of the constants A, B, C: $A + B + C = 0.81$, $A0.81z^{-2} = 0.81z^{-2}$, and $A[-0.9e^{-j\frac{2\pi}{3}} - -0.90e^{j\frac{2\pi}{3}})]$ $-0.9e^{-j\frac{2\pi}{3}}B - 0.9e^{j\frac{2\pi}{3}}C = 0$.

Solving the three equations yields the constant values, $A = 1$, $B = 1.167e^{-j1.49}$ and $C = 1.167e^{j1.49}$.

Hence,

$X_b(z) = 1 + \frac{1.167e^{-j1.49}}{(1-0.90e^{j1.042})} + \frac{1.167e^{j1.49}}{(1-0.90e^{j1.042})}$

Thus,

$x_b[n] = \delta[n]+1.167e^{-j1.49}(0.90e^{j1.042})^n u[n]+1.167e^{j1.49}(0.90e^{0j1.042})^n u[n]$

$(c)X_c(z) = \frac{1+z^{-1}}{1+0.1z^{-1}-0.72z^{-2}} = \frac{A}{(1-0.9z^{-1})} + \frac{B}{(1+0.8z^{-1})}$

$\implies (1 + z^{-1}) = A(1 + 0.8z^{-1}) + B(1 - 0.9z^{-1})$

Equating the coefficients on RHS and LHS: $A + B = 1$ and $0.8A - 0.9B = 1$.

On solving the simultaneous pair of equations, $A = 1.11764$ and$B = -0.11764$.

Hence, $X_c(z) = \frac{1.11764}{(1-0.9z^{-1})} + \frac{(-0.11764)}{(1+0.8z^{-1})}$.