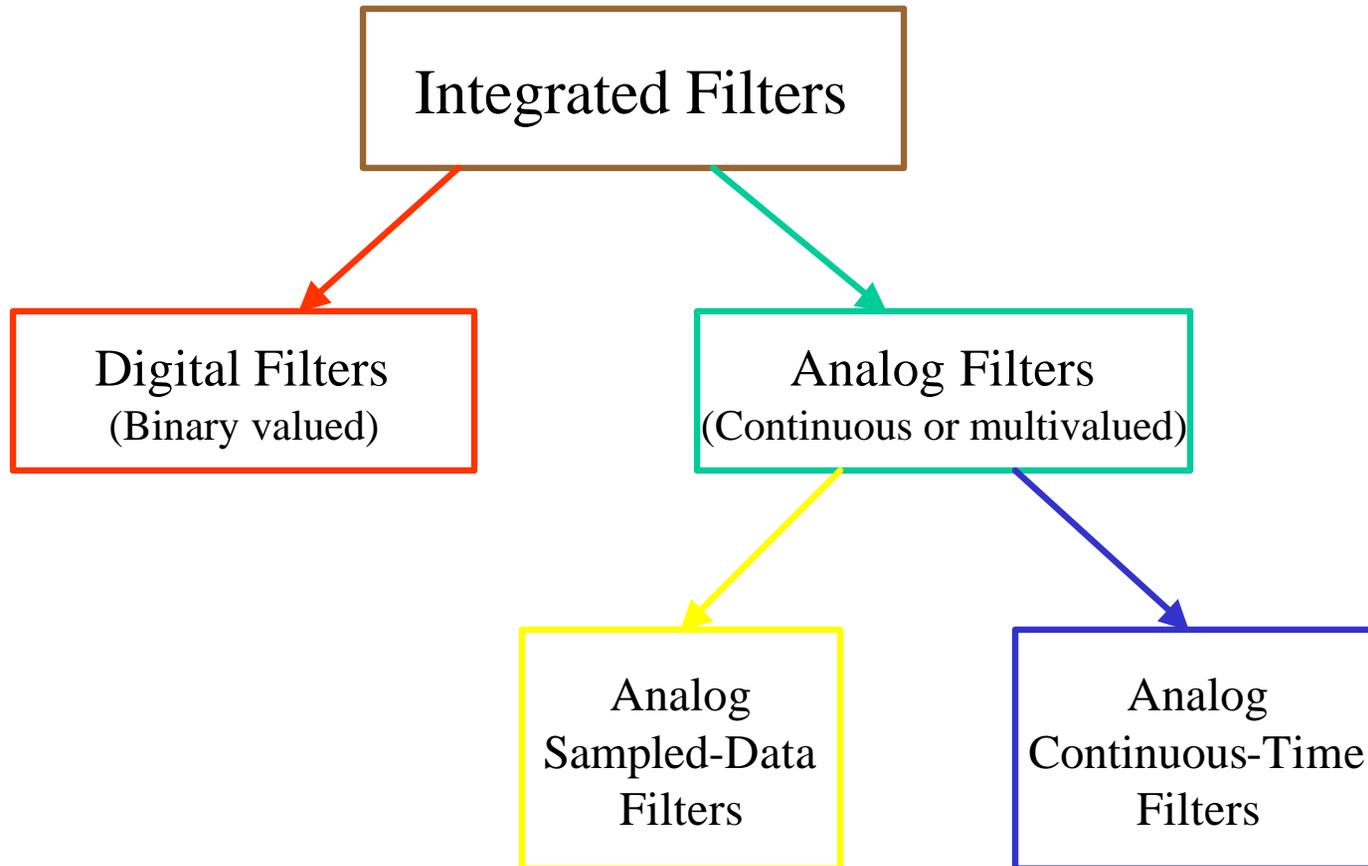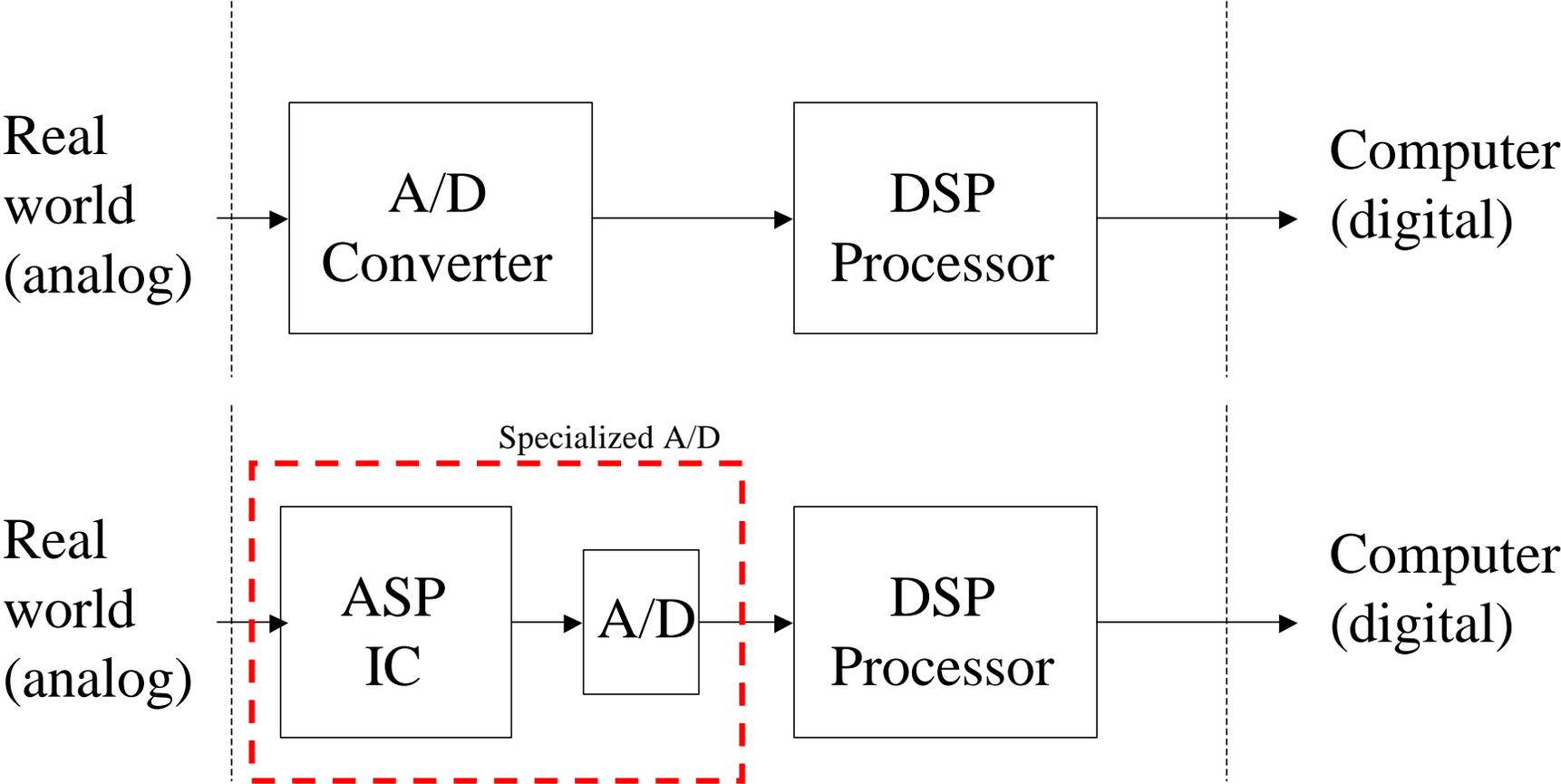# Signal Processing Design of Integrated Analog and Digital Filters

Prof. Paul Hasler

Types of Integrated Filters
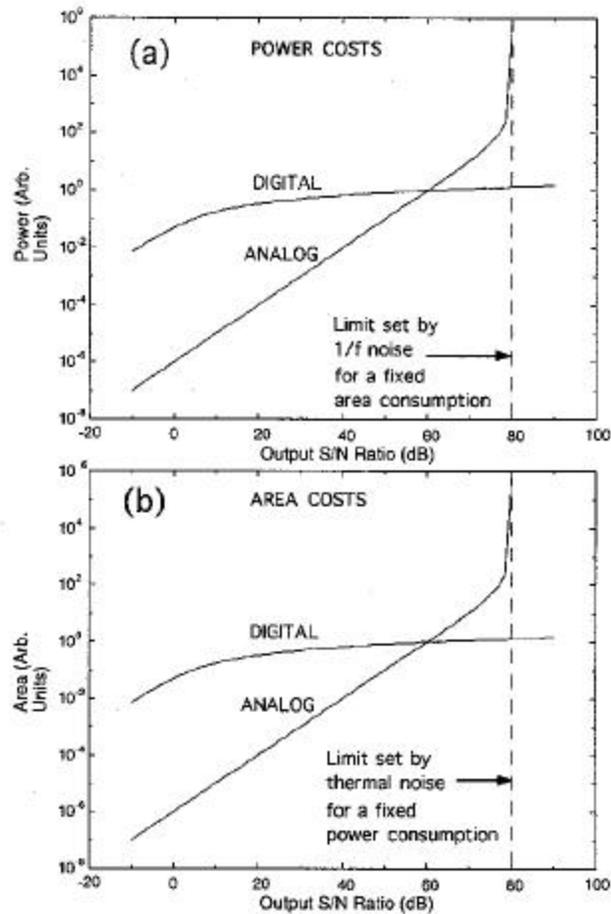
Where to divide Analog and Digital?

Analog-Digital Comparison
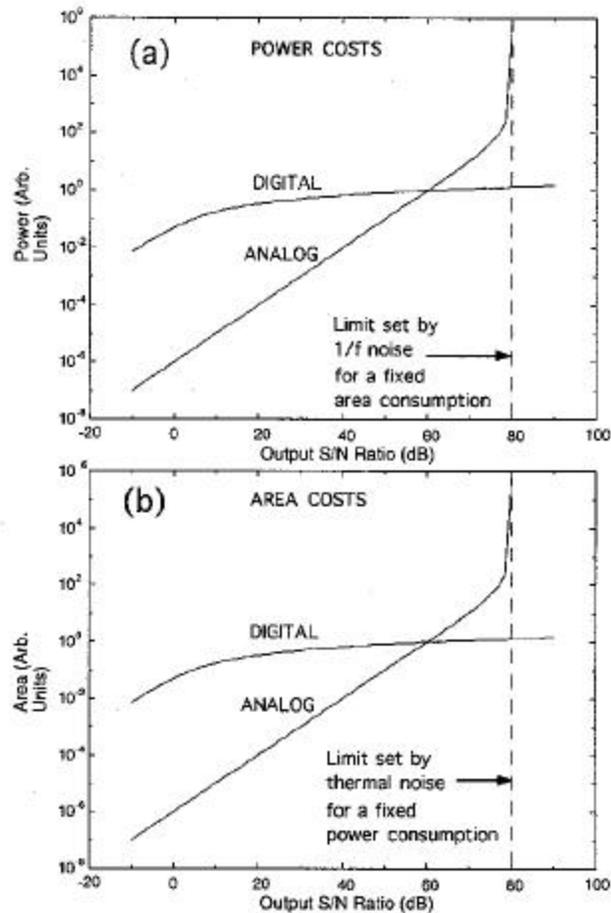
Low SNR: Analog / High SNR: Digital

[Sarpeskar 1997]

Analog-Digital Comparison

Low SNR: Analog / High SNR: Digital

(a) POWER COSTS — DIGITAL, ANALOG, Limit set by 1/f noise for a fixed area consumption

(b) AREA COSTS — DIGITAL, ANALOG, Limit set by thermal noise for a fixed power consumption

[Sarpeskar 1997]

Practical Interpretation of Cost

Input → 16bit A/D → FFT → DSP Application Program (DSP), SNR < 10bits

Input → filter → 10 bit A/D → 10 → DSP Application Program (DSP)
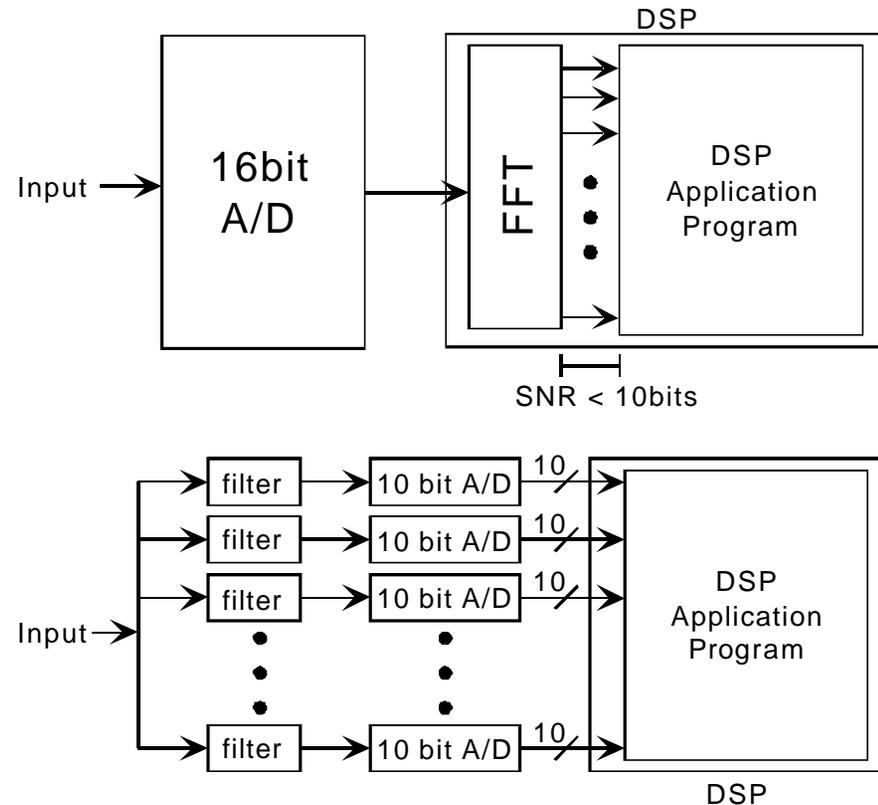
- What if exponentially spaced FFT?

Circuit Issues for Filters

Programmability / Tunability: flexibility and complexity

Available for digital (clocks/ crystals) as well as
some analog (e.g. Floating-Gate) filters

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High Signal-to-Noise Ratio (resolution):

Ratio of the largest signal and the smallest signal

Largest signal: Harmonic Distortion (continuous-time filters), Range limitations
Smallest Signal: Noise

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Insensitivity to environmental fluctuations:
Power-supply: Power Supply Rejection Ratio (**PSRR**)
Temperature, etc.

Not a problem for digital filters,
but can be the cause of several
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Typically, sampling in amplitude / time results in,
- the more complexity is needed (S/H blocks, anti-aliasing filters),
- more power / lower-frequency / area

Design of Analog Filters

- Find the transfer function for a given filter

- "Partition" the transfer function,
  or an approximation,
  into simple parts that can be implemented.

- Implement the transfer function in
  a particular circuit technology

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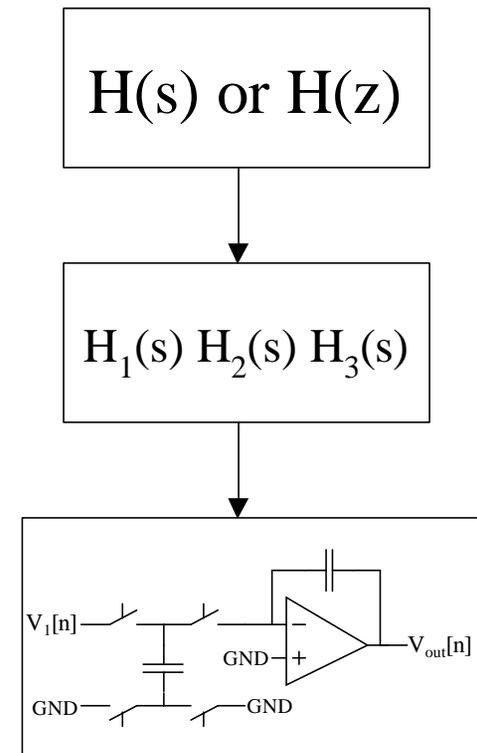   a particular circuit technology
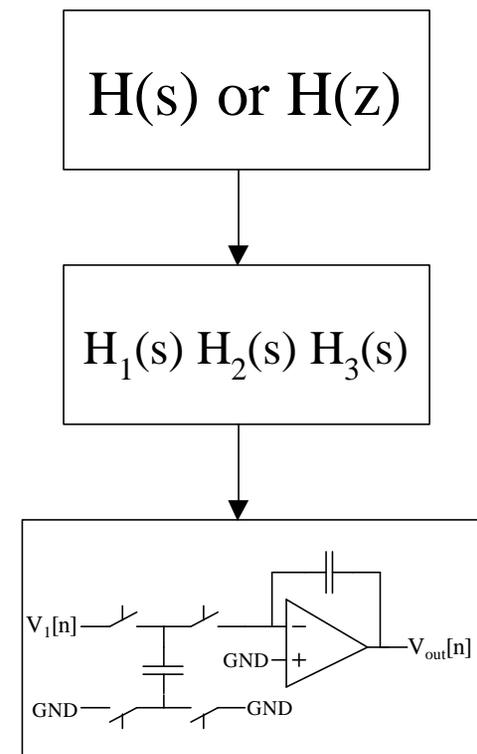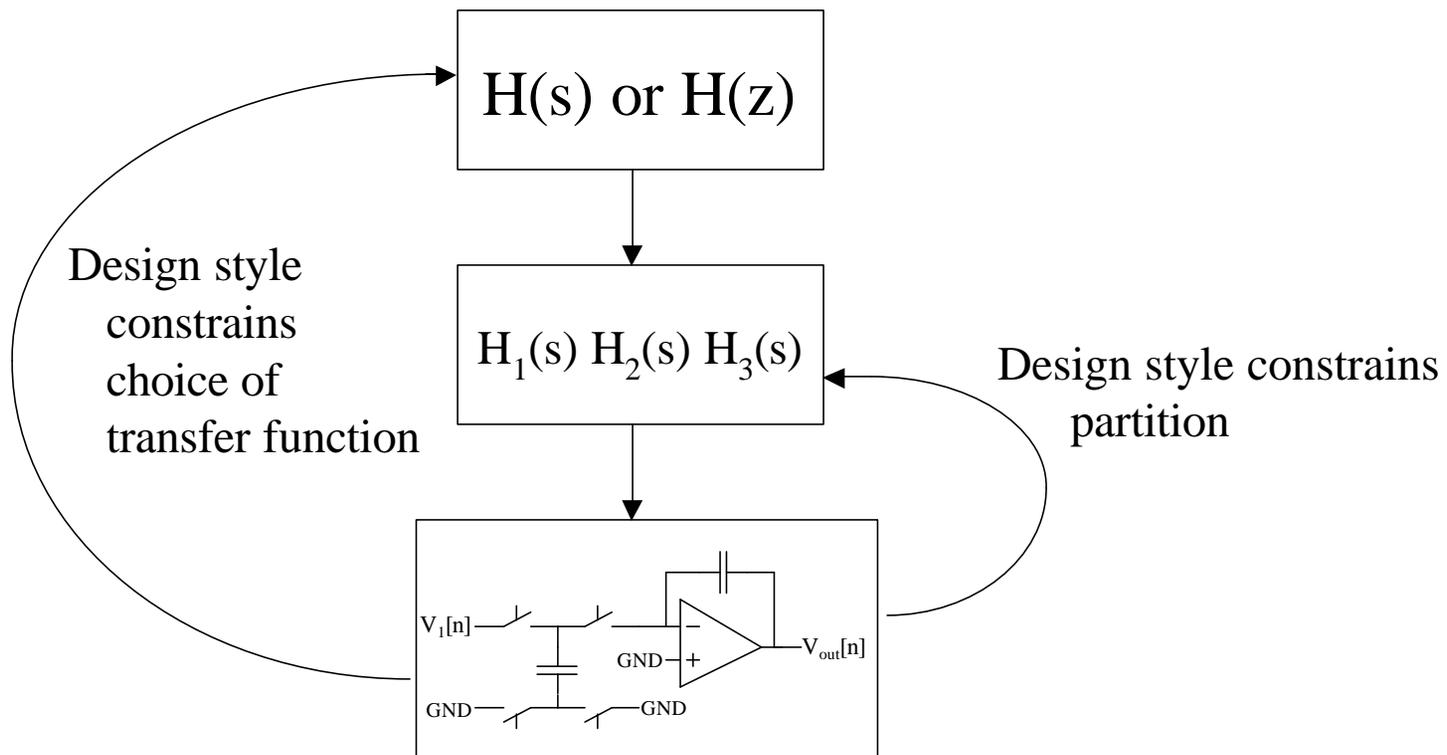
Design of Analog Filters

As basic building blocks we have
- integrators, delay elements
- first-order (low-pass / bandpass)
- second order functions
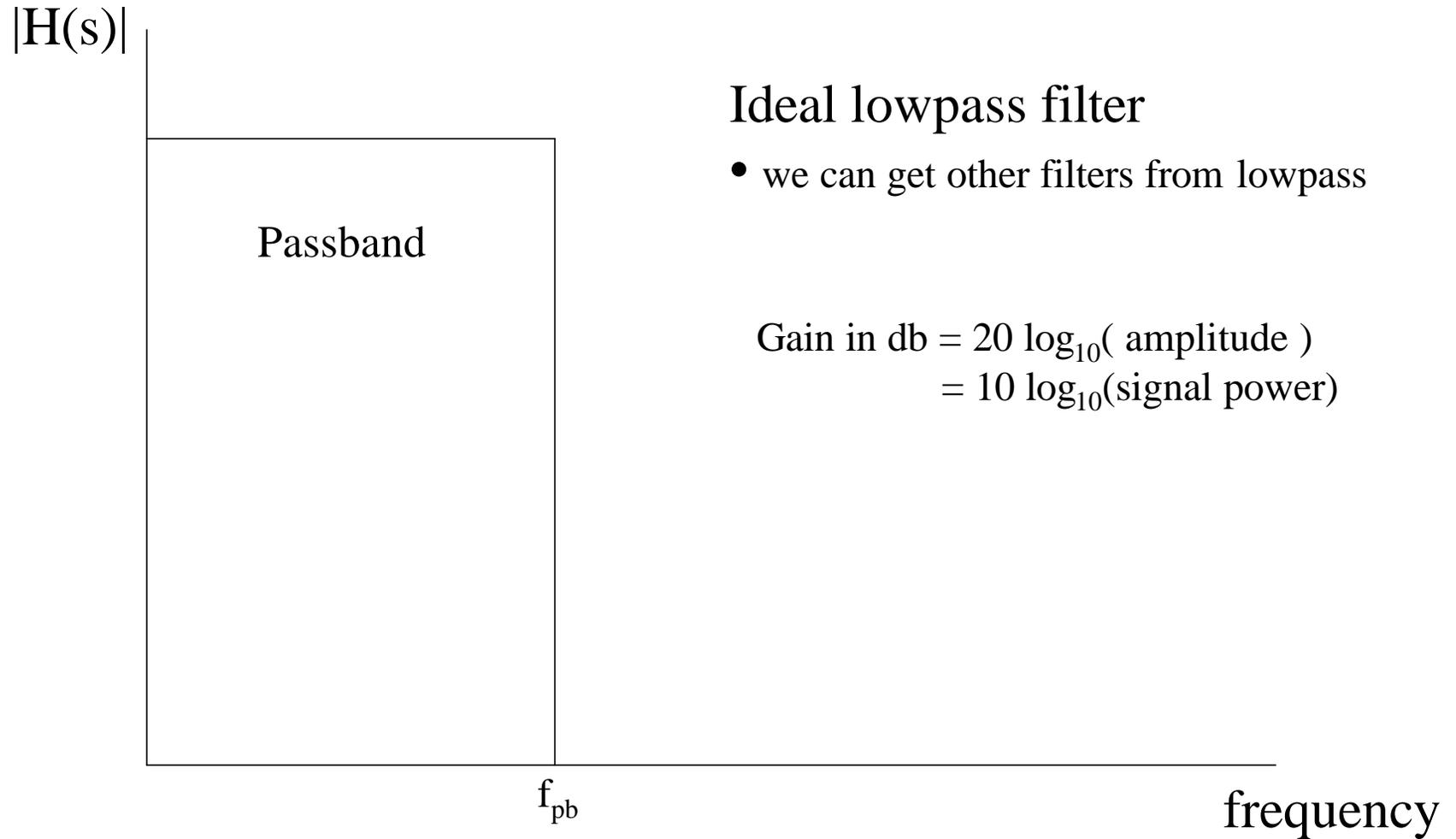
  (low-pass / bandpass / highpass)

The "circuit" design question is

  how to make these functions
  what inputs / outputs / internal variables
  should be voltages / currents, etc.

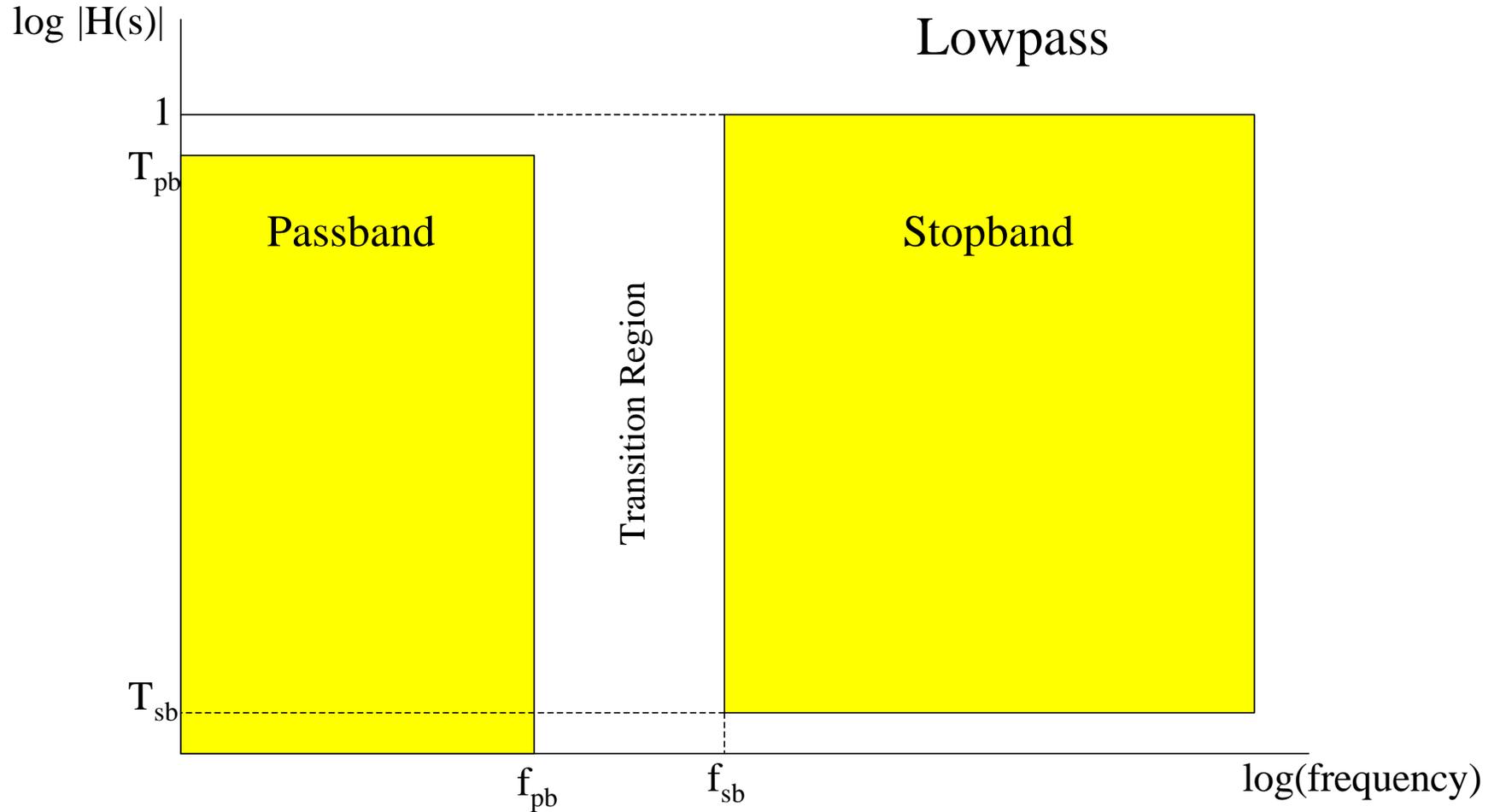# Design of Analog Filters

Choosing H(s) or H(z) for a filter

$|H(s)|$

Ideal lowpass filter

- we can get other filters from lowpass

Passband

Gain in db $= 20 \log_{10}($ amplitude $)$
$= 10 \log_{10}($signal power$)$

f_{pb}

frequency

H(s) or H(z) for a lowpass filter

log |H(s)|

Lowpass

1

T_{pb}

Passband

T_{sb}

Stopband

Transition Region

f_{pb} f_{sb}

log(frequency)

H(s) or H(z) for a lowpass filter
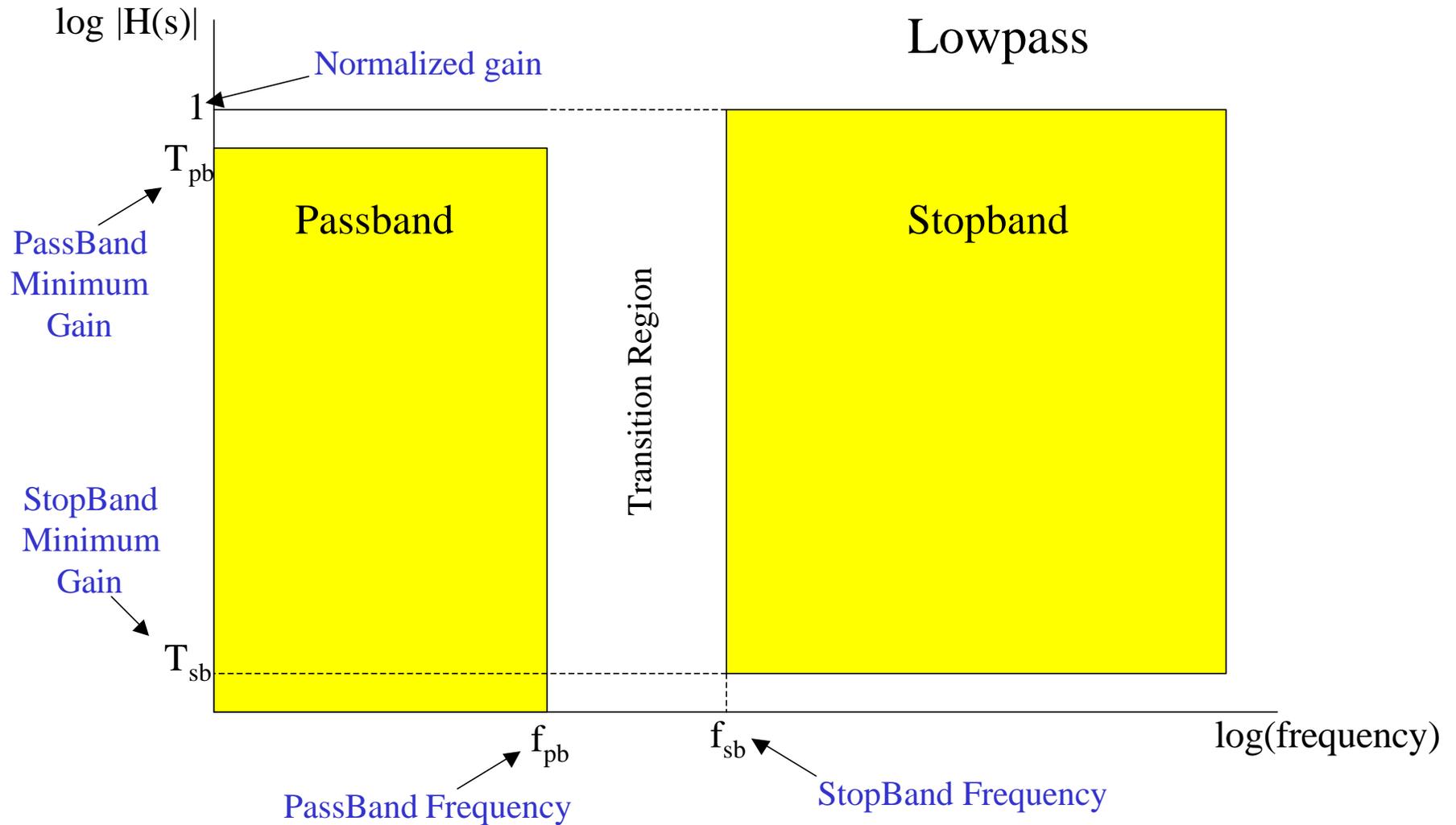
H(s) or H(z) for a Highpass filter
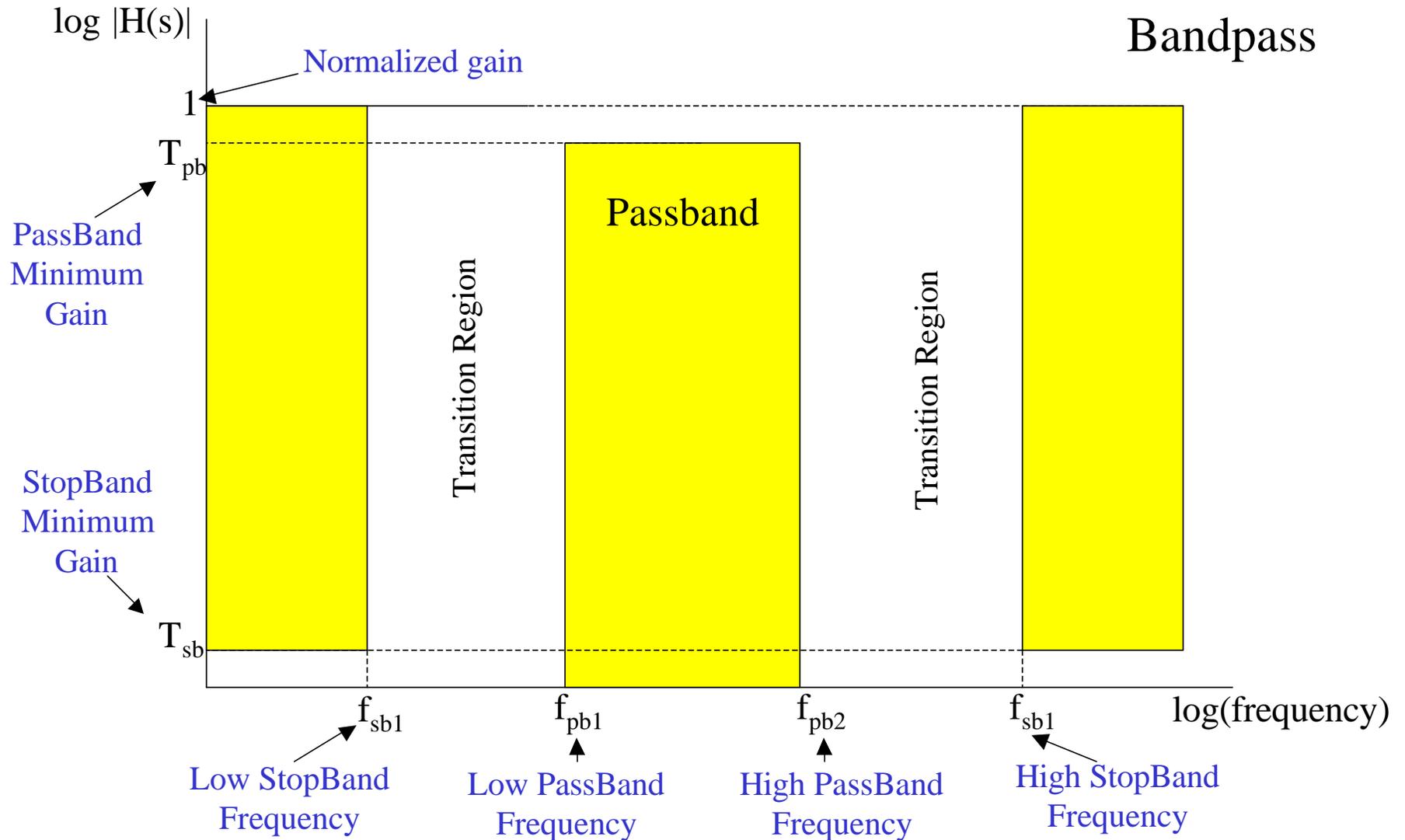
log |H(s)|

Normalized gain

Bandpass

1

T_{pb}

PassBand
Minimum
Gain

Passband

Transition Region

Transition Region

StopBand
Minimum
Gain

T_{sb}

f_{sb1} f_{pb1} f_{pb2} f_{sb1} log(frequency)

Low StopBand
Frequency

Low PassBand
Frequency

High PassBand
Frequency

High StopBand
Frequency

Four Canonical Cont-Time Filters

Classic Analog Filters (IIR digital filters):

Butterworth: Maximally flat in passband…moderate rolloff

Chebyshev : Faster rolloff by allowing ripples in passband or stopband

Elliptic: Fastest rollff by appowing ripples in both passband and stopband

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Other FIR (digital) filters….

Other filter design (H(s) or H(z)) techniques: Optimization approaches

Should we choose H(s) or H(z) for our representation?

Partially due to particular circuit tradeoffs

(tunability? tools? continuous tunability? Accuracy? power consumption?)
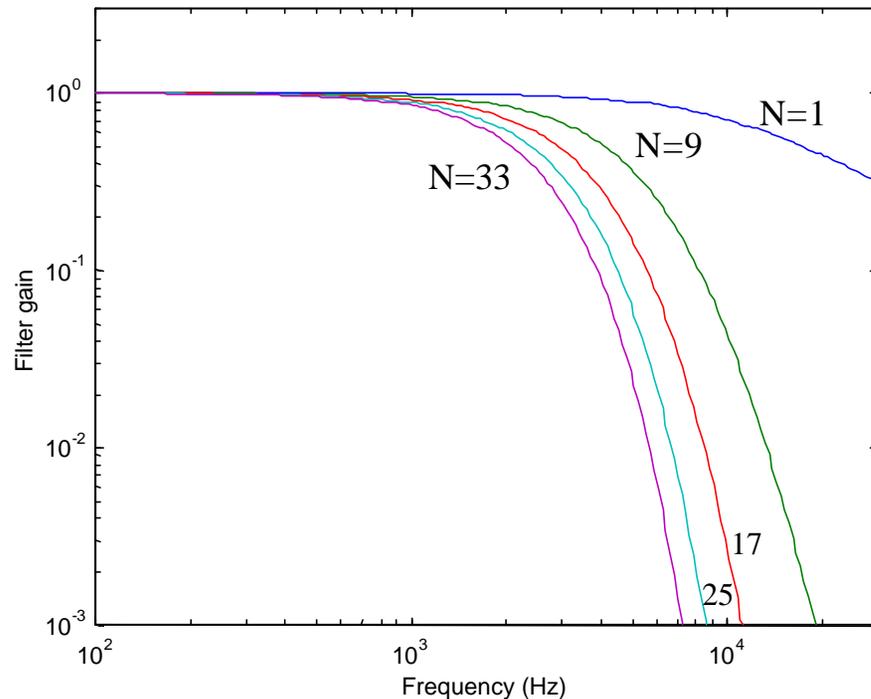
What happens if we just cascade first order stages?

$$T(s) = \frac{1}{(1 + j s\tau)^N}$$

What happens if we just cascade first order stages?

$$T(s) = \frac{1}{(1 + j\, s\tau)^N}$$

Corner shifts with N

Rolloff is not initially sharp…..

Butterworth Filter Design

Transfer Function: (low-pass)

$$T(s) = \frac{1}{1 + (-1)^{N+1}\, \varepsilon\, s^N \tau^N}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2\, \omega^{2N} \tau^{2N}}}$$

Definition of ε:

$$T_{pb} = \frac{1}{\sqrt{1 + \varepsilon^2}} \qquad (f_{sb}\, \tau = 2\pi)$$

$$T_{pb} = 1/\sqrt{2} \quad \text{for } \varepsilon = 1$$

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Need to solve to meet the specification of T_{sb} at f_{sb} : Filter Order (N)

$$T_{sb}^2 \left(\ 1 \ \ + \varepsilon^2 (f_{sb} \, / \, f_{pb} \,)^{2N} \,\right) = 1$$

To meet specifications, one chooses the next largest integer

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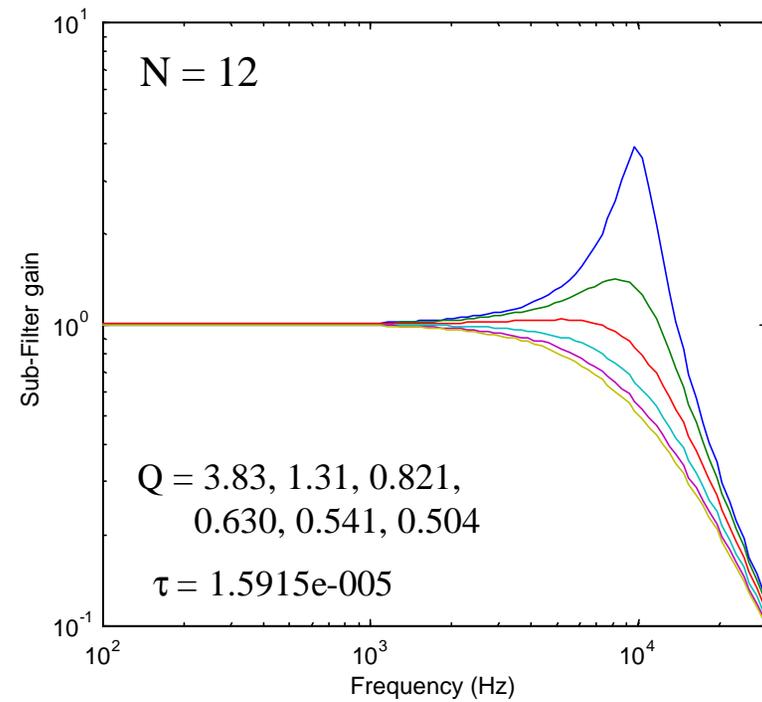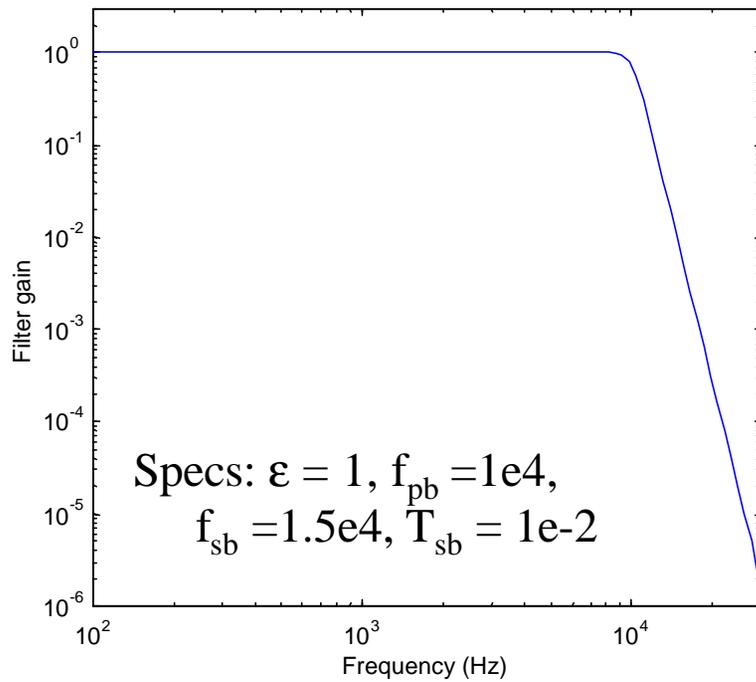$$T_{sb}{}^2 \left(\; 1 \;\; + \varepsilon^2(f_{sb}\,/\,f_{pb}\,)^{2N}\,\right) = 1$$

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Pole locations: ($\varepsilon = 1$)

$$1/\tau_k = (1/\tau)\big(\sin(\,(2k-1)\pi/(2N)\,) \; + j\,\cos(\,(2k-1)\pi/(2N)\,)\,\big),\ k=1\ldots N$$

$$T(j\omega) = \frac{1}{(1 + s\,\tau_1\,)\,(1 + s\,\tau_2\,)\,\ldots\,(1 + s\,\tau_N\,)}$$

Butterworth Filter Design

Specs: $\varepsilon = 1$, $f_{pb} = 1e4$,
$f_{sb} = 1.5e4$, $T_{sb} = 1e-2$

N = 12

Q = 3.83, 1.31, 0.821,
0.630, 0.541, 0.504

$\tau = 1.5915e-005$

Chebyshev Filter Design

Transfer Function(low-pass)

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 \cos^2(N \cos^{-1}(\omega\tau))}} \qquad \omega\tau > 1$$

$$= \frac{1}{\sqrt{1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(\omega\tau))}} \qquad \omega\tau < 1$$

Definition of ε:

$$T_{pb} = \frac{1}{\sqrt{1 + \varepsilon^2}}$$

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$$(1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(f_{sb} / f_{pb}))) T_{sb} = 1$$

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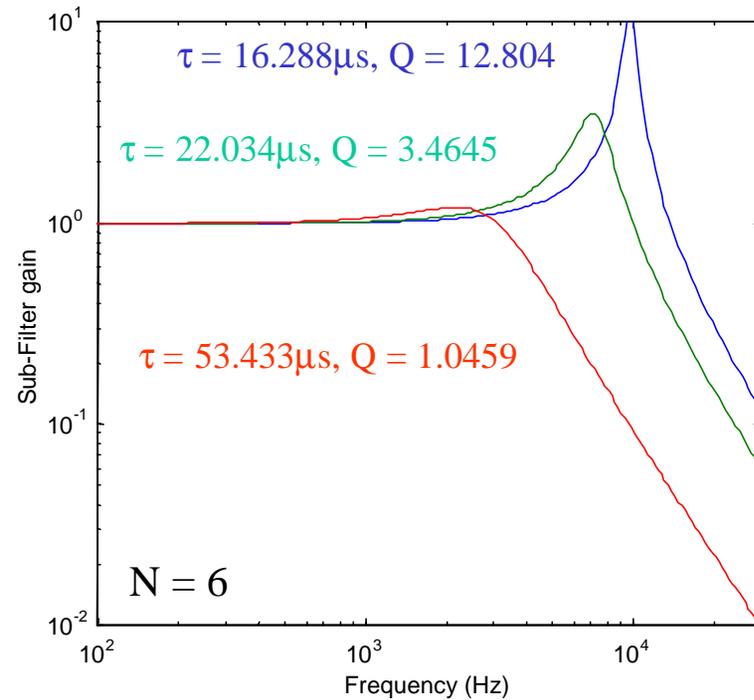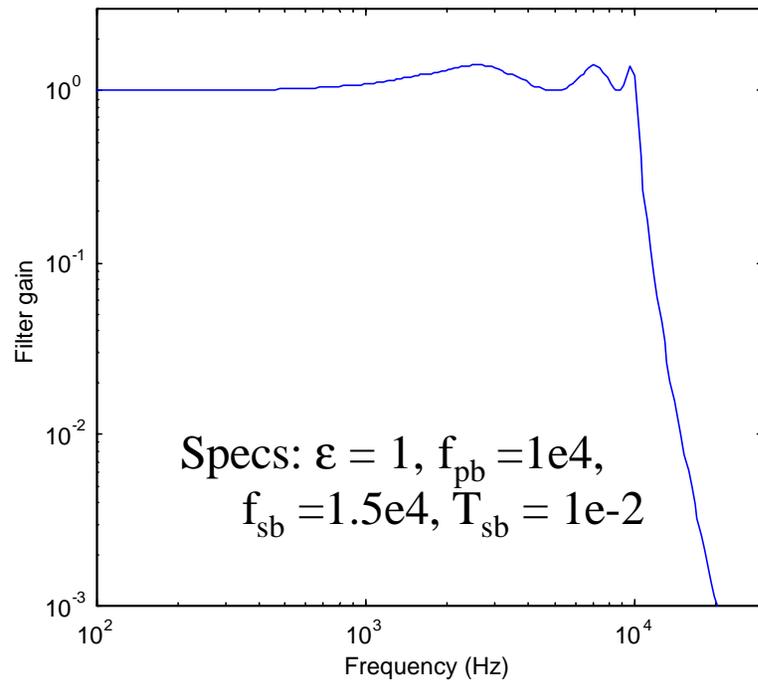
$$(1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(f_{sb} / f_{pb}))) T_{sb}^2 = 1$$

Pole locations:

$$1/\tau_k = (1/\tau)\left(\sin((2k\text{-}1)\pi/(2N)) \sinh((1/N) \sinh^{-1}(1/\varepsilon))\right.$$

$$\left. + j \cos((2k\text{-}1)\pi/(2N)) \cosh((1/N) \sinh^{-1}(1/\varepsilon))\right)$$

Where k goes from 1, 2, ….N

Chebyshev Filter Design

Specs: $\varepsilon = 1$, $f_{pb} = 1e4$,
$f_{sb} = 1.5e4$, $T_{sb} = 1e-2$

$\tau = 16.288\mu s$, $Q = 12.804$

$\tau = 22.034\mu s$, $Q = 3.4645$

$\tau = 53.433\mu s$, $Q = 1.0459$

$N = 6$

Fewer stages, higher Qs….

Transformations between s and z

$$z^{-1} = e^{-sT}$$

T = sampling period

Simple Transformation

$$s \sim \frac{1}{T} (1 - z^{-1})$$

$$z^{-1} \sim 1 - sT$$

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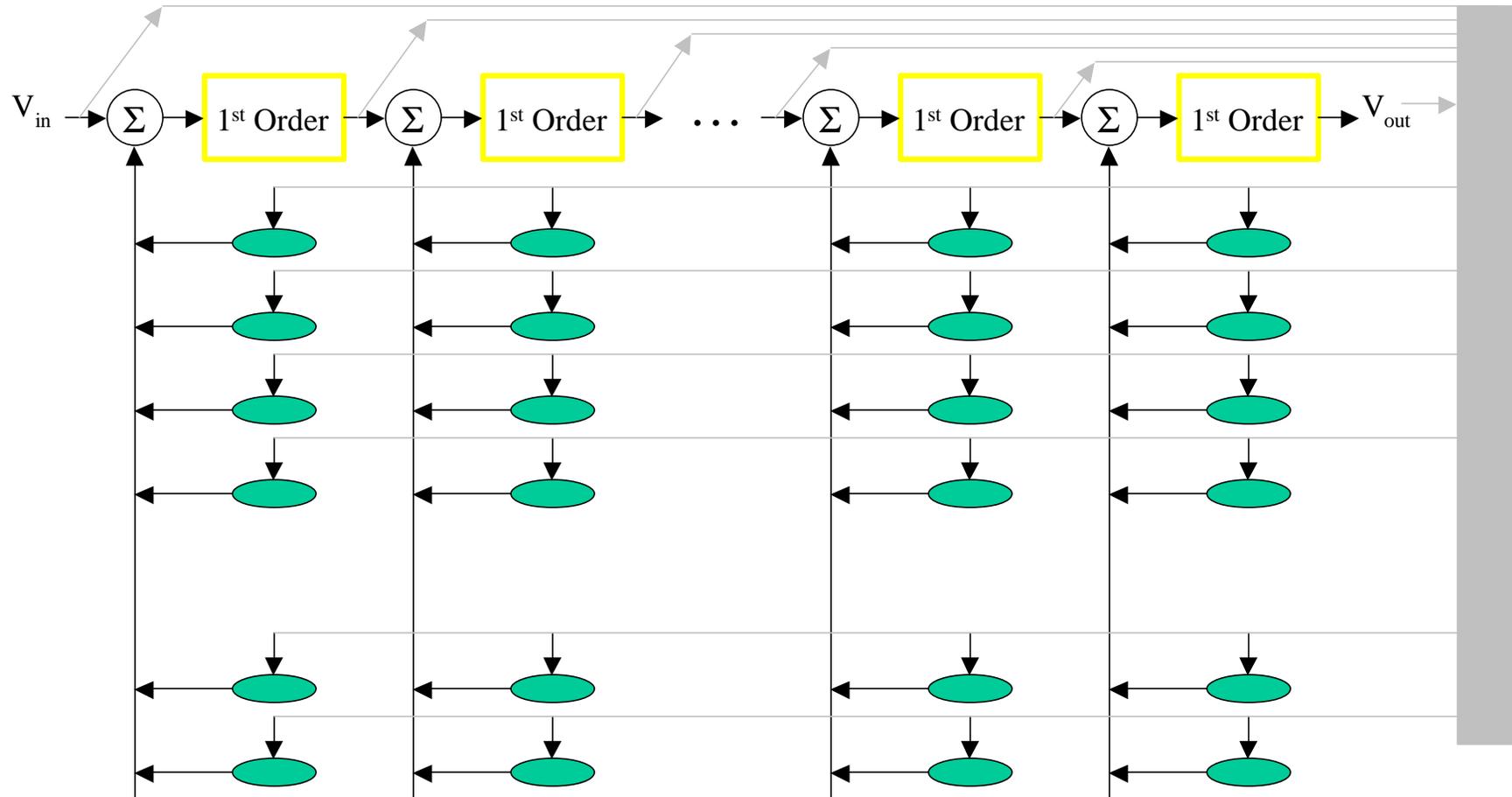
Simple Transformation

$$s \sim \frac{1}{T}(1 - z^{-1})$$

$$z^{-1} \sim 1 - sT$$

Bilinear transform

$$s \sim \frac{1}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \sim \frac{1}{T}\frac{1 - e^{-sT}}{1 + e^{-sT}}$$

$$z^{-1} \sim \frac{1 - sT}{1 + sT} \qquad \sim \frac{1}{T}\frac{1 - (1 - sT + 0.5*(sT)^2 - \ldots)}{1 + 1 - sT + 0.5*(sT)^2 - \ldots} \sim s$$

General Filter Topology

General Filter Topology

N*N parameters

N poles, N zeros for N-th order filter

Problem is underspecified…therefore can optimize
for SNR, complexity, etc…

Partitioning H(s) for Circuit Implementation

"Partition" the transfer function into simple parts

- Factorization into first order and second order terms
- Nearest neighbor feedback (~LC ladder filter network)
- Addition of factors
    (maybe out of an approximation of an FIR filter)
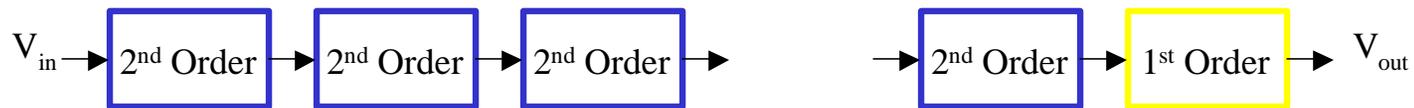- Additional feedback / feedforward terms

Similar for either H(s) or H(z)

Typical Filter Topologies

Cascade of First and Second-Order Sections

V_{in} → [2nd Order] → [2nd Order] → [2nd Order] → ... → [2nd Order] → [1st Order] → V_{out}
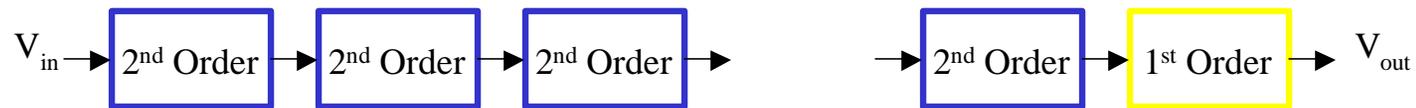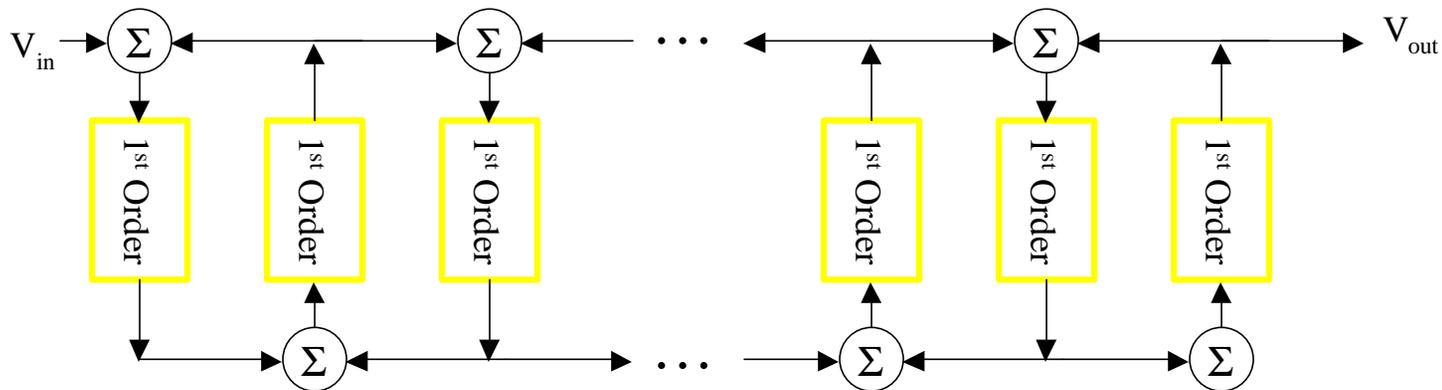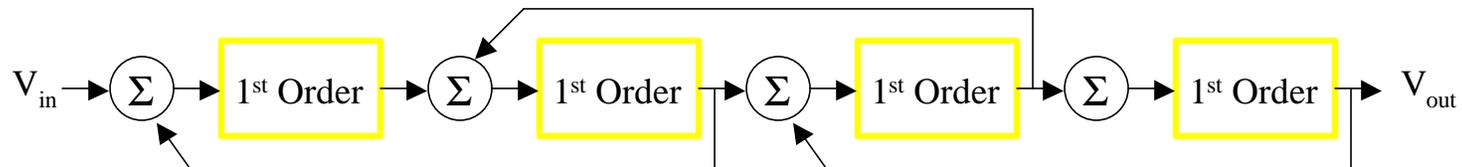
Typical Filter Topologies

Cascade of First and Second-Order Sections

Nearest Neighbor Feedback (Inspired by LC ladder network filters)

Conclusions

Basic directions for integrated circuit filters

• Continuous or Discrete: Time and/or Amplitude

• High level specifications of filters

• Obtaining a filter function (H(s) or H(z))

• Implementing the filter function into basic blocks

       (first and second-order filter sections, integrators, delays, etc.)