Name: _____

AC Circuit Analysis and Frequency Response of First and Second-Order Circuits

Objective: To gain experience with
- Measuring frequency response
- Resonance

Table of Contents:

Pre-Lab Assignment
1) Download the NI ELVISmx software and install it to your PC.

   http://joule.ni.com/nidu/cds/view/p/id/2157/lang/en

2) View the video on measuring frequency response using the myDAQ.
3) Read the Background section.
4) Perform hand calculations on the circuits in part A, B, and C to determine the expected values for the empirically determined unknowns.

Background
Frequency Response of Circuits
A circuit is a system with input x(t) and output y(t). Usually, the input is a voltage source x(t) =v_s(t), but a current source may also be used, x(t) = i_s(t). The output is identified as a voltage or current in the circuit.

Phasor representation is used in AC circuit analysis to understand the steady-state response of sinusoidal inputs signals.

The frequency response of a circuit is the behavior of the system as it processes sinusoids of different frequencies. The frequency response function $H(\omega)$ is found mathematically using the impedance method or through experimental measurements by comparing the amplitude and phases of the input and output waveforms.

Experimental Determination of Frequency Response
Examine Figure 1, which shows the input-output behavior of a circuit with frequency response function $H(\omega)$ and sinusoidal input with amplitude A_i and frequency ω_1 rad/sec. The steady-state output is also a sinusoid with the same frequency ω_1 but with a scaled amplitude A_o and phase lag ϕ. The frequency response plot is
     1) a plot of the amplitude scaling $|H(\omega)| = A_o/A_i$ versus the frequency ω and
     2) a plot of the phase lag ϕ versus frequency ω.
The data is commonly plotted using log scales for magnitude and frequency, resulting in a Bode plot representation of the frequency response.

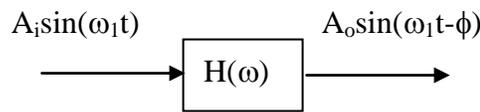$$A_i \sin(\omega_1 t) \quad\quad\quad\quad A_o \sin(\omega_1 t - \phi)$$

$$\longrightarrow \boxed{H(\omega)} \longrightarrow$$

Figure 1: Input Output behavior of a linear circuit in steady state.

The output amplitude, A_o, and the output phase, ϕ, depend on $H(\omega)$ as follows:

$$\mathbf{A_o = A_i |H(\omega_1)|} \quad\quad \text{and} \quad\quad \phi = \angle H(\omega_1)$$

To determine $H(\omega)$ from experimental measurements of a circuit, find the steady-state sinusoidal response to a wide range of input frequencies. From each sinusoidal response, measure A_o and ϕ. Fill in a table such as the one below for each input frequency.
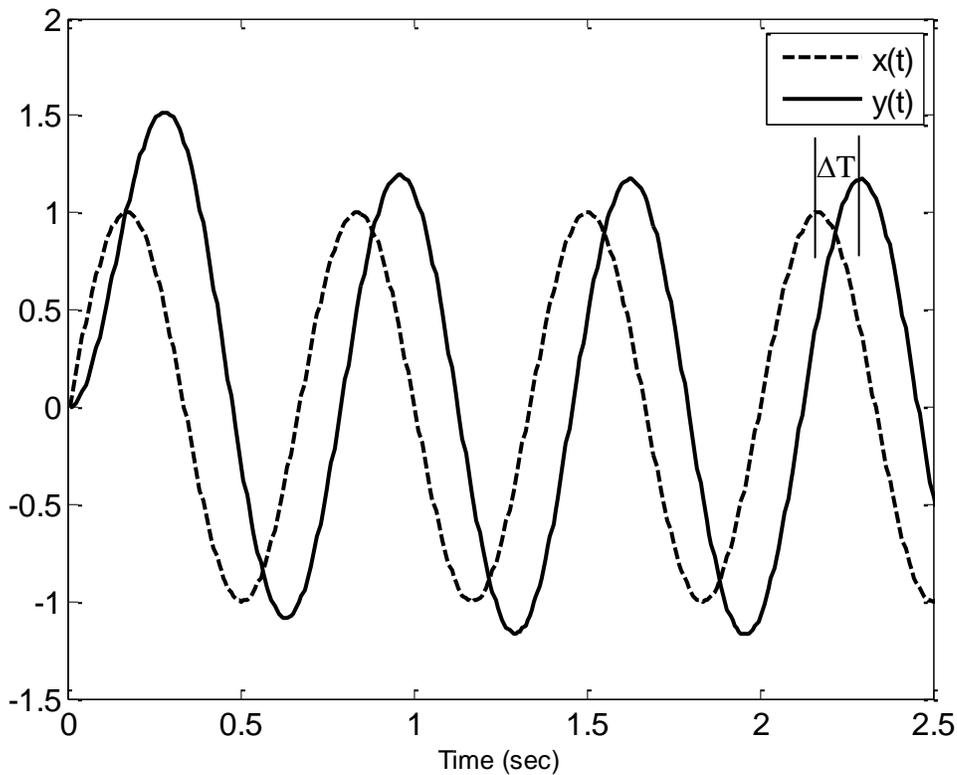
ω_1	$\phi = \angle H(\omega_1)$	$|H(\omega_1)| = A_o/A_i$	$20\log|H(\omega_1)|$

Consider, for example, the response, $y(t)$, of a system with input $x(t) = \sin(3\pi t)$ shown below. For this input, $\omega_1 = 3\pi$, and $A_i = 1$. For the signal in steady-state, measure A_o and ΔT, the time lag. The phase lag in degrees is calculated as

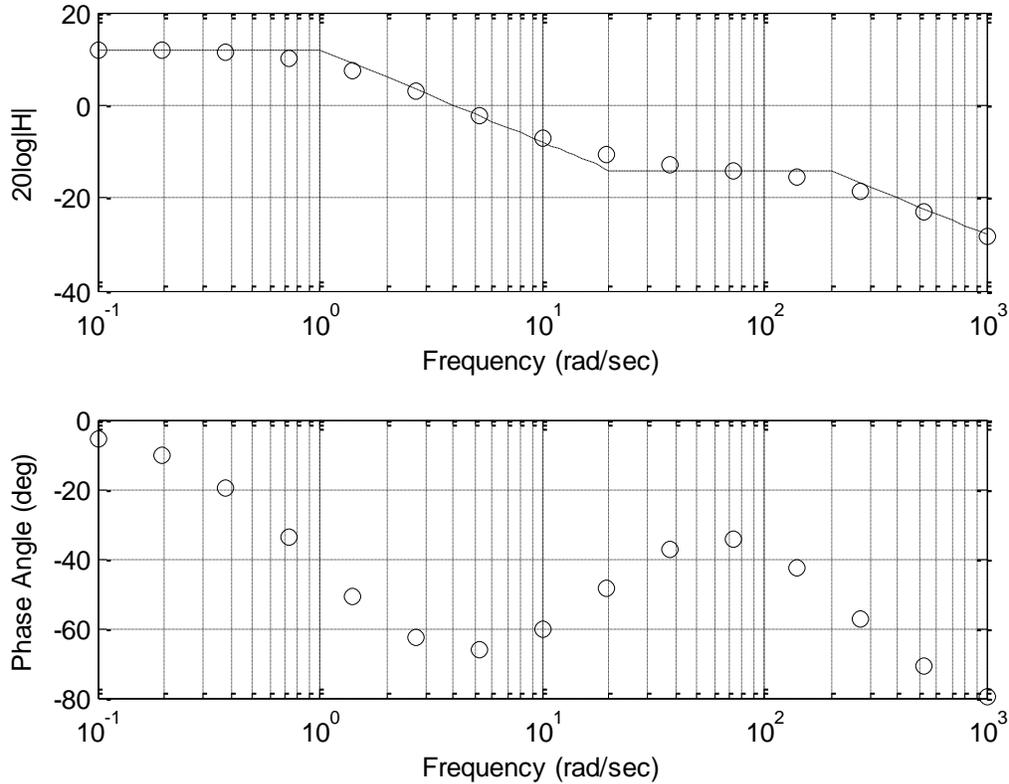$$\phi = -\frac{\Delta T}{T} * 360^o$$

where T is the period of the signal.

The period $T = 2\pi/\omega_1 = 0.67$. From the plot, $\Delta T = 0.12s$ and $A_o = 1.17$. So $|H(\omega_1)| = A_o/A_i = 1.17$ and $\phi = -0.12(360)/T = -64.7^o$.

ω_1	$\phi=\angle H(\omega_1)$	$|H(\omega_1)|=A_o/A_i$	20log $|H(\omega_1)|$
3π	-64.7^o	1.17	1.36dB

Select a wide range of values of input frequencies that fully characterize the frequency response.

Once the data is collected, plot all the points on semilog paper to obtain the Bode plot of the system.

Bandwidth

When a magnitude plot starts flat at low frequencies, then drops off as the frequency increases, then the bandwidth is defined as the frequency at which the magnitude is equal to 0.707 of the DC value, that is $|H(\omega_B)| = 0.707|H(0)|$. On the Bode plot, the log magnitude, $20\log|H|$, is 3dB below that of the DC value, $20\log|H(0)|$.

Resistors

The resistance of physical resistors is denoted by four color bands on the resistor. The color code for bands 1-3 is

Color	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Purple	7
Grey	8
White	9

1st band and 2nd band give the first two significant numbers of the resistance

3rd band gives the base 10 multiplier, $\times 10^n$

4th band gives the tolerance (silver is ±10% and gold is ±5%)

A resistor with bands (yellow, red, orange, silver) is a 42,000 Ω resistor with a tolerance of ±10%.

*National Instruments myDAQ Input/Output Interfaces:*
The myDAQ has an interface on one side with +15v, -15v, Analog Output (AO), Analog Input (AI), and digital input/outputs. The part of the interface that will be used in this experiment is shown below.

+15v	-15v	AGND	0	1	AGND	0+	0-	1+	1-
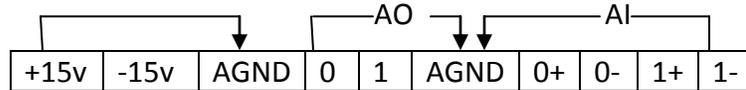
The Scope is used to plot the inputs in (AI) Channels 0 and 1, that is, the potential between AI0- and AI0+ and between AI1- and AI1+.

Op Amp Buffers Circuits
Since op amps have their own power source, they can be used to build a unity gain buffer circuit whose purpose is to boost the power without changing the voltage waveform. The myDAQ analog output, AO, channels are limited to 2mA, so the AO channels can be passed through a unity gain op amp in order to deliver higher current levels. The pin diagram and schematic of an integrated chip (IC) containing an op amp is shown in the figures below. The IC pin number 1 is the one closest to the circle stamped into the top of the IC.
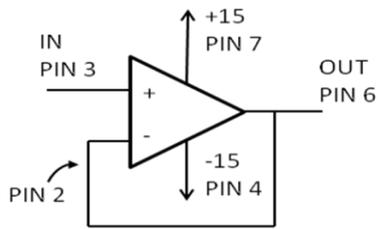
Schematic of unity gain buffer
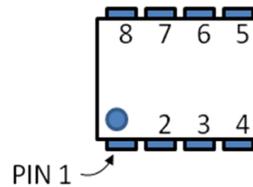
PIN diagram for LF351
Op Amp IC

Lab Procedures

1. Setup
   - Connect the myDAQ to your computer using the USB connector.
   - Start up the NI ELVISmx Instrument Launcher software.
   - Start up the function generator FGEN and the SCOPE by clicking on their icons.

Part A) Using an Op Amp Buffer Circuit

2. Connect a 1kΩ resistor to the function generator (AO wires) and to Channel 0 of the scope (AI wires) as shown in the figure.

Scope
Channel 0   FGEN
AI: 0+   AO: 0

1kΩ

AO:
AI: 0-   AGND
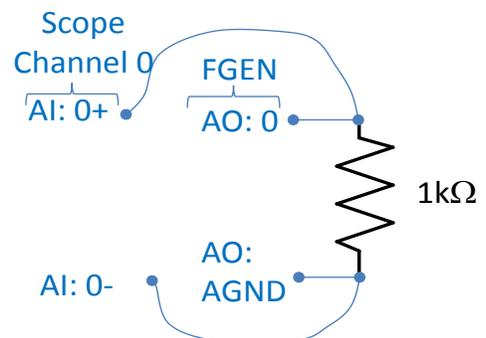
3. Apply a 1 volt peak to peak sinusoidal signal across the resistor using FGEN, and view the sine wave on the SCOPE. The corresponding instrument settings are below.
   a. Function Generator
      - Produce a sine wave voltage with frequency f= 500 Hz (=1000π rad/sec) and 1 V peak-to-peak with a dc offset of 0 V. That is, $v_s = 0.5 \sin(1000\pi t)$.
   b. Oscilloscope
      - Set the trigger type to edge, the level to 0.1 V, and the horizontal position to 10%
      - Set scale for Volts/Div to 1V
      - Set the Time/Div setting to 2ms

4. Click Run on both the function generator and the oscilloscope.

5. Does the voltage waveform across the resistor look as expected (shape and amplitude)?

   (Not working? See the comments below)

6. Slowly increase the amplitude of the sine wave from FGEN until the voltage signal starts clipping (ie, the peaks flatten). Note the amplitude of the clipping. Sketch the shape of the waveform below and explain what is happening (hint: read the background section on buffer circuits).

7. Insert the buffer op amp circuit between the myDAQ output AO channel 0 and the resistor as shown in the schematic.
   - Connect pins 4 and 7 of the op amp IC to the myDAQ +15v and -15v power supply (use the op amp pin diagram in the background section as a guide).
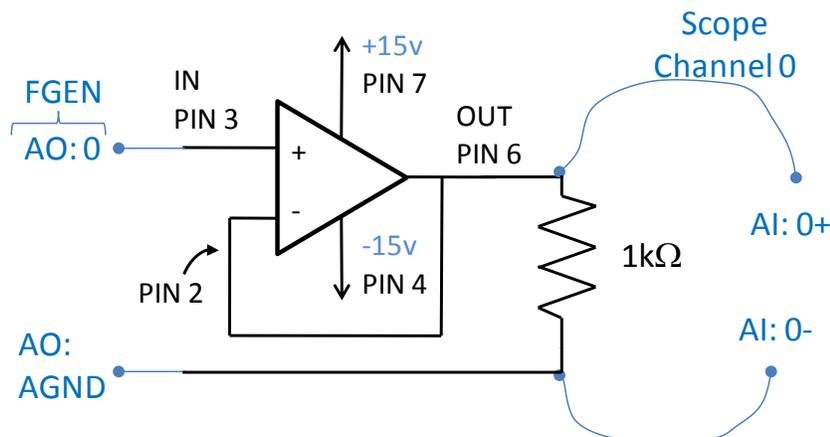
7. Increase the amplitude of the sine wave up to 10 volts peak to peak. Does the signal clip? If not why not? Click "Stop" on the FGEN when finished.

Not Working?
- Make sure that the interface is pushed into to the myDAQ unit all the way. If it comes loose, good electrical connections may not be present.
- The wire leads often come out of the protoboard, make sure they are inserted completely
- It is easy to have wires inserted into the rows next to the ones intended, so look carefully.
- Double-check your pins against the pin diagram, and make sure that the power supply lines are connected properly.
- Double-check the resistor is the correct value.

Part B) Measuring Sinusoidal Steady-State Behavior

8. Use the protoboard to build the circuit below.

Voltage source provided by the function generator, FGEN, passed through the buffer circuit
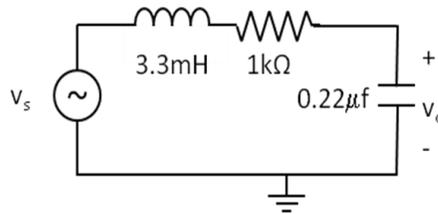
The enhanced schematic showing the op amp and the myDAQ connections:
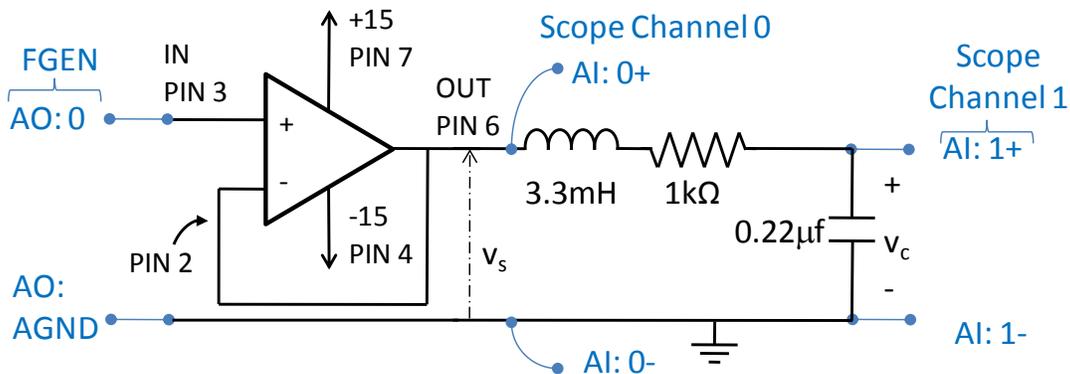
9. Using the impedance method, calculate the steady-state amplitude and phase lag (in degrees) of v_c to an input $v_s = \sin(2\pi ft)$ where f = 500 Hz ($\omega = 2\pi f$).

A = _____   Phase lag, ϕ = _____degrees

10. Measure the steady-state output v_c sine wave of your circuit to an input of $v_s = \sin(2\pi ft)$ for f = 500 hz. Follow these steps for this measurement:
    a. FGEN Settings
       - Set the frequency on FGEN to 500 Hz
       - Set the amplitude to 2 Vpp (that is, peak to peak)
    b. SCOPE Settings
       - Enable Channel 1 and set to AI 1
       - Time/Div = 500 us
       - Scale Volt/Div = 1v on both channels
       - Vertical position = 0v (ie, no DC bias)
       - Turn on the cursors (box is in the bottom left of the SCOPE) and set C1 to CH0 and C2 to CH1
    c. Measure the amplitude and lag of v_c.
       - Drag the line for cursor 2 (C2) from the left axis until the "+" sign is aligned with the peak of the CH 1 (blue) sine wave. Read the corresponding voltage for C2 (below the plot). This is the amplitude:

       A = _____

       - Drag cursor 1 (C1) from the left axis until the "+" sign is aligned with the peak of the CH 0 (green) sine wave. Read the time difference between the input sine wave peak (C1) and output sine wave peak (C2), that is, the dT value under the plots:

       dT = _____. From the background section, compute the phase lag.

       Phase lag, ϕ = _____ degrees.

11. Compare the experimental results to the analytical prediction of the amplitude and phase lag computed in Step 9.

Part C) Frequency Response Data

12. To determine the frequency response experimentally using the method described in the background section, you must take amplitude and phase lag measurements at a number of input frequencies. Start by recording the **ΔT** and amplitude measurements for f = 500 hz measured in Step 10 into the table below. Note that $|H(f)| = A_o/A_i = A_o$ when $A_i = 1$. Repeat the procedure done for f = 500 hz for all of the frequencies in the table.

f(Hz)	ΔT	$\phi = \angle H(f)$	$\|H(f)\| = A_o/A_i$	20log $\|H(f)\|$
100				
200				
500				
1000				
2000				
6000				
10,000				

> Adjust the time scale on the scope to get clear sinusoidal images at each frequency

> Increase A_i to get a measureable output amplitude

13. Compute the log magnitude 20log $\|H(f)\|$ and phase lag ϕ at each frequency to complete the table (this can be completed later, at home).

14. **At home,** you will transfer these data points to semi-log paper. Draw the straight-line approximation for the Bode plot on the semi-log paper and note the difference between the actual data and the approximation.

Part D) Responses in the Time Domain and in the Frequency Domain

This section allows you to see the filtering effects of an RLC circuit in the time domain and in the frequency domain. You will also see the effect of having a small value of R, which results in low damping in the time domain and high resonant peaks.

15. With the same RLC circuit used in Part B), slowly increase the frequency of the input signal in FGEN (change the time/div scale on the SCOPE as needed to see the sine wave clearly). This procedure is called a *sine sweep.* Describe the effect on the amplitude and phase lag of the output signal behavior as the frequency is increased:

    Find the bandwidth, that is, the frequency at which the output amplitude = 0.707 of its low frequency value.

    Bandwidth = _____

16. Change the input waveform to a square wave, 2 Vpp, frequency of approximately 250 – 350 hz and set the time division on the scope to 500usec/div.
    - Capture the SCOPE screen image (the Prnt Scrn key; you may need to press simultaneously the fn key; open Paint, paste the image and crop it).
    - Save the image to a file called HighDampingSquareResponse (as a jpeg).

17. Replace the 1kΩ resistor with a 50Ω resistor and repeat Step 15. Pay particular attention to the behavior between 2k hz and 8k hz as you vary the input frequency. Describe the behavior:

    Bandwidth = _____
    Determine the approximate resonant frequency, where the maximum output amplitude occurs.

    Resonant frequency = _____

18. Repeat Step 16 with the 50Ω resistor and save the image file as LowDampingSquareResponse.jpg.

19. The myDAQ software has a Bode plot tool that automatically does a sine sweep of different frequencies and records the data, essentially automating the procedures that you did in Part C).
    • Close the FGEN and SCOPE windows and open the BODE window.
    • Choose 15 steps per decade and run the Bode analyzer.
    • Capture the screen image (the Prnt Scrn key; you may need to press simultaneously the fn key; open Paint, paste the image and crop it).
    • Save the image to a file called LowDampingBode.jpg
    • Determine the bandwidth (3dB below the low frequency value)

    Bandwidth = _____
    • Determine from the plot the frequency at which the maximum gain occurs:

    Resonant frequency = _____

    And compare these answers to those found in Step 17.

20. Replace the 50Ω resistor with the 1kΩ resistor and repeat step 18 to find the Bode plot of this circuit. Save the results to a file called HighDampingBode.jpg.

    Bandwidth = _____

Extra Time" Try other combinations of resistors (other resistor values or putting them in parallel or series) to see the effect of R on the step response and on the frequency response.

More Time? Put the resistor in parallel with the capacitor and redo some of the experiments.

Name: _____

To Turn in:

Each person should turn in his/her own lab report including:

1. This lab handout with the answers and discussion questions completed.

2. Bode plot of the high damping circuit obtained Part C). Verify that this plot is consistent with the Bode plot obtained automatically using the Bode tool in Part D).

3. Image files, labeled as low or high damping along with the corresponding value of R.

    - LowDampingSquareResponse.jpg
    - LowDampingBode.jpg
    - HighDampingSquareResponse.jpg
    - HighDampingBode.jpg

4. Briefly summarize the filtering effects of an RLC circuit in the time domain and in the frequency domain (bandwidth, amplitude, etc.) Compare the bandwidth to the value of $\sqrt{1/LC}$. Discuss the effect of having a small value of R in the time domain and in the frequency domain.