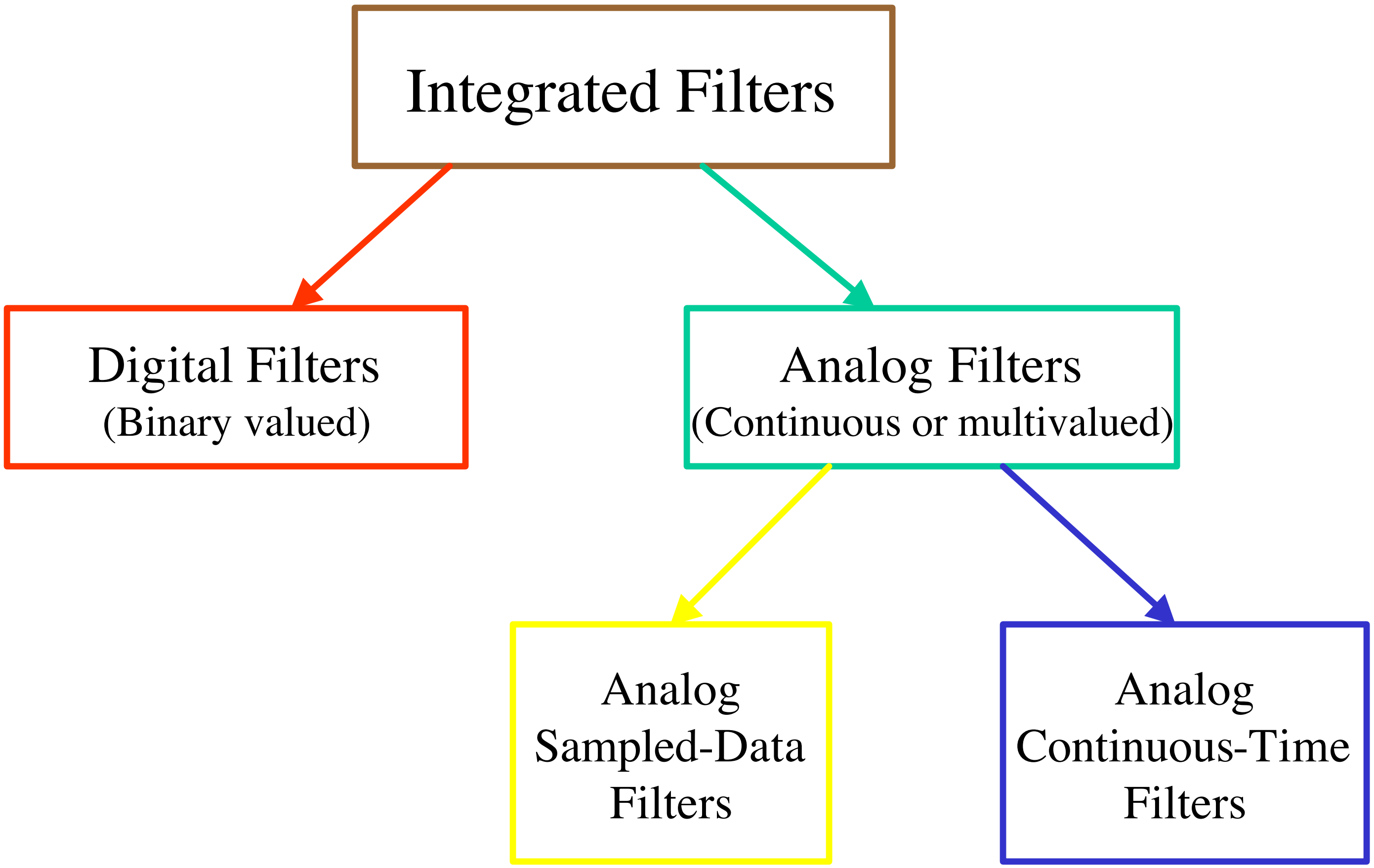
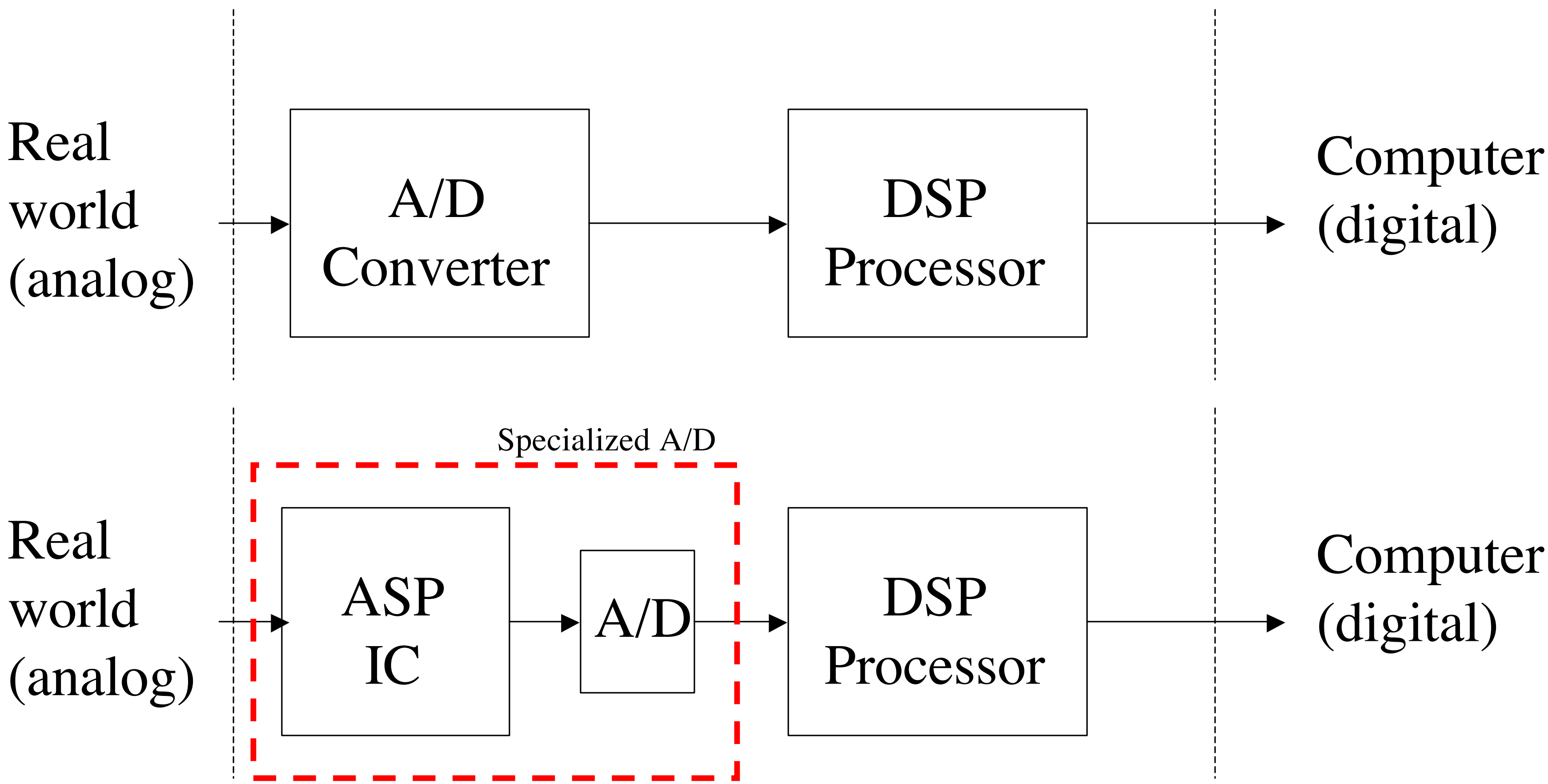
Types of Integrated Filters

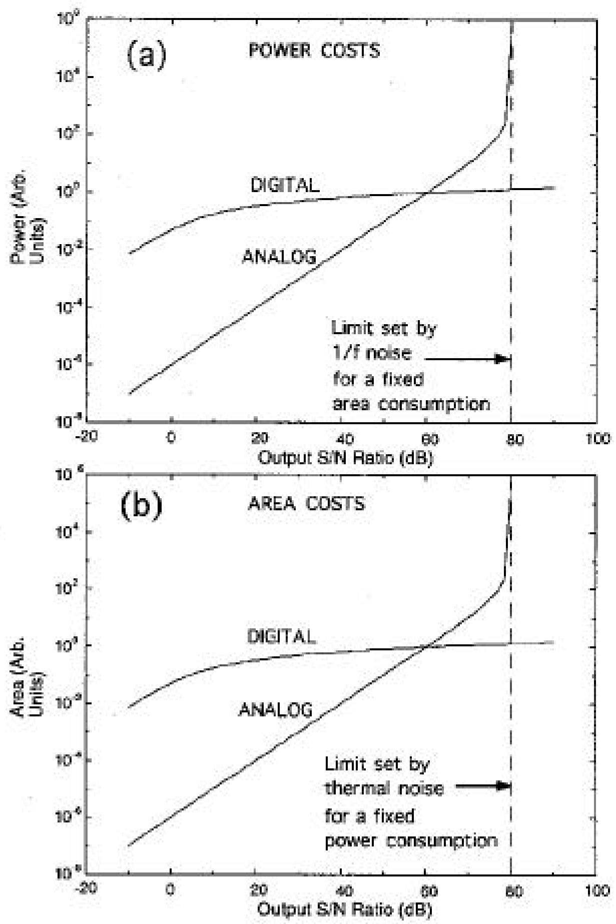


Where to divide Analog and Digital?



Analog-Digital Comparison

Low SNR: Analog / High SNR: Digital



[Sarpeskar 1997]

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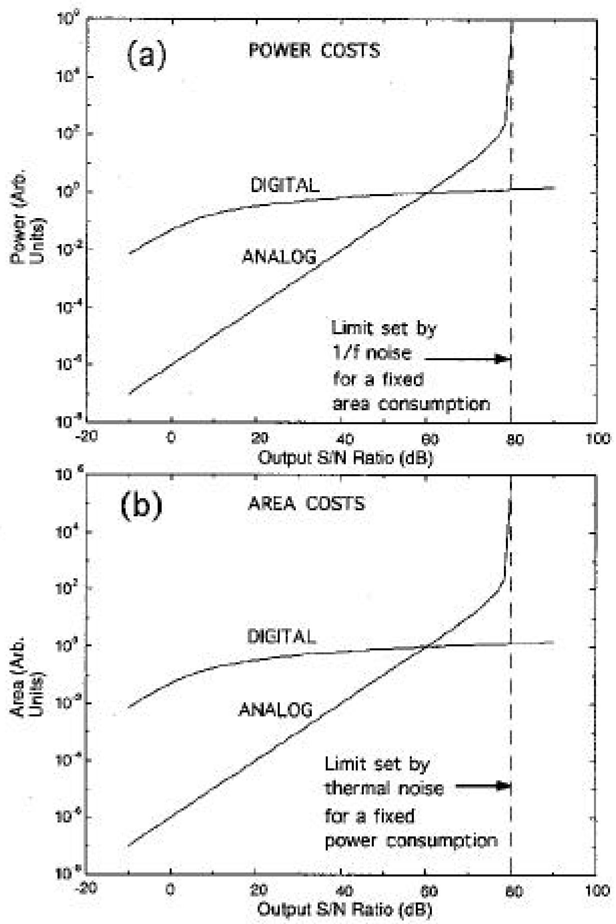
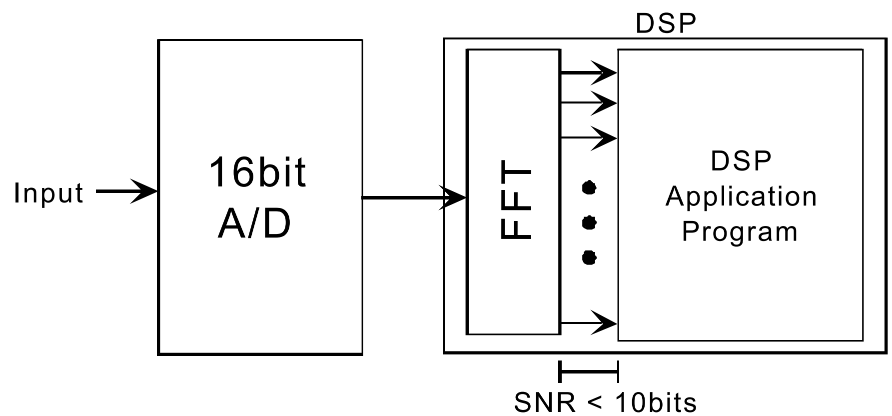
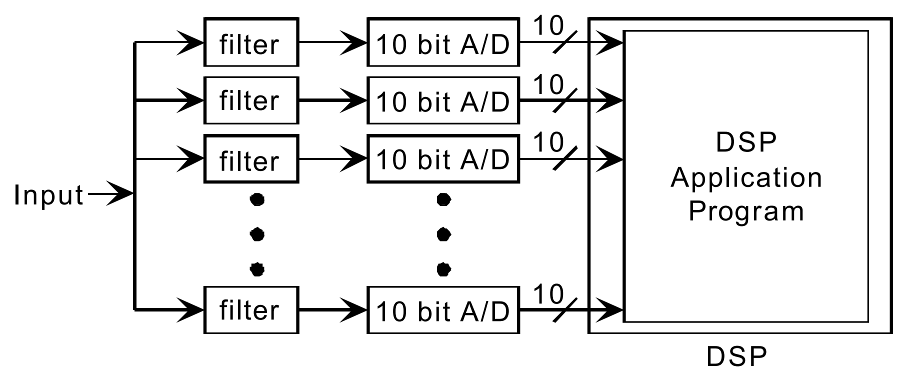


[Sarpeskar 1997]

Practical Interpretation of Cost





- What if exponentially spaced FFT?

Circuit Issues for Filters

Programmability / Tunability: flexibility and complexity

Available for digital (clocks/ crystals) as well as some analog (e.g. Floating-Gate) filters

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Power-supply: Power Supply Rejection Ratio (**PSRR**)
Temperature, etc.

Not a problem for digital filters,
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Typically, sampling in amplitude / time results in,

- the more complexity is needed (S/H blocks, anti-aliasing filters),
- more power / lower-frequency / area

Design of Analog Filters

- Find the transfer function for a given filter

- "Partition" the transfer function,
  or an approximation,
  into simple parts that can be implemented.

- Implement the transfer function in
  a particular circuit technology

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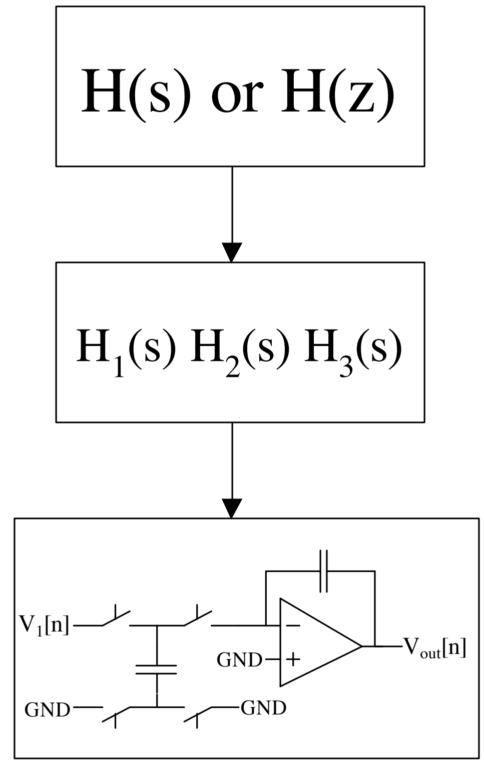
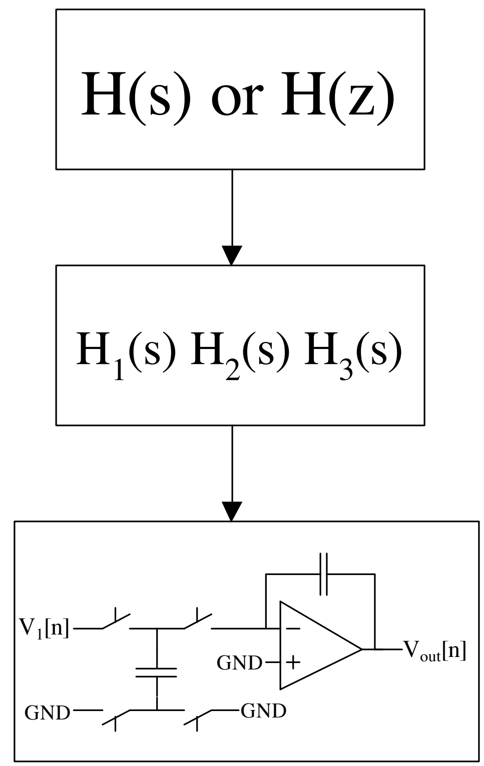


Design of Analog Filters

As basic building blocks we have

- integrators, delay elements
- first-order (low-pass / bandpass)
- second order functions
  (low-pass / bandpass / highpass)

The "circuit" design question is
how to make these functions
what inputs / outputs / internal variables
should be voltages / currents, etc.



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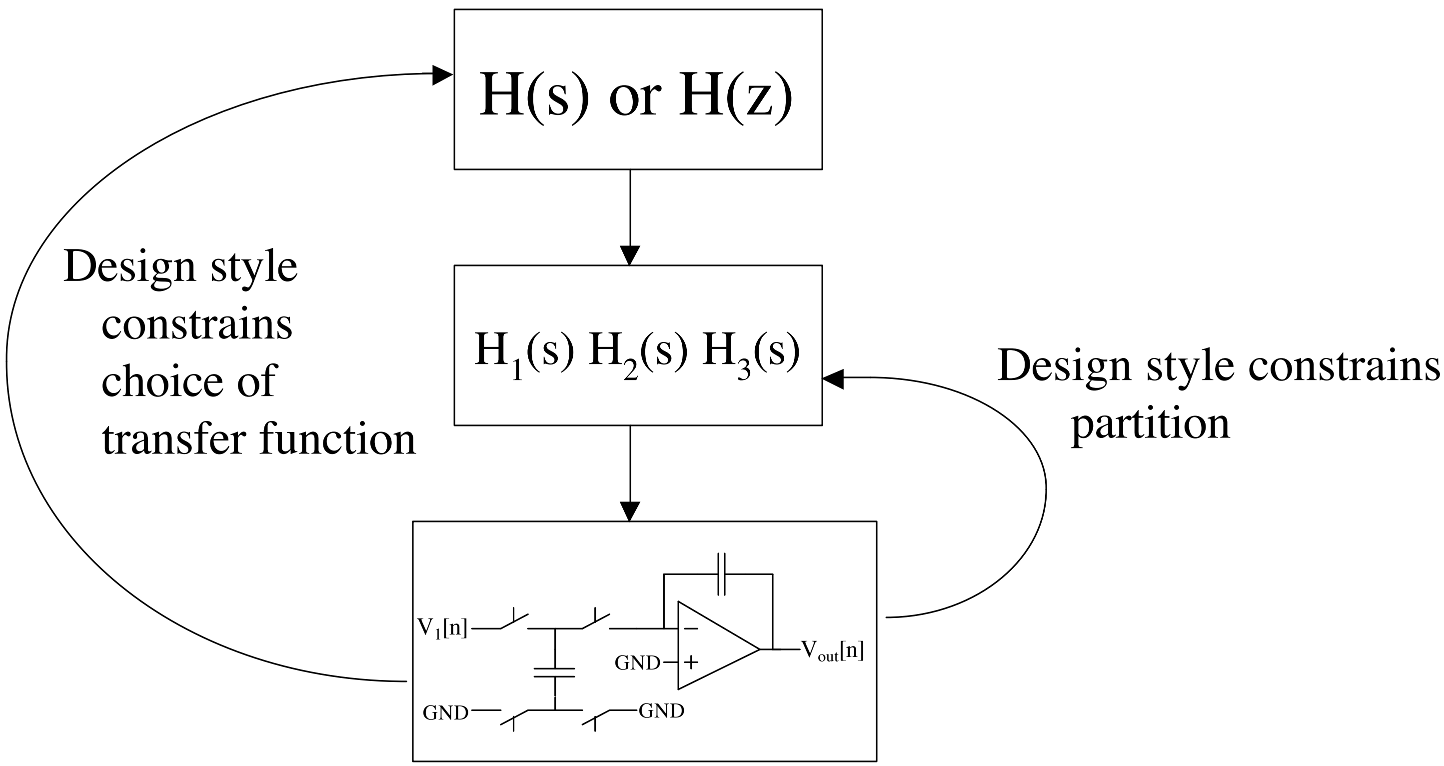
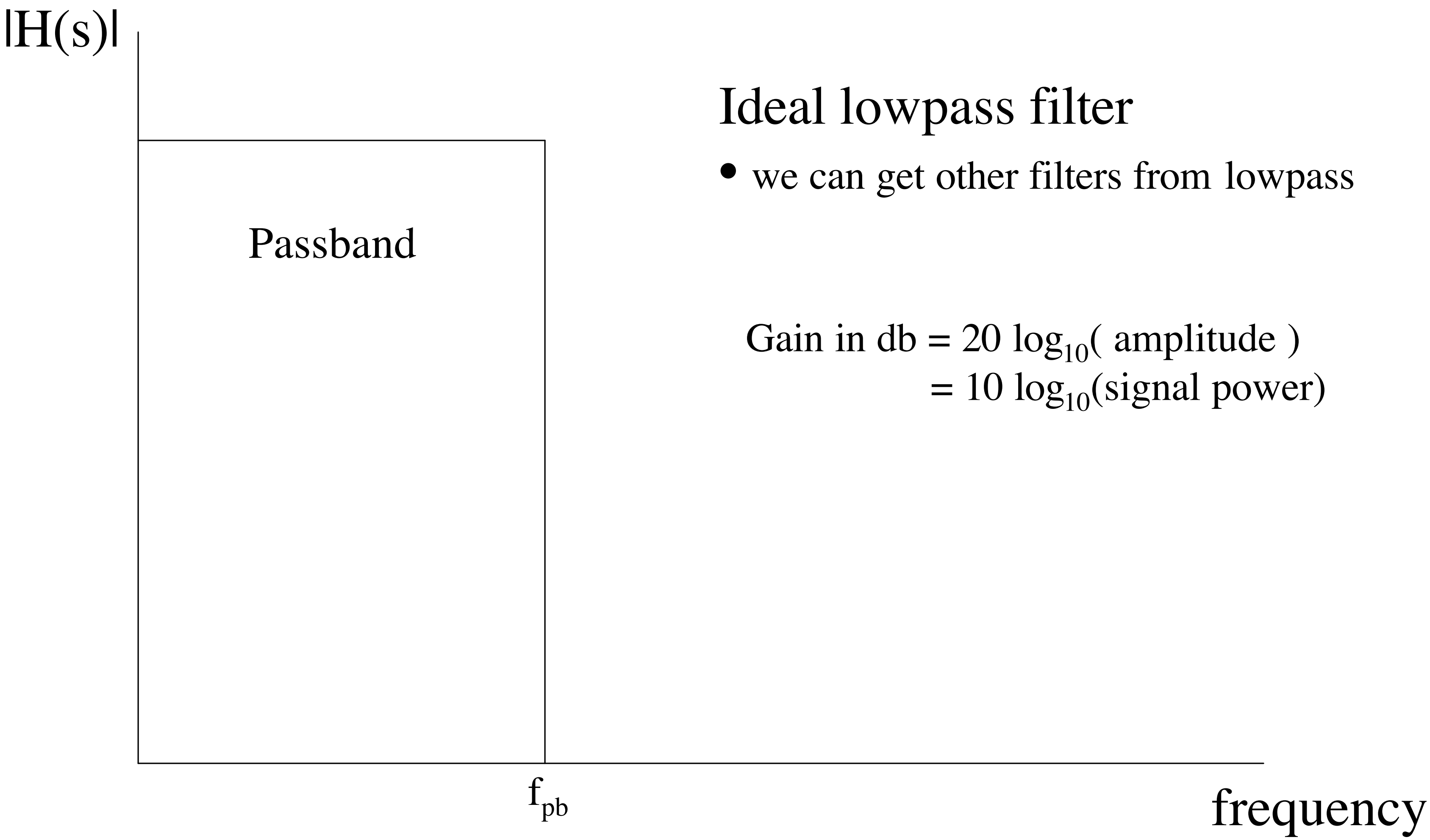


Choosing H(s) or H(z) for a filter

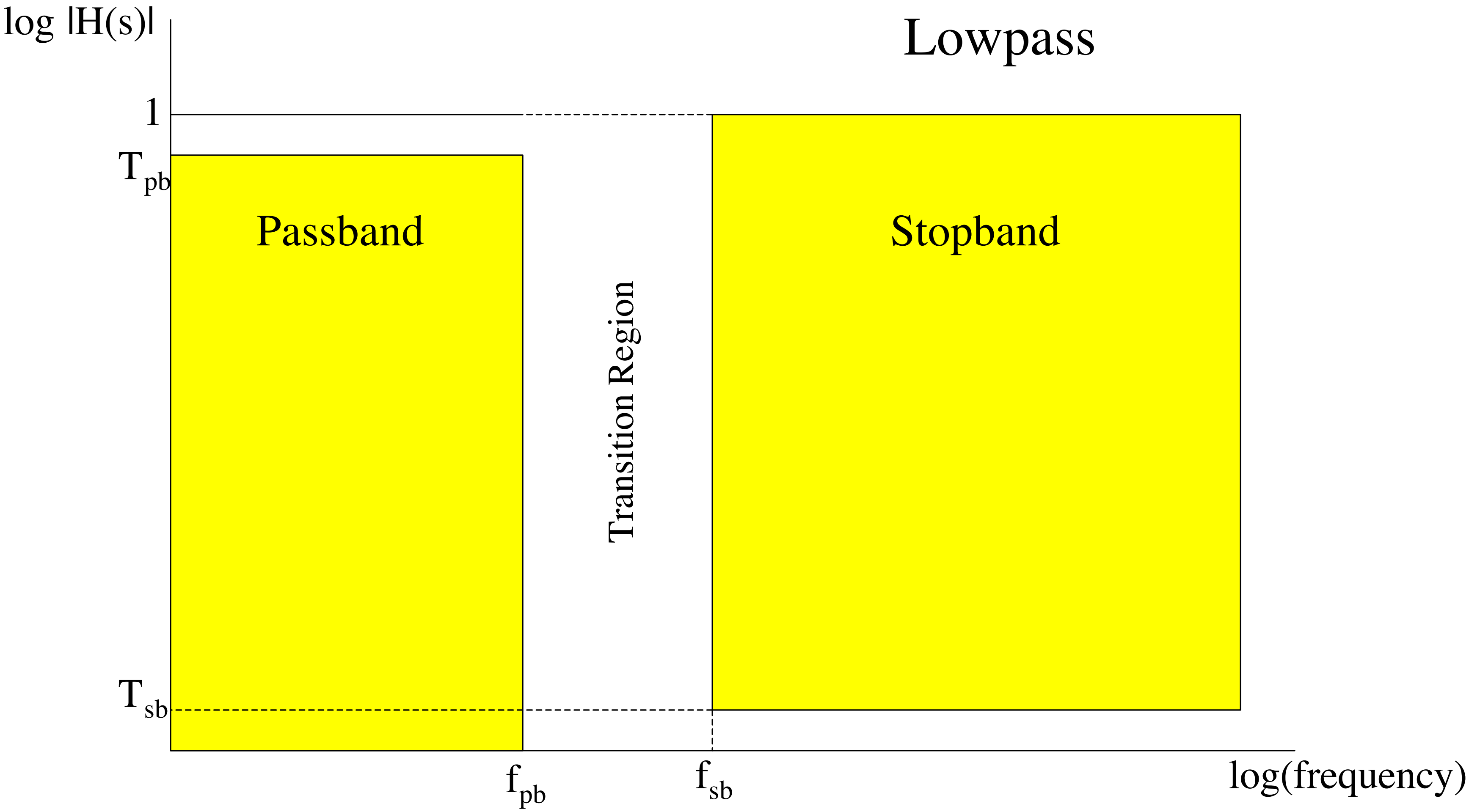


Ideal lowpass filter

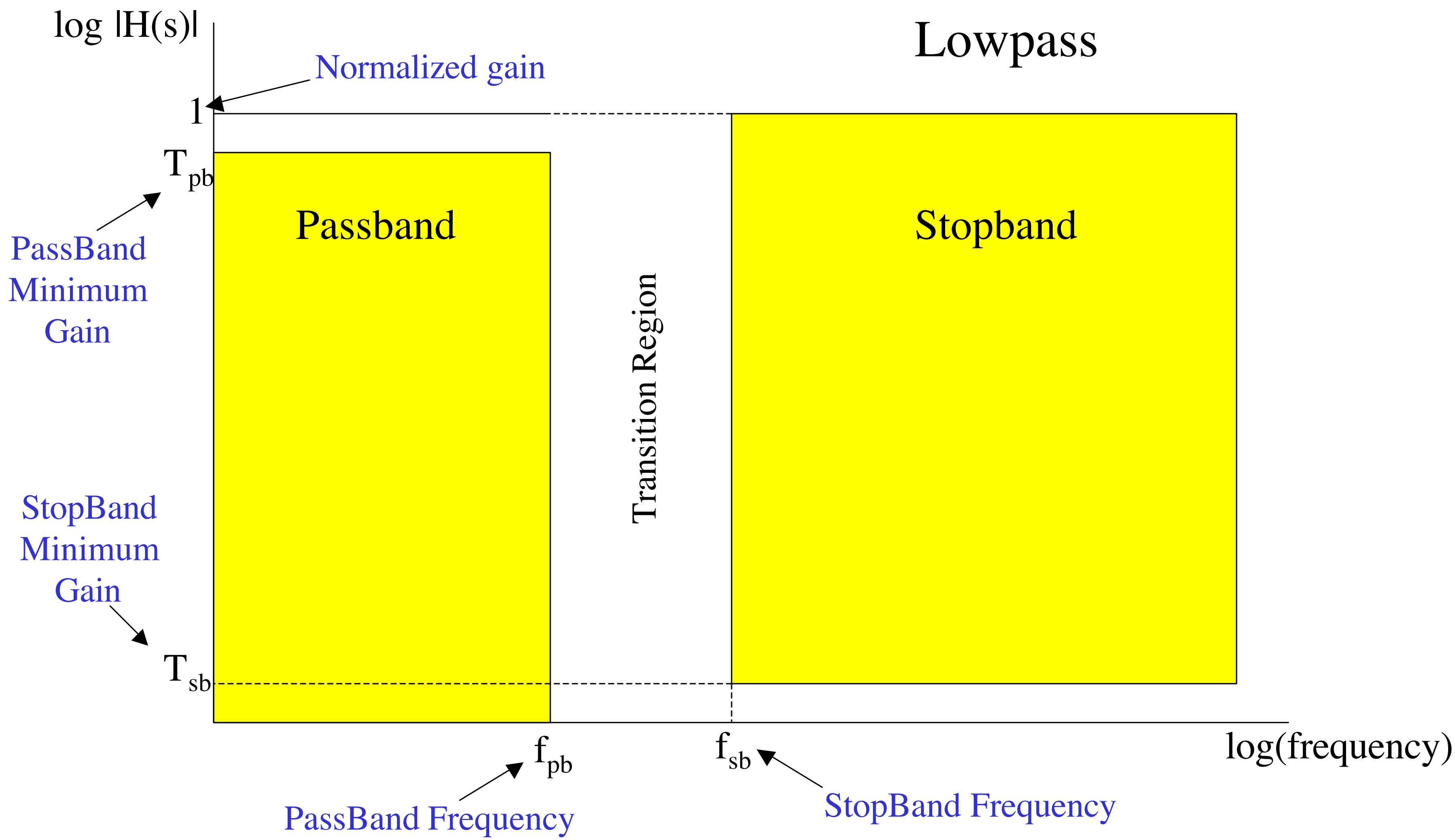
- we can get other filters from lowpass

$$\text{Gain in db} = 20 \log_{10}(\text{ amplitude }) = 10 \log_{10}(\text{signal power})$$

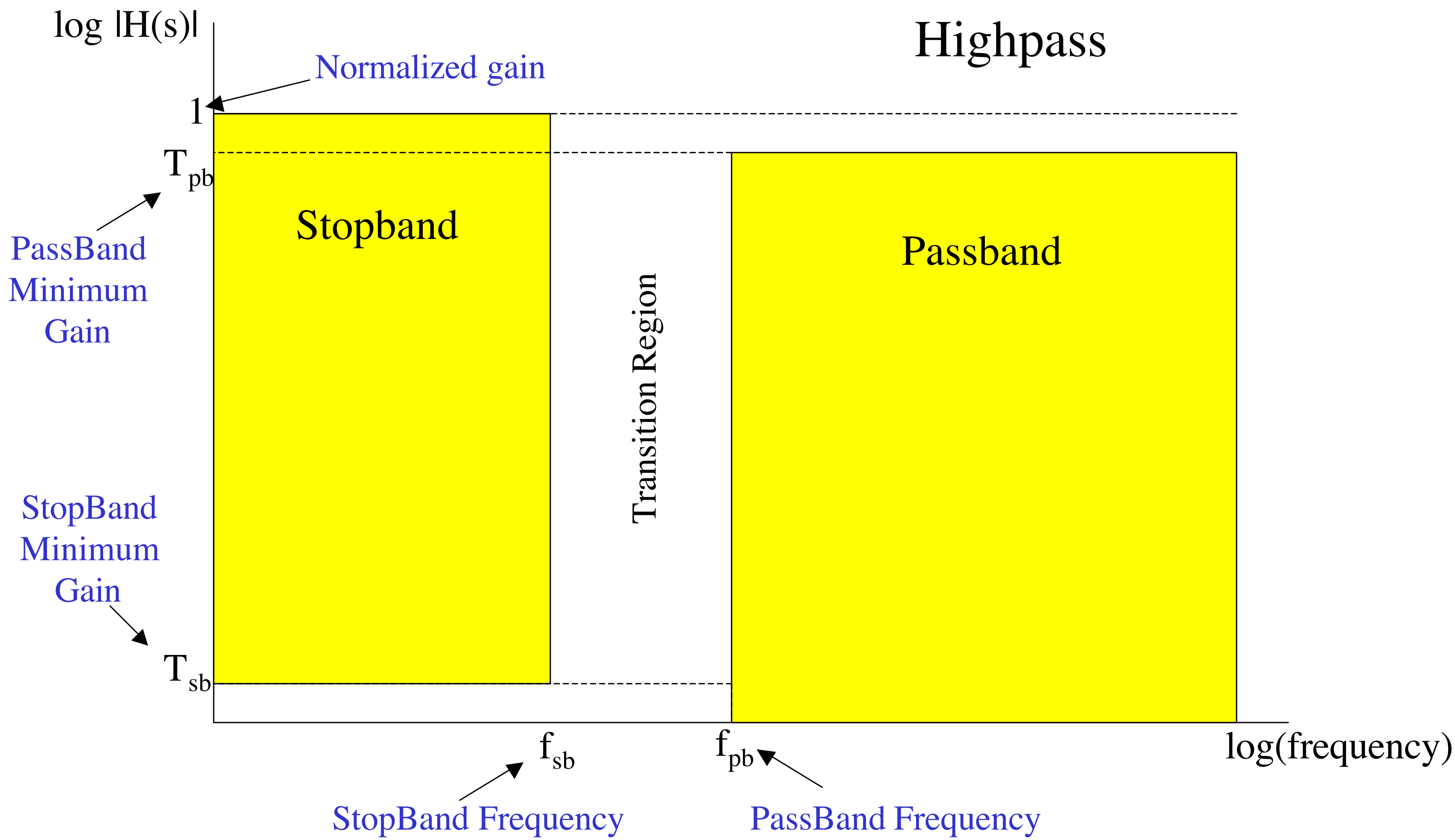
H(s) or H(z) for a lowpass filter



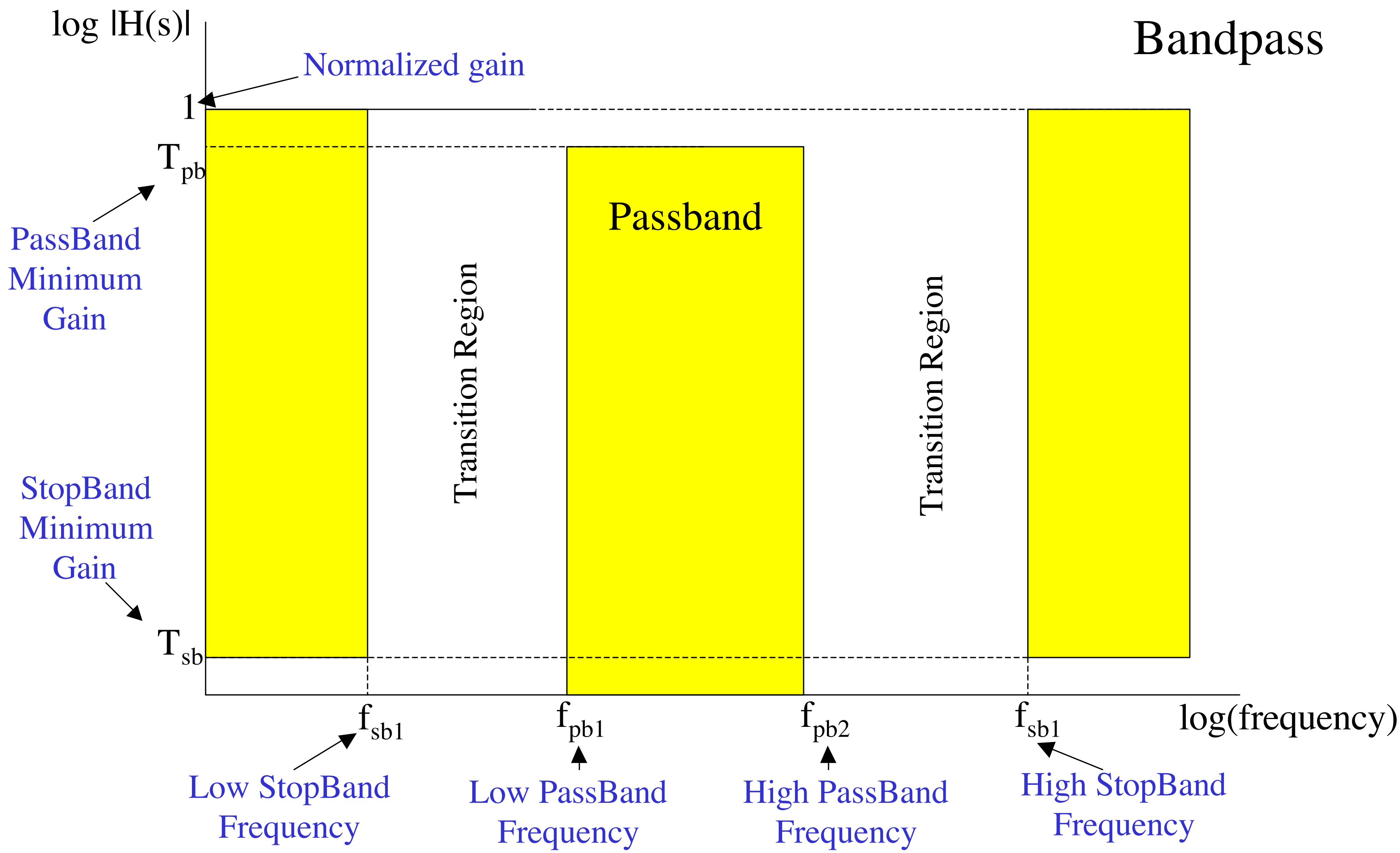
H(s) or H(z) for a lowpass filter



H(s) or H(z) for a Highpass filter



H(s) or H(z) for a Highpass filter



Four Canonical Cont-Time Filters

Classic Analog Filters (IIR digital filters):

Butterworth: Maximally flat in passband…moderate rolloff

Chebyshev : Faster rolloff by allowing ripples in passband or stopband

Elliptic: Fastest rollff by appowing ripples in both passband and stopband

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Other FIR (digital) filters….

Other filter design (H(s) or H(z)) techniques: Optimization approaches

Should we choose H(s) or H(z) for our representation?

Partially due to particular circuit tradeoffs

(tunability? tools? continuous tunability? Accuracy? power consumption?)

What happens if we just cascade first order stages?

$$T(s) = \frac{1}{(1 + j\,s\tau)^N}$$

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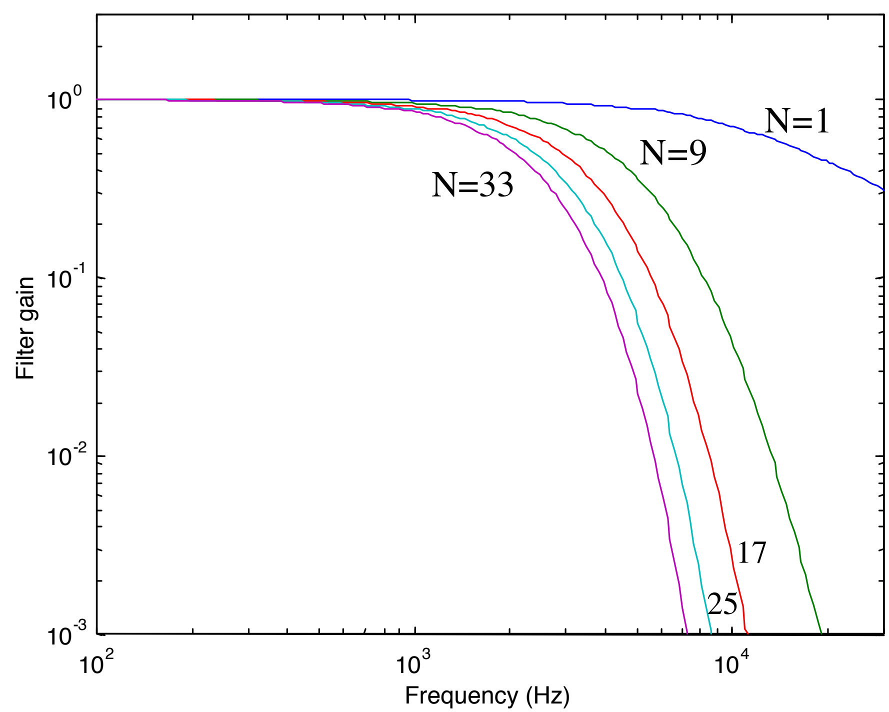
Corner shifts with N

Rolloff is not initially sharp…..



Butterworth Filter Design

<u>Transfer Function: (low-pass)</u>

$$T(s) = \frac{1}{1 + (-1)^{N+1}\, \varepsilon\, s^N \tau^N}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2\, \omega^{2N} \tau^{2N}}}$$

<u>Definition of ε:</u>

$$T_{pb} = \frac{1}{\sqrt{1 + \varepsilon^2}} \qquad (f_{sb}\,\tau = 2\pi\,)$$

$T_{pb} = 1/\sqrt{2}$ for $\varepsilon = 1$

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Need to solve to meet the specification of T_{sb} at f_{sb} : Filter Order (N)

$$T_{sb}^2 \left(1 + \varepsilon^2 (f_{sb} / f_{pb})^{2N} \right) = 1$$

To meet specifications, one chooses the next largest integer

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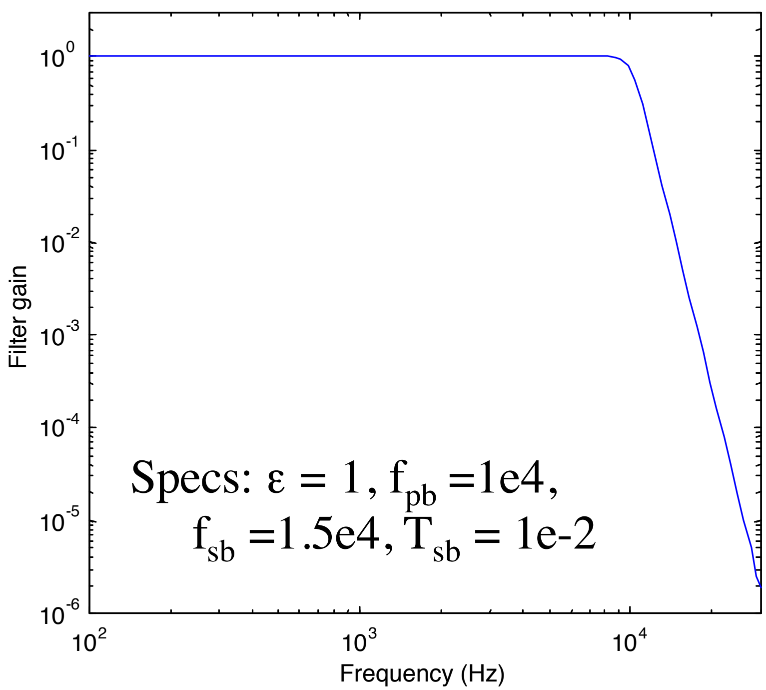
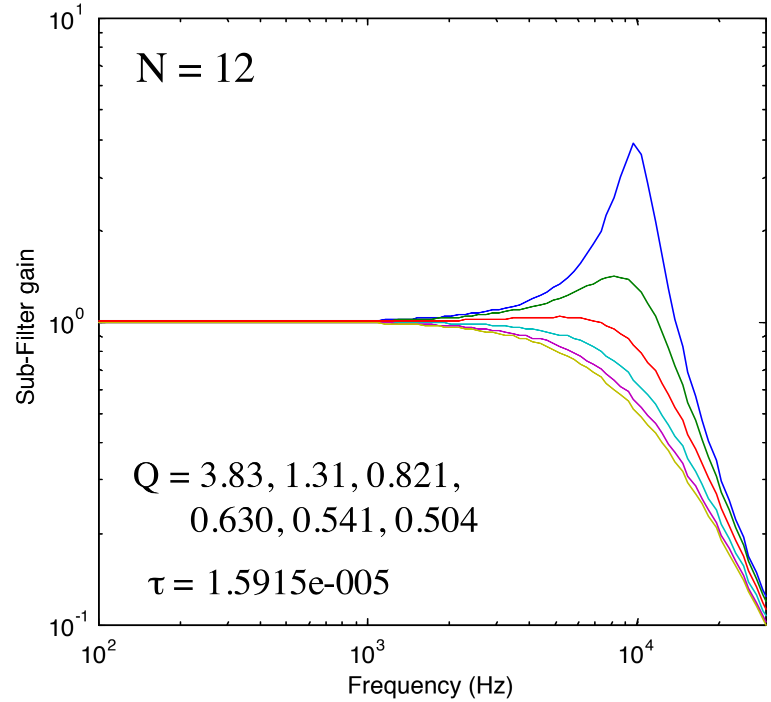
Pole locations: ($\varepsilon = 1$)

$$1/\tau_k = (1/\tau)\Big(\sin(\,(2k-1)\pi/(2N)\,) + j\cos(\,(2k-1)\pi/(2N)\,)\Big), \ k=1\ldots N$$

$$T(j\omega) = \frac{1}{(1 + s\,\tau_1)\,(1 + s\,\tau_2)\,\ldots\,(1 + s\,\tau_N)}$$

Butterworth Filter Design





Chebyshev Filter Design

Transfer Function(low-pass)

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 \cos^2(\, N \cos^{-1}(\, \omega\tau\,)\,)}} \qquad \omega\tau > 1$$

$$= \frac{\quad}{\sqrt{1 + \varepsilon^2 \cosh^2(\, N \cosh^{-1}(\, \omega\tau\,)\,)}} \qquad \omega\tau < 1$$

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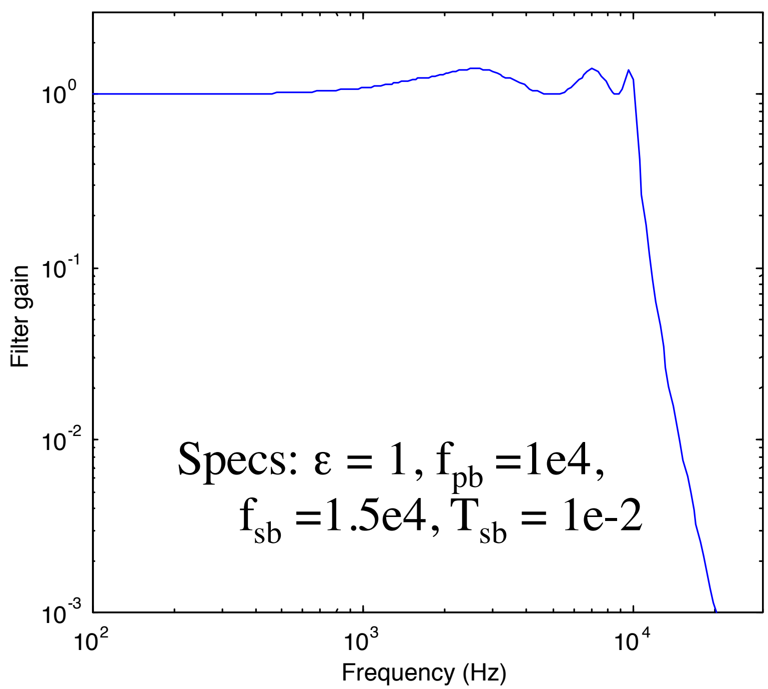
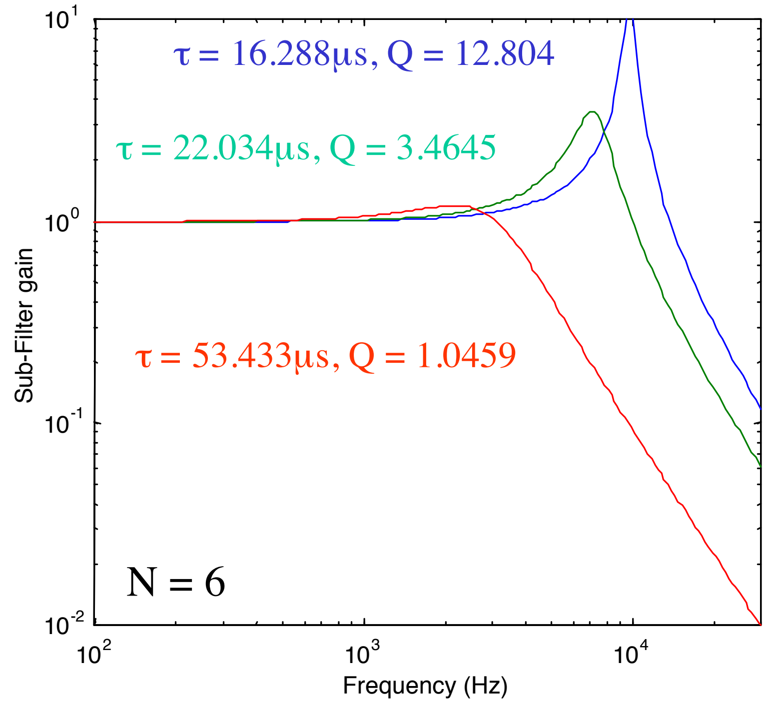
$$+\, j\cos(\,(2k-1)\pi/(2N)\,)\cosh(\,(1/N)\sinh^{-1}(1/\,\varepsilon)\,)\Big)$$

Where k goes from 1, 2, ….N

Chebyshev Filter Design





Fewer stages, higher Qs….

Transformations between s and z

$$z^{-1} = e^{-sT}$$

T = sampling period

Simple Transformation

$$s \sim \frac{1}{T}(1 - z^{-1})$$

$$z^{-1} \sim 1 - sT$$

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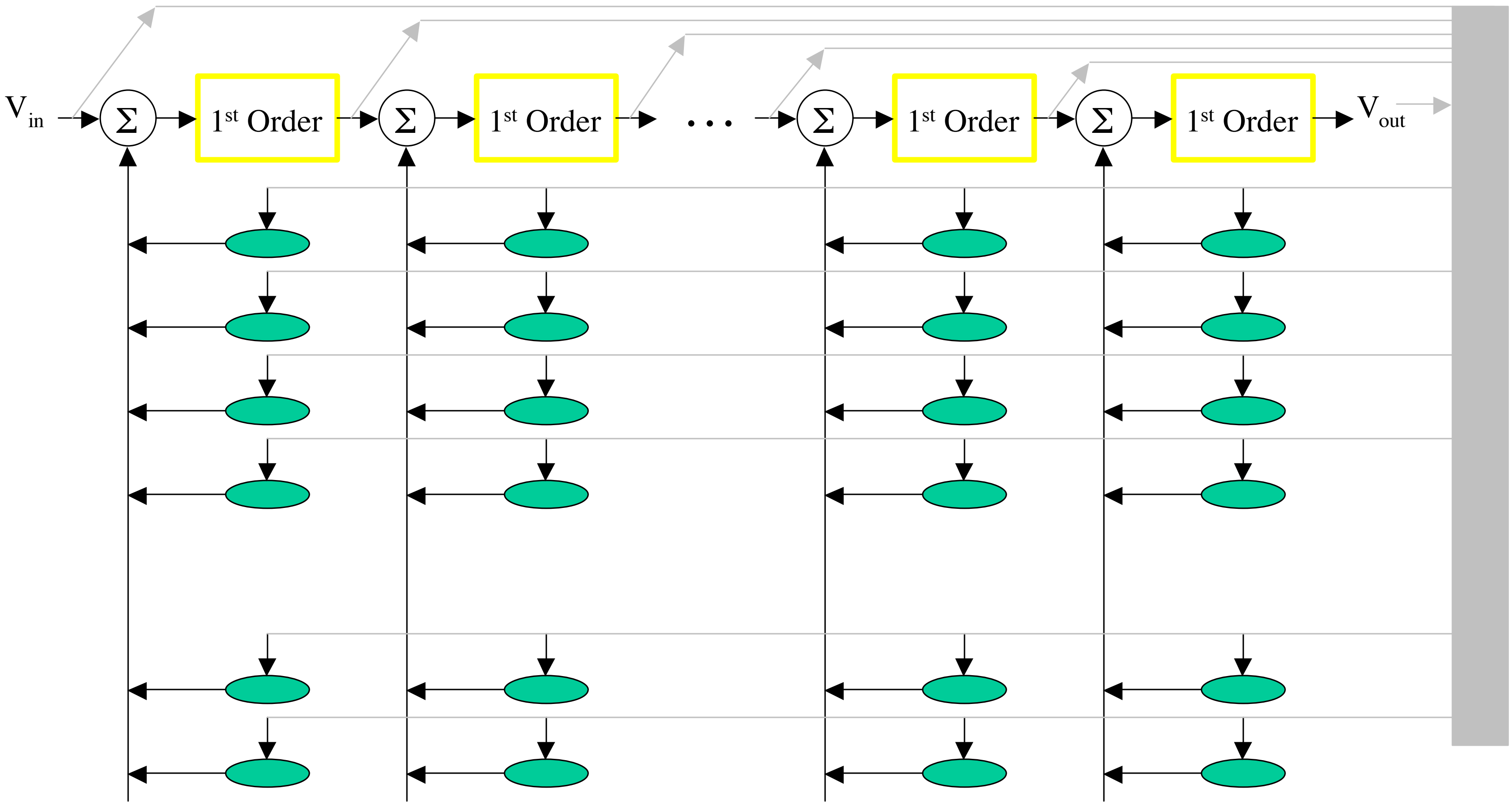
Bilinear transform

$$s \sim \frac{1}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \sim \frac{1}{T}\frac{1 - e^{-sT}}{1 + e^{-sT}}$$

$$z^{-1} \sim \frac{1 - sT}{1 + sT} \qquad \sim \frac{1}{T}\frac{1 - (1 - sT + 0.5*(sT)^2 - \ldots)}{1 + 1 - sT + 0.5*(sT)^2 - \ldots} \sim s$$

General Filter Topology



General Filter Topology

N*N parameters

N poles, N zeros for N-th order filter

Problem is underspecified…therefore can optimize
for SNR, complexity, etc…

Partitioning H(s) for Circuit Implementation

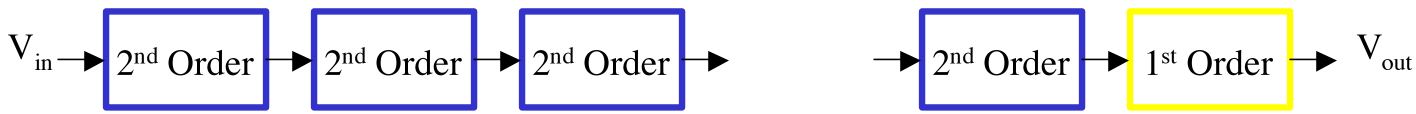
"Partition" the transfer function into simple parts

- Factorization into first order and second order terms
- Nearest neighbor feedback (~LC ladder filter network)
- Addition of factors
  (maybe out of an approximation of an FIR filter)
- Additional feedback / feedforward terms

Similar for either H(s) or H(z)

Typical Filter Topologies

Cascade of First and Second-Order Sections



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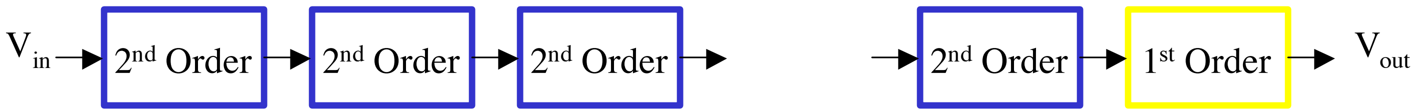
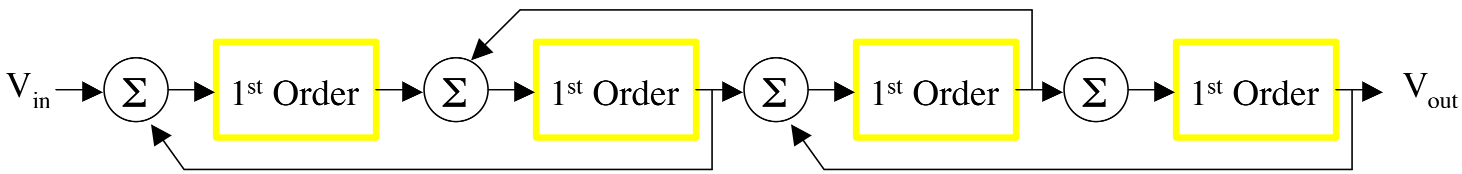
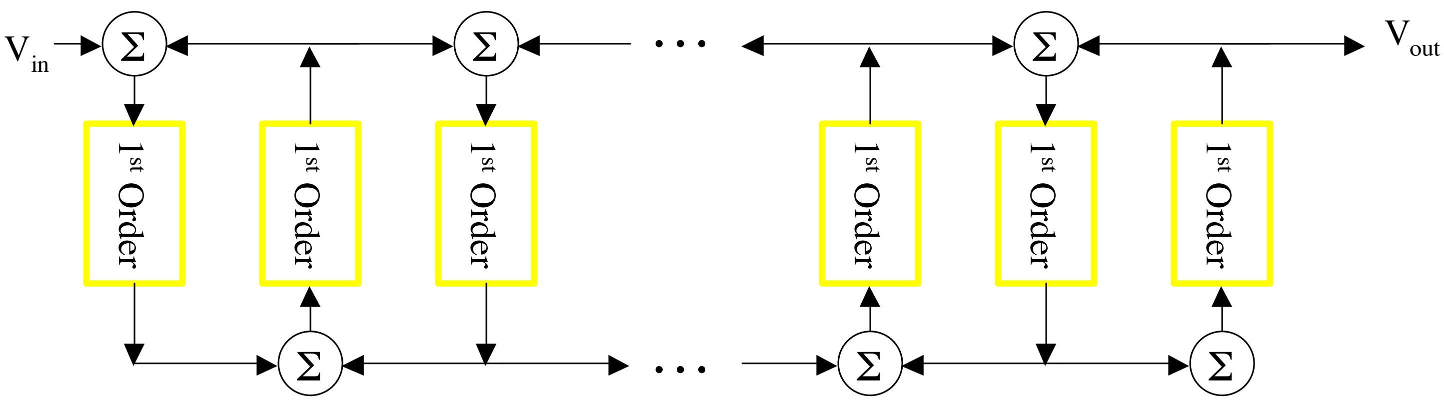


Nearest Neighbor Feedback (Inspired by LC ladder network filters)





Conclusions

Basic directions for integrated circuit filters

- Continuous or Discrete: Time and/or Amplitude
- High level specifications of filters
- Obtaining a filter function ($H(s)$ or $H(z)$)
- Implementing the filter function into basic blocks

  (first and second-order filter sections, integrators, delays, etc.)