**BLOCK DIAGRAM OF A DIGITAL-ANALOG CONVERTER**

$b_1$ is the most significant bit (MSB)

The MSB is the bit that has the most (largest) influence on the analog output

$b_N$ is the least significant bit (LSB)

The LSB is the bit that has the least (smallest) influence on the analog output
10.1 - CHARACTERIZATION OF DIGITAL-ANALOG CONVERTERS

STATIC CHARACTERISTICS

OUTPUT-INPUT CHARACTERISTICS

Ideal input-output characteristics of a 3-bit DAC

![Graph showing ideal input-output characteristics of a 3-bit DAC]
DEFINITIONS

• Resolution of the DAC is equal to the number of bits in the applied digital input word.

• The full scale (FS):

\[
FS = \text{Analog output when all bits are 1} - \text{Analog output all bits are 0}
\]

\[
FS = \left( V_{\text{REF}} - \frac{V_{\text{REF}}}{2^N} \right) - 0 = V_{\text{REF}} \left( 1 - \frac{1}{2^N} \right)
\]

• Full scale range (FSR) is defined as

\[
FSR = \lim_{N \to \infty} FS = V_{\text{REF}}
\]

• Quantization Noise is the inherent uncertainty in digitizing an analog value with a finite resolution converter.
MORE DEFINITIONS

- **Dynamic Range (DR)** of a DAC is the ratio of the FSR to the smallest difference that can be resolved (i.e. an LSB)

\[ DR = \frac{FSR}{\text{LSB change}} = \frac{FSR}{(FSR/2^N)} = 2^N \]

or in terms of decibels

\[ DR(\text{dB}) = 6.02N \text{ (dB)} \]

- **Signal-to-noise ratio (SNR)** for the DAC is the ratio of the full scale value to the \( \text{rms} \) value of the quantization noise.

\[
\text{rms(quantization noise)} = \sqrt{\frac{1}{T} \int_{0}^{T} \left( \frac{t}{T} - 0.5 \right)^2 dt} = \frac{LSB}{\sqrt{12}} = \frac{FSR}{2^N\sqrt{12}}
\]

\[
\therefore \quad \text{SNR} = \frac{v_{\text{OUT} (\text{rms})}}{(FSR/\sqrt{12} 2^N)}
\]

- **Maximum SNR (SNR\text{max})** is defined as

\[
\text{SNR}_{\text{max}} = \frac{v_{\text{OUT}_{\text{max}} (\text{rms})}}{(FSR/\sqrt{12} 2^N)} = \frac{FSR(2\sqrt{2})}{FSR(\sqrt{12} 2^N)} = \frac{\sqrt{6} 2^N}{2}
\]

or in terms of decibels

\[
\text{SNR}_{\text{max}}(\text{dB}) = 20 \log_{10} \left( \frac{\sqrt{6} 2^N}{2} \right) = 10 \log_{10}(6) + 20 \log_{10}(2^N) - 20 \log_{10}(2)
\]

\[
= 7.78 \text{ dB} - 6.02 \text{ dB} + 6.02N \text{ dB} = 1.76 \text{ dB} + 6.02N \text{ dB}
\]
EVEN MORE DEFINITIONS

- *Effective number of bits (ENOB)* can be defined from the above as

\[
ENOB = \frac{SNR_{Actual} - 1.76}{6.02}
\]

where \(SNR_{Actual}\) is the actual SNR of the converter.

Comment:

The \(DR\) is the amplitude range necessary to resolve \(N\) bits regardless of the amplitude of the output voltage.

However, when referenced to a given output analog signal amplitude, the \(DR\) required must include 1.76 dB more to account for the presence of quantization noise.

Thus, for a 10-bit DAC, the \(DR\) is 60.2 dB and for a full-scale, \(rms\) output voltage, the signal must be approximately 62 dB above whatever noise floor is present in the output of the DAC.
OFFSET AND GAIN ERRORS

An offset error is a constant difference between the actual finite resolution characteristic and the infinite resolution characteristic measured at any vertical jump.

A gain error is the difference between the slope of an actual finite resolution and an infinite resolution characteristic measured at the right-most vertical jump.

![Offset Error in a 3-bit DAC](image)

![Gain Error in a 3-bit DAC](image)
INTEGRAL AND DIFFERENTIAL NONLINEARITY

- *Integral Nonlinearity (INL)* is the maximum difference between the actual finite resolution characteristic and the ideal finite resolution characteristic measured vertically (% or LSB).

- *Differential Nonlinearity (DNL)* is a measure of the separation between adjacent levels measured at each vertical jump (% or LSB).

\[
DNL = \left( \frac{V_{cx} - V_s}{V_s} \right) \times 100\% = \left( \frac{V_{cx}}{V_s} - 1 \right) \text{LSBs}
\]

where \(V_{cx}\) is the actual voltage change on a bit-to-bit basis and \(V_s\) is the ideal change (\(V_{FSR}/2^N\)).

Example of a 3-bit DAC:
EXAMPLE OF INL AND DNL OF A NONIDEAL 4-BIT DAC

Find the $\pm INL$ and $\pm DNL$ for the 4-bit DAC shown.
DYNAMIC CHARACTERISTICS OF DIGITAL-ANALOG CONVERTERS

Dynamic characteristics include the influence of time.

DEFINITIONS

• Conversion speed is the time it takes for the DAC to provide an analog output when the digital input word is changed.

Factor that influence the conversion speed:

- Parasitic capacitors (would like all nodes to be low impedance)
- Op amp gainbandwidth
- Op amp slew rate

• Gain error of an op amp is the difference between the desired and actual output voltage of the op amp (can have both a static and dynamic influence)

\[
\text{Actual Gain} = \text{Ideal Gain} \times \left( \frac{\text{Loop Gain}}{1 + \text{Loop Gain}} \right)
\]

\[
\text{Gain error} = \text{Ideal Output} - \text{Actual Output} = \frac{\text{Ideal Gain} - \text{Actual Gain}}{\text{Ideal Gain}} = \frac{1}{1 + \text{Loop Gain}}
\]
EXAMPLE OF INFLUENCE OF OP AMP GAIN ERROR ON DAC PERFORMANCE

Assume that a DAC using an op amp in the inverting configuration with $C_1 = C_2$ and $A_{vd}(0) = 1000$. Find the largest resolution of the DAC if $V_{REF}$ is 1V and assuming worst case conditions.

Solution

The loop gain of the inverting configuration is $LG = \frac{C_2}{C_1+C_2} A_{vd}(0) = 0.5 \cdot 1000 = 500$. The gain error is therefore $1/501 \sim 0.002$. The gain error should be less than the quantization noise of $\pm 0.5$ LSB which is expressed as

$$\text{Gain error} = \frac{1}{501} \sim 0.002 = \frac{V_{REF}}{2^{N+1}}$$

Therefore the largest value of $N$ that satisfies this equation is $N = 7$. 
INFLUENCE OF THE OP AMP GAINBANDWIDTH

Single-pole response:

\[ v_{out}(t) = A_{CL}[1 - e^{-\omega_H t}]v_{in}(t) \]

where

\[ A_{CL} = \text{closed-loop gain} \]

\[ \omega_H = GB \left( \frac{R_1}{R_1 + R_2} \right) \text{ or } GB \left( \frac{C_2}{C_1 + C_2} \right) \]

To avoid errors in DACs (and ADCs), \( v_{out}(t) \) must be within ±0.5LSB of the final value by the end of the conversion time.

Multiple-pole response:

Typically the response is underdamped like the following (see Appendix C of text).
EXAMPLE OF THE INFLUENCE OF GB AND SETTLING TIME ON DAC PERFORMANCE

Assume that a DAC uses a switched capacitor noninverting amplifier with $C_1 = C_2$ and $GB = 1$MHz. Find the conversion time of an 8-bit DAC if $V_{REF}$ is 1V.

Solution

From the analysis in Secs. 9.2 and 9.3, we know that

$$\omega_H = \left( \frac{C_2}{C_1+C_2} \right) GB = (2\pi)(0.5)(10^6) = 3.141 \times 10^6$$

and $A_{CL} = 1$. Assume that the ideal output is equal to $V_{REF}$. Therefore the value of the output voltage which is 0.5LSB of $V_{REF}$ is

$$1 - \frac{1}{2^{N+1}} = 1 - e^{-\omega_H T}$$

or

$$2^{N+1} = e^{\omega_H T}$$

Solving for $T$ gives

$$T = \left( \frac{N+1}{\omega_H} \right) ln(2) = 0.693 \left( \frac{N+1}{\omega_H} \right) = \left( \frac{9}{3.141} \right) 0.693 = 1.986 \mu s$$
TESTING OF DACs

INPUT-OUTPUT TEST

Test setup:

![Diagram](image)

Comments:

Sweep the digital input word from 000...0 to 111...1.

The ADC should have more resolution by at least 2 bits and be more accurate than the errors of the DAC

\( INL \) will show up in the output as the presence of 1’s in any bit.

If there is a 1 in the Nth bit, the \( INL \) is greater than ±0.5\( LSB \)

\( DNL \) will show up as a change between each successive digital error output.

The bits which are greater than \( N \) in the digital error output can be used to resolve the errors to less than ±0.5\( LSB \)
SPECTRAL TEST

Test setup:

Comments:
Digital input pattern is selected to have a fundamental frequency which has a magnitude of at least 6N dB above its harmonics.
Length of the digital sequence determines the spectral purity of the fundamental frequency.
All nonlinearities of the DAC (i.e. INL and DNL) will cause harmonics of the fundamental frequency.
The THD can be used to determine the SNR dB range between the magnitude of the fundamental and the THD. This SNR should be at least 6N dB to have an INL of less than ±0.5 LSB for an ENOB of N-bits.
Note that the noise contribution of $V_{REF}$ must be less than the noise floor due to nonlinearities.
If the period of the digital pattern is increased, the frequency dependence of INL can be measured.