History of Starting with Signal Processing (SP) First

Classically (e.g. 1950s to 1990s),
Electrical Engineering started with circuits
Signal processing through circuits (historical start)

In the 1990s, Digital SP: 4th year or Grad course

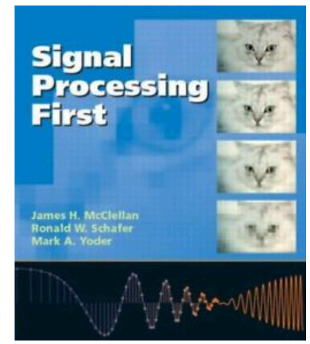
Why not start with Signal Processing?
Signal Processing first,
then circuits enabled through SP

Ron Schafer

Jim McClellan

And others at GT, those who founded DSP
and some good friends at Rice, Rose-Hulman

Fall 1999 at GT:
Full implementation
Requirement for
all ECE students



SP First (GT 1999)

Each week:
- Two large lecture sessions
- Faculty Led Recitation Session
- Weekly computer (MATLAB) exercises

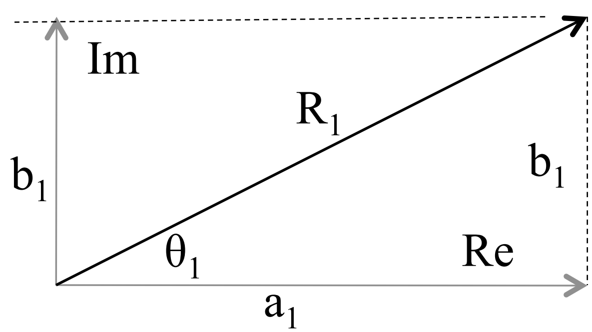
Complex Numbers = Real + j $\underbrace{\text{Imaginary}}_{\text{like a unicorn}}$

$$j = \sqrt{-1}$$

Unicorn → divide by "j" ⟶

$$C_1 = \underbrace{a_1 + jb_1}_{\text{(rectangular)}} = \underbrace{R_1 e^{j\theta_1}}_{\text{(polar)}}$$

$$C_2 = a_2 + jb_2 = R_2 e^{j\theta_2}$$



$$a_1 = R_1 \cos\theta_1$$

$$b_1 = R_1 \sin\theta_1$$

$$R_1 = \sqrt{a_1^2 + b_1^2}$$

$$\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

Complex Operations

Complex Addition: (rectangular)

$$C_1 + C_2 = (a_1 + jb_1) + (a_2 + jb_2) = a_1 + a_2 + j(b_1 + b_2)$$

Complex Multiplication: (polar)

$$C_1 C_2 = R_1 e^{j\theta_1} R_2 e^{j\theta_2} = R_1 R_2 e^{j(\theta_1 + \theta_2)}$$

One often converts between rectangular and polar forms

Euler's Formula:

$$\underline{e^{j\theta} = \cos\theta + j\sin\theta} \qquad j = \sqrt{-1}$$

$$e^{j0} = e^{j2\pi} = -e^{j\pi} = 1$$

$$e^{j\pi/2} = -e^{j3\pi/2} = j$$

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta \rightarrow \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta \rightarrow \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\begin{aligned}\cos^2(\theta) &= \left(\frac{e^{j\theta} + e^{-j\theta}}{2}\right)^2\\ &= \frac{e^{j2\theta} + 2 + e^{-j2\theta}}{4}\\ &= \frac{1 + \cos(2\theta)}{2}\end{aligned}$$

$$\begin{aligned}\sin^2(\theta) &= \left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)^2\\ &= \frac{-e^{j2\theta} + 2 - e^{-j2\theta}}{4}\\ &= \frac{1 - \cos(2\theta)}{2}\end{aligned}$$

$$\begin{aligned}\cos(\theta)\sin(\theta) &= \left(\frac{e^{j\theta} + e^{-j\theta}}{2}\right)\left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)\\ &= \frac{-e^{j2\theta} - e^{-j2\theta}}{4j}\\ &= -\sin(2\theta)/2\end{aligned}$$

$$\begin{aligned}\cos^3(\theta) &= \left(\frac{e^{j\theta} + e^{-j\theta}}{2}\right)^3\\ &= \frac{1}{8}\left(e^{j3\theta} + 3e^{j\theta} + 3e^{-j\theta} + e^{-j3\theta}\right)\\ &= \frac{1}{8}\left(e^{j3\theta} + e^{-j3\theta} + 3e^{j\theta} + 3e^{-j\theta}\right)\\ &= \frac{3}{4}\cos(\theta) + \frac{1}{4}\cos(3\theta)\end{aligned}$$

$$\begin{aligned}\sin^3(\theta) &= \left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)^3\\ &= \frac{1}{8j}\left(e^{j3\theta} - e^{-j3\theta} - 3e^{j\theta} + 3e^{-j\theta}\right)\\ &= \frac{1}{8j}\left(e^{j3\theta} - 3e^{j\theta} + 3e^{-j\theta} - e^{-j3\theta}\right)\\ &= \frac{1}{4}\sin(3\theta) - \frac{3}{4}\sin(\theta)\end{aligned}$$

Sinusoidal functions

$$y_1(t) = \underbrace{A_1}_{\text{(Amplitude)}} \cos(2\pi f t + \underbrace{\theta_1}_{\text{(phase)}}) \rightarrow A_1 e^{j(2\pi f t + \theta_1)}$$

f = frequency of the sinusoid (Hz)

T = 1/f = period of the sinusoid (s)

2π f = radians of the sinusoid (rad)

$$Re\left\{A_1 e^{j(2\pi f t+\theta_1)}\right\} = Re\left\{A_1 \cos(2\pi f t+\theta_1) + j\sin(2\pi f t+\theta_1)\right\} =$$
$$= A_1 \cos(2\pi f t + \theta_1)$$

$$y_2(t) = A_2 \cos(2\pi f t + \theta_2) \rightarrow A_2 e^{j(2\pi f t + \theta_2)}$$

Summation (y_1(t) + y_2(t)):

(Same frequency)

$$\boxed{Re\left\{\sum_{k=1}^{n} X_k\right\} = \sum_{k=1}^{n} Re\left\{X_k\right\}}$$

$$y_1(t) + y_2(t) = A_1 \cos(2\pi f t + \theta_1) + A_2 \cos(2\pi f t + \theta_2)$$
$$= Re\left\{A_1 e^{j(2\pi f t+\theta_1)} + A_2 e^{j(2\pi f t+\theta_2)}\right\}$$
$$= Re\left\{\left(A_1 e^{j\theta_1} + A_2 e^{j\theta_2}\right) e^{j2\pi f t}\right\}$$
$$Ae^{j\theta} = A_1 e^{j\theta_1} + A_2 e^{j\theta_2}$$
$$y_1(t) + y_2(t) = Re\left\{Ae^{j\theta} e^{j2\pi f t}\right\} = A\cos(2\pi f t + \theta)$$

$$y_1(t) + y_2(t) = 5\cos(200t + \pi/3)$$
$$+5\sin(200t + \pi/6)$$
$$= Re\left\{5e^{j(200t+\pi/3)} + 5e^{j(200t+2\pi/3)}\right\}$$
$$= Re\left\{\left(5e^{j\pi/3} + 5e^{j2\pi/3}\right) e^{j200t}\right\}$$
$$= Re\left\{\left(5\sqrt{3}e^{j\pi/2}\right) e^{j200t}\right\}$$
$$= 5\sqrt{3}\cos(200t + \pi/2)$$

Linear Functions: f(x)

$$f(ax+by) = af(x) + bg(y)$$

$$f\left(ag(x) + bh(y)\right) = af\left(g(x)\right) + bf\left(g(y)\right)$$

Example functions

Differentiation

$$\frac{d}{dt}\left(ag(x) + bh(y)\right) = a\frac{d}{dt}g(x) + b\frac{d}{dt}h(y)$$

Integration

$$\int \left(ag(x) + bh(y)\right) dt = a\int g(x)dt + b\int h(y)dt$$

Linear gain factor:

$$f(ax) = af(x) \rightarrow \frac{d}{dt}\left(ag(x)\right) = a\frac{d}{dt}g(x), \int \left(ag(x)\right) dt = a\int g(x)dt$$

Linear (Time-Independent) Systems are characterized by sinusoids and exponentials

Single frequency input → Single frequency ouput

$$\sin(\omega t) \rightarrow \boxed{\text{Linear System}} \rightarrow H\sin(\omega t + \theta)$$