First-Order Dynamical Circuits

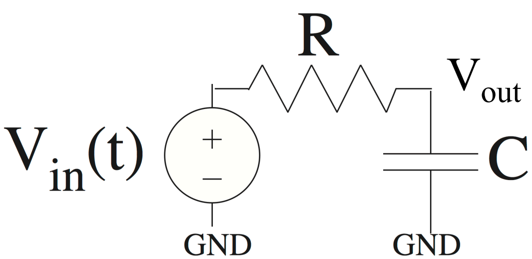
Typically an R + C or L

Need to find equivalent R, C, L

First order → one state variable

→ storage (e.g. Q) point.

R + C Prototype (lowpass)

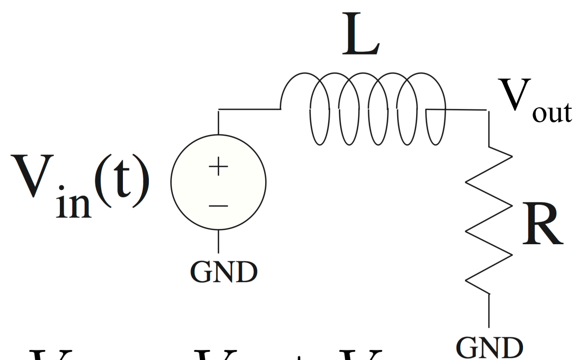


$$V_{in} = V_R + V_{out}$$

$$\frac{V_R}{R} = C\frac{dV_{out}}{dt}$$

$$\tau = RC \rightarrow V_1 = \tau\frac{dV_{out}}{dt}$$

R + L Prototype (lowpass)



$$V_{in} = V_L + V_{out}$$

$$i = \frac{V_{out}}{R} \quad \tau = \frac{L}{R}$$

$$V_L = L\frac{di}{dt} = \frac{L}{R}\frac{di}{dt}$$

$$\tau\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

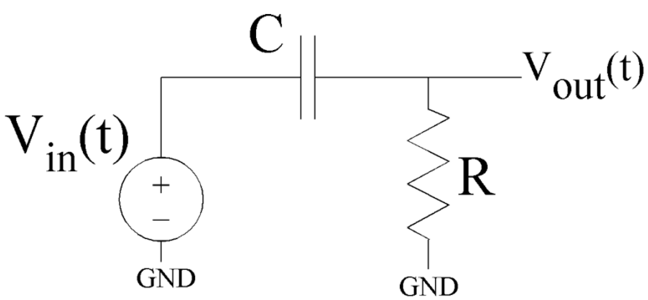
1st-Order High-Pass Circuits

Typically an R + C or L

Need to find equivalent R, C, L

First order → one state variable
→ storage (e.g. Q) point.

R + C Prototype (lowpass)

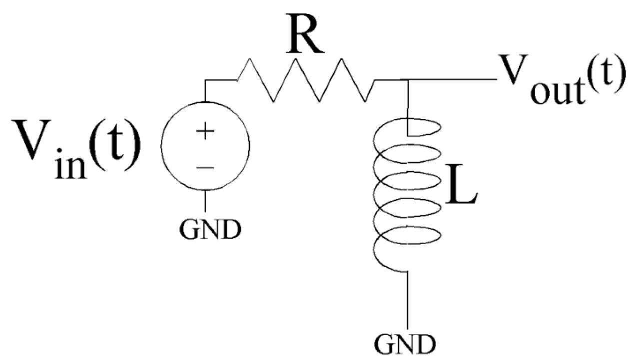


$$V_{in} = V_C + V_{out}$$

$$\frac{V_{out}}{R} = C\frac{d(V_{in} - V_{out})}{dt}$$

$$\tau = RC \rightarrow V_{out} = \tau\frac{d(V_{in} - V_{out})}{dt}$$

R + L Prototype (lowpass)

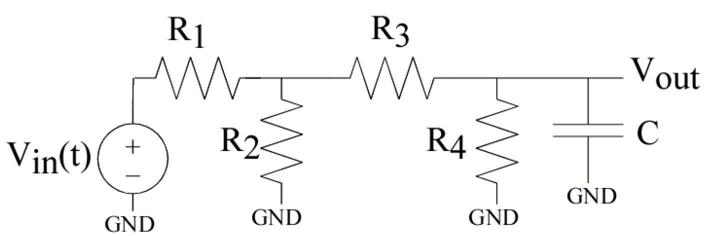


$$I = \frac{V_{in} - V_{out}}{R}$$

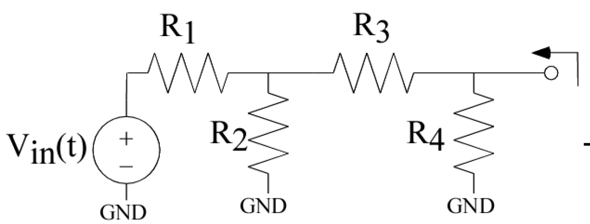
$$V_{out} = L\frac{dI}{dt} = \frac{L}{R}\frac{d(V_{in} - V_{out})}{dt}$$

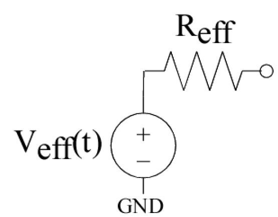
$$\tau = \frac{L}{R}$$

$$\boxed{\tau\frac{dV_{out}}{dt} + V_{out} = \tau\frac{dV_{in}}{dt}}$$



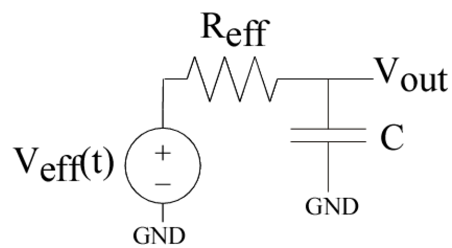
$$R_1 = 10k\Omega$$
$$R_2 = 10k\Omega$$
$$R_3 = 5k\Omega$$
$$R_4 = 10k\Omega$$
$$C = 200nF$$





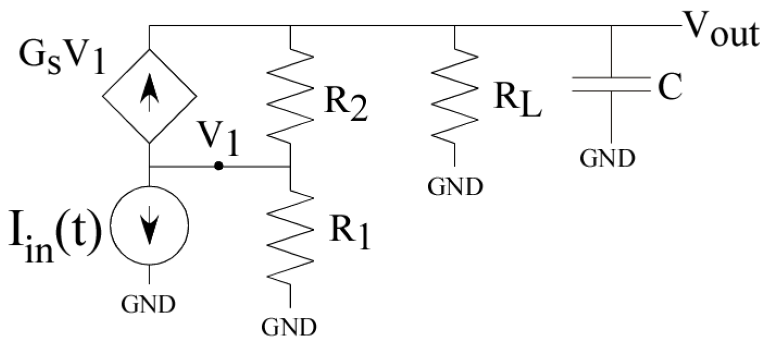
$$V_{eff} = \frac{1}{4} V_{in}(t)$$

$$R_{eff} = 5k\Omega$$

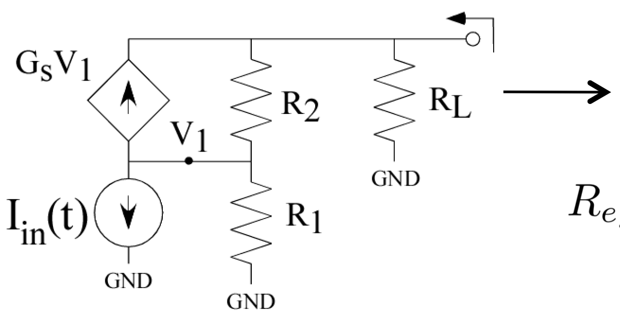


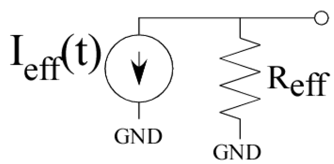
$$\tau = R_{eff} C = 1ms$$

$$\tau \frac{dV_{out}}{dt} + V_{out} = \frac{1}{4} V_{in}$$

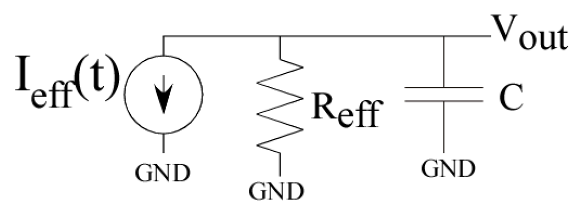


$$G_s = 1/1k\Omega$$
$$R_1 = 100k\Omega$$
$$R_2 = 100k\Omega$$
$$R_L = 1M\Omega$$
$$C = 1nF$$





$$R_{eff} = R_L//(R_1 G_s R_2)$$
$$= 1M\Omega//10M\Omega \approx 1M\Omega$$
$$I_{eff}(t) \approx I_{in}(t)$$



$$\tau = R_{eff}C = 1ms$$

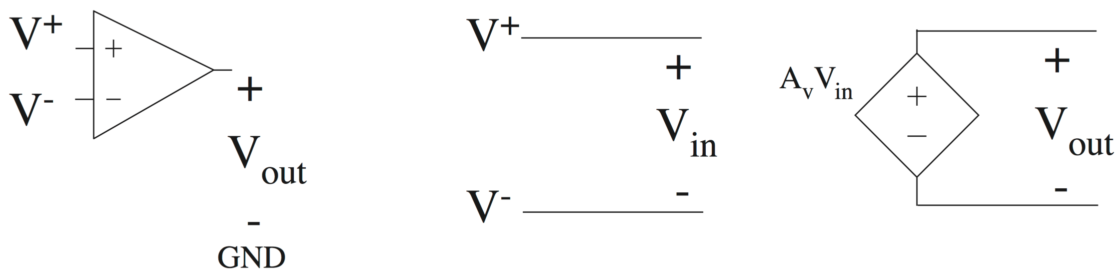
$$\tau \frac{dV_{out}}{dt} + V_{out} = -R_{eff} I_{in}$$

Op-Amp Basics

$V_{out} = A_v(V^+ - V^-)$

Typical A_v:

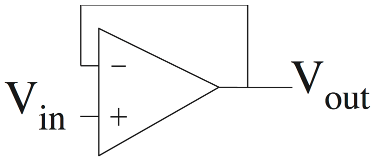
300 – 1000000 → infinite



Inputs equal (V^+, V^-), and no connection between them ????

Core Op-Amp Circuits

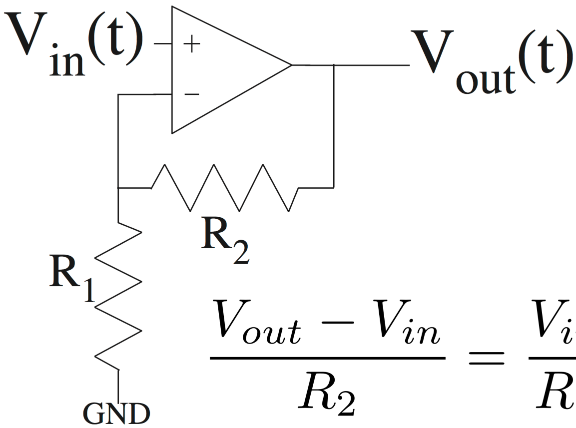
Voltage follower



$V_{out} \sim V_{in}$

Buffer

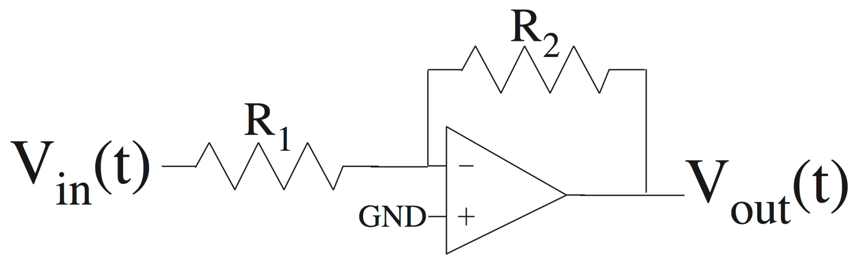
Non-Inverting Amplifier



$$\frac{V_{out} - V_{in}}{R_2} = \frac{V_{in}}{R_1}$$

$$\boxed{\frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}}$$

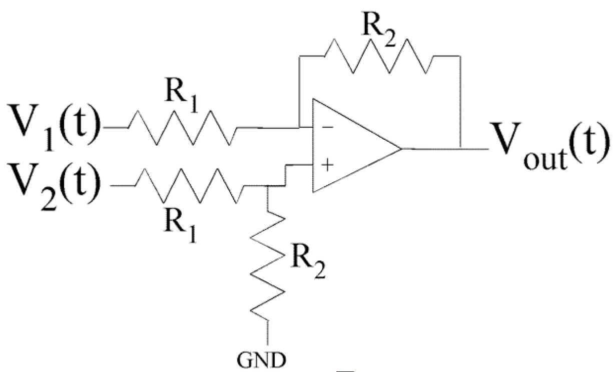
Inverting Amplifier Configuration



$$\frac{V_{in}}{R_1} = \frac{-V_{out}}{R_2}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

Difference Configuration



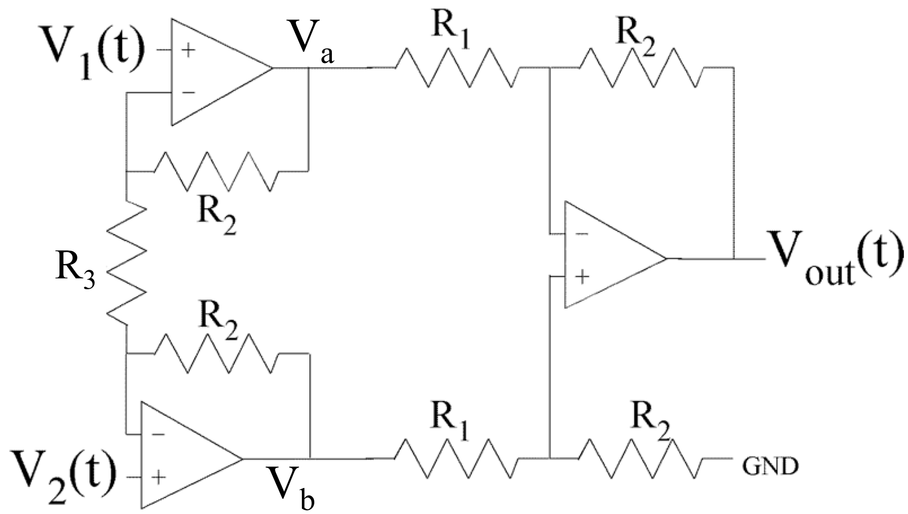
$$V_2(t) = 0V, V_{out} = -\frac{R_2}{R_1}V_1(t)$$

$$V_1(t) = 0V, V_{out} =$$

$$\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_2}{R_1 + R_2}\right)V_2(t) = \frac{R_2}{R_1}V_2(t)$$

$$V_{out} = \frac{R_2}{R_1}(V_2(t) - V_1(t))$$

Instrumentation Amplifier



Symmetric Circuit

($R_3 = R_3/2 + R_3/2$)

Symmetry Point =0V

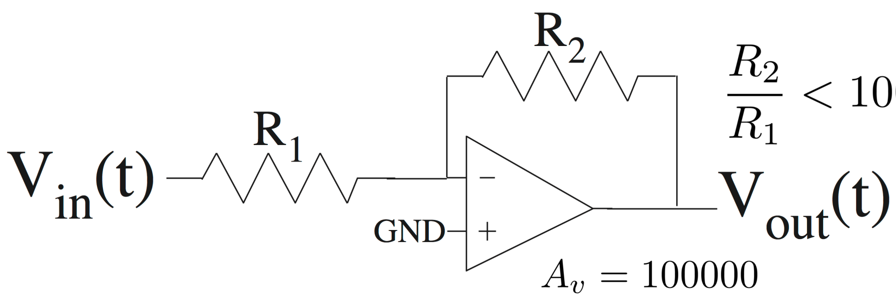
(diff circuit)

Difference Configuration

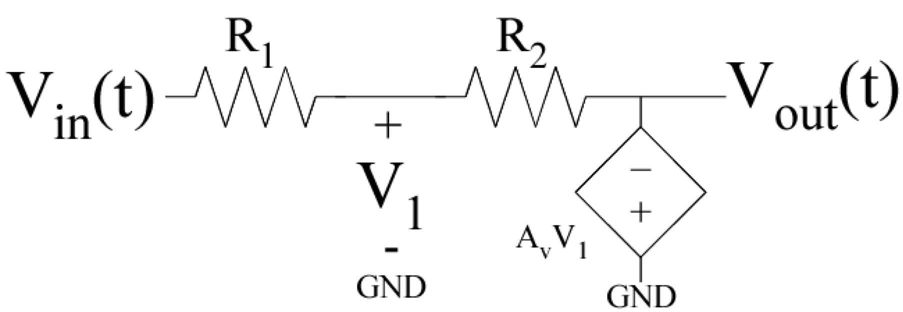
$$V_{out} = \frac{R_2}{R_1}(V_b(t) - V_a(t))$$

$$V_b - V_a = \left(1 + \frac{2R_2}{R_3}\right)(V_2 - V_1)$$

$$V_b - V_a = (V_2 - V_1) \cdot \frac{R_2}{R_1}\left(1 + \frac{2R_2}{R_3}\right)$$



$$\frac{R_2}{R_1} < 100$$



$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{A_v(R_1//R_2)} - \frac{V_{out}}{R_2}$$

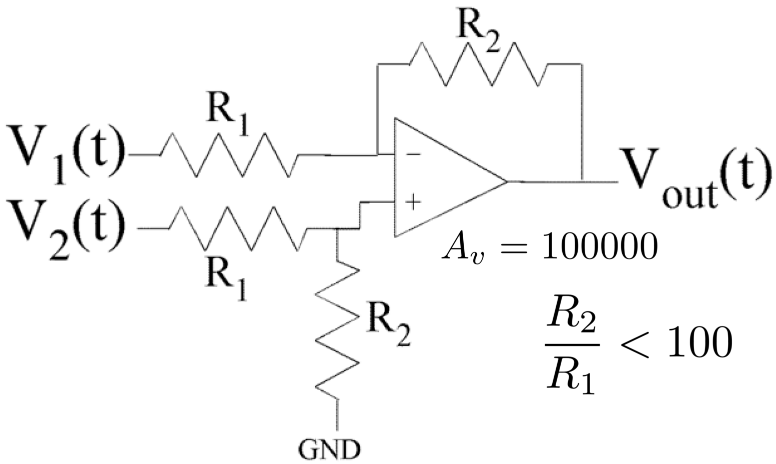
$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\frac{1}{1+\frac{R_1}{A_v(R_1+R_2)}}$$

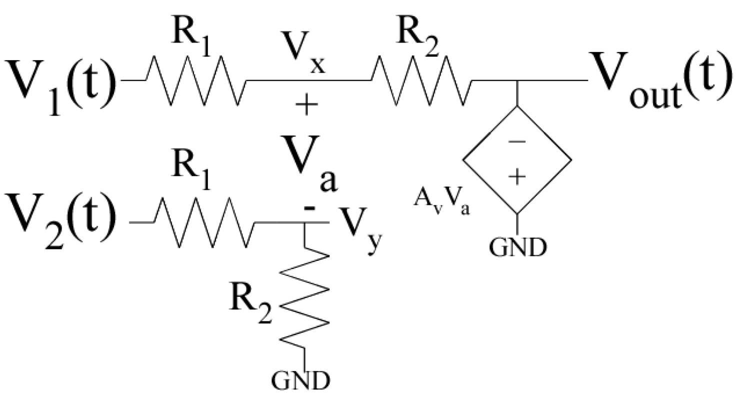
$$\approx -\frac{R_2}{R_1}$$

Invert this term

$$V_{out} = -A_v V_1 \qquad \frac{V_{in}-V_1}{R_1} = \frac{V_1 - V_{out}}{R_2}$$

$$\begin{matrix} R_1 = 10k\Omega \\ R_2 = 100k\Omega \end{matrix} \longrightarrow \frac{V_{out}}{V_{in}} = -10$$





$$\frac{V_1 - V_x}{R_1} = \frac{V_x - V_{out}}{R_2} \longrightarrow \frac{V_1}{R_1} + \frac{V_{out}}{R_2} = \frac{V_x}{R_1//R_2}$$

$$V_y = \frac{R_2}{(R_1 + R_2)} V_2 \qquad V_x = (R_1//R_2)\left(\frac{V_1}{R_1} + \frac{V_{out}}{R_2}\right)$$

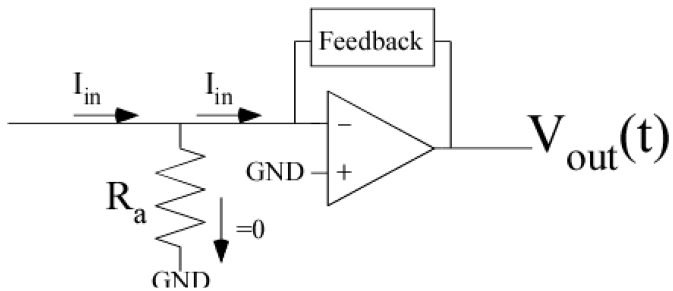
$$V_{out} = -A_v V_a = A_v (V_y - V_x)$$

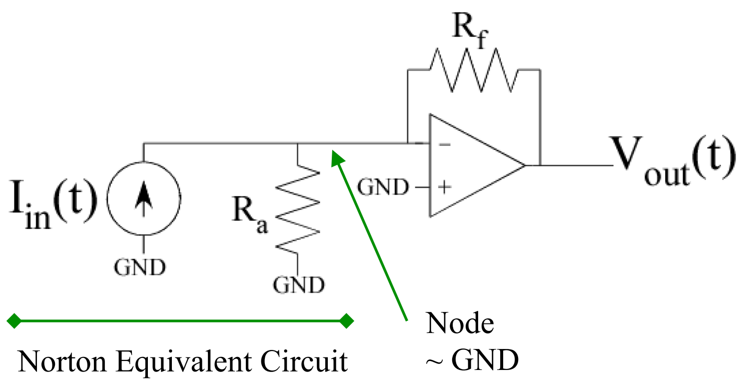
$$\frac{V_{out}}{A_v} = V_y - V_x = \frac{R_2}{R_1 + R_2} V_2 - \frac{R_2}{R_1 + R_2} V_1 - \frac{R_1}{R_1 + R_2} V_{out}$$

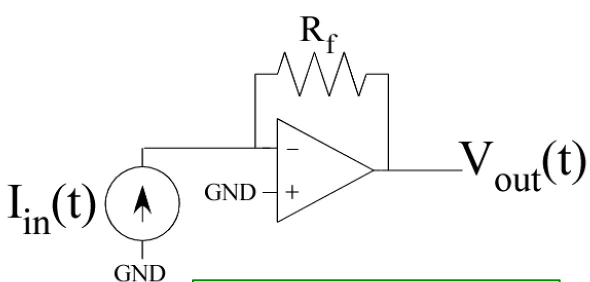
$$V_{out}\left(1 + \frac{R_1 + R_2}{A_v R_1}\right) = \frac{R_2}{R_1}(V_2 - V_1)$$

$$V_{out} \approx -\frac{R_2}{R_1}(V_2 - V_1)$$

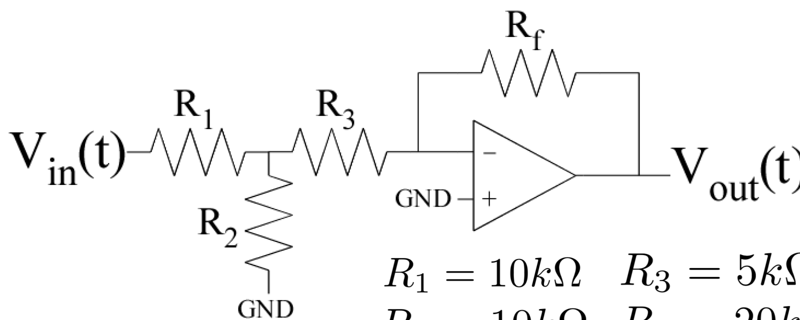
$$R_1 = 10k\Omega$$
$$R_2 = 100k\Omega$$
$$\longrightarrow V_{out} \approx -10(V_2 - V_1)$$





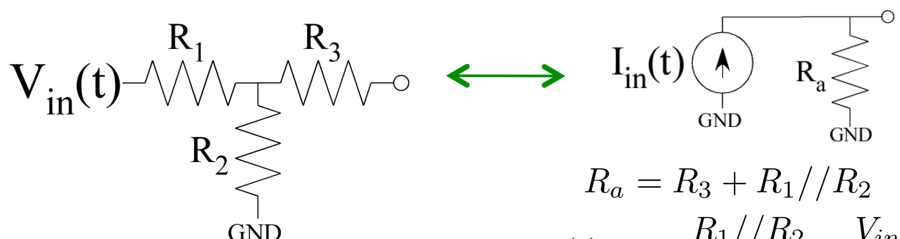


$V_{out}(t) = -R_f\, I_{in}(t)$



$R_1 = 10k\Omega \quad R_3 = 5k\Omega$

$R_2 = 10k\Omega \quad R_f = 20k\Omega$



$$R_a = R_3 + R_1//R_2$$

$$I_{in}(t) = \frac{R_1//R_2}{R_3 + R_1//R_2} \frac{V_{in}}{R_1}$$

$$V_{out}(t) = -\frac{R_f}{R_1} \frac{R_1//R_2}{R_3 + (R_1//R_2)} V_{in}$$

$$R_a = 10k\Omega \qquad I_{in}(t) = \frac{V_{in}}{20k\Omega} \qquad V_{out}(t) = -V_{in}$$

How to handle general linear differential equations with arbitrary inputs (e.g. signals)?

$$\frac{d^4y}{dt^4} + 3\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 25y = x_{in}(t)$$

Change d/dt → s (Heavyside)

$$s^4y + 3s^3y + 7s^2y + 4sy + 25y = x_{in}(t)$$

and solve in some different ways

Laplace Transform (should be Heavyside Transform)

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

$s = \sigma + j\omega$ Complex exponential Basis functions

$$f(t) \rightarrow F(s) \qquad F(s) \rightarrow f(t)$$

Laplace Transform Table

Time Domain (t)	Laplace Domain (s)
$\frac{df(t)}{dt}$	$sF(s)$
$\int f(t)dt$	$\frac{1}{s}F(s)$
$u(t)$	$\frac{1}{s}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin(\omega_1 t)\,u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$
$\cos(\omega_1 t)\,u(t)$	$\frac{s}{s^2+\omega_1^2}$
$f(t \rightarrow \infty)$	$\lim_{t\rightarrow 0} sF(s)$
$f(t \rightarrow 0)$	$\lim_{t\rightarrow \infty} sF(s)$

$$\tau\frac{dV}{dt}+V(t)=V_{in}(t)$$

$$s\tau V(s)+V(s)=V_{in}(s)$$

$$V(s)=\frac{1}{s\tau+1}V_{in}(s)$$

$$V_{in}(t)=u(t)\rightarrow V_{in}(s)=\frac{1}{s}$$

$$V(s)=\frac{1}{s\tau+1}\frac{1}{s}=\frac{1}{s}-\frac{\tau}{s\tau+1}$$

$$V(s)=\frac{1}{s}-\frac{1}{s+1/\tau}$$

$$V(t)=u(t)-e^{-t/\tau}u(t)=\left(1-e^{-t/\tau}\right)$$

$$V(t\rightarrow 0^+)=\lim_{s\rightarrow\infty}s\left(\frac{1}{s\tau+1}\frac{1}{s}\right)$$

$$=\lim_{s\rightarrow\infty}\frac{1}{s\tau+1}=0$$

$$V(t\rightarrow\infty)=\lim_{s\rightarrow 0}s\left(\frac{1}{s\tau+1}\frac{1}{s}\right)$$

$$=\lim_{s\rightarrow 0}\frac{1}{s\tau+1}=1$$

$$V_{in}(t)=\sin(\omega t) \qquad s=\sigma+j\omega\rightarrow j\omega$$

$$\frac{V(s)}{V_{in}(s)}=\frac{1}{s\tau+1} \qquad \frac{V(j\omega)}{V_{in}(j\omega)}=\frac{1}{j\omega\tau+1}$$

$$V_{in}(t)=\frac{1}{\sqrt{1+\omega^2\tau^2}}\sin\left(\omega t-\tan^{-1}(\tau\omega)\right)$$

$$V_{in}(t)=e^{-at/\tau}u(t)\rightarrow V_{in}(s)=\frac{1}{s+a/\tau}$$

$$V(s)=\frac{1}{s\tau+1}\frac{\tau}{s\tau+a}$$

$$V(s)=\frac{\tau}{a-1}\frac{1}{s\tau+1}-\frac{\tau}{a-1}\frac{1}{\tau s+a}$$

$$V(s)=\frac{1}{a-1}\frac{1}{s+1/\tau}-\frac{1}{a-1}\frac{1}{s+a/\tau}$$

$$V(t)=\frac{1}{a-1}\left(e^{-t/\tau}-e^{-at/\tau}\right)u(t)$$

$$\tau^2 \frac{d^2V}{dt^2} + 3\tau \frac{dV}{dt} + 2V(t) = V_{in}(t)$$

$$s^2\tau^2 V(s) + 3s\tau V(s) + 2V(s) = V_{in}(s)$$

$$V(s) = \frac{1}{s^2\tau^2 + 3s\tau + 2} V_{in}(s)$$

$$V(s) = \frac{1}{(s\tau + 2)(s\tau + 1)} V_{in}(s)$$

$$V_{in}(t) = u(t) \rightarrow V_{in}(s) = \frac{1}{s}$$

$$V(s) = \frac{1}{s\tau + 2} \frac{1}{s\tau + 1} \frac{1}{s}$$

$$V(s) = A\frac{\tau}{s\tau + 2} + B\frac{\tau}{s\tau + 1} + C\frac{1}{s}$$

$$V(s) = \frac{1}{2}\frac{1}{s + 2/\tau} - \frac{1}{s + 1/\tau} + \frac{1}{2}\frac{1}{s}$$

$$V_{out}(t) = \left(\frac{1}{2}\left(1 + e^{-2t/\tau}\right) - e^{-t/\tau}\right) u(t)$$

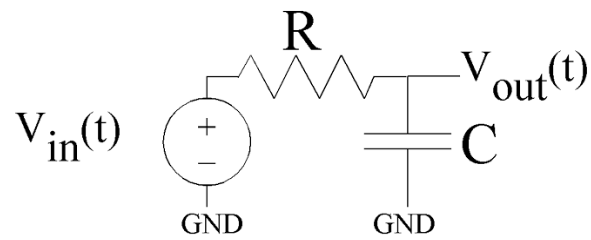
$$\frac{V(s)}{V_{in}(s)} = \frac{1}{s^2\tau^2 + 3s\tau + 2}$$

$$V_{in}(t) = \sin(\omega t)$$

$$s = \sigma + j\omega \rightarrow j\omega$$

$$\frac{V(j\omega)}{V_{in}(j\omega)} = \frac{1}{(j\omega)^2\tau^2 + 3(j\omega)\tau + 2}$$

$$V_{in}(t) = \frac{1}{\left((2 - \omega^2\tau^2)^2 + 9\omega^2\tau^2\right)^{1/2}} \sin\left(\omega t - \tan^{-1}\left(\frac{3\omega\tau}{2 - \omega^2\tau^2}\right)\right)$$

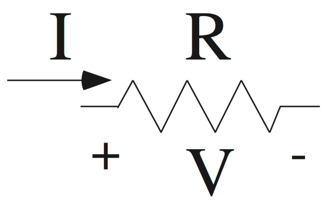


Voltage Divider:

$$V_{out}(s) = V_{in}(s)\frac{1/sC}{R+1/sC}$$
$$= V_{in}(s)\frac{1}{1+sRC}$$
$$= V_{in}(s)\frac{1}{1+s\tau}$$

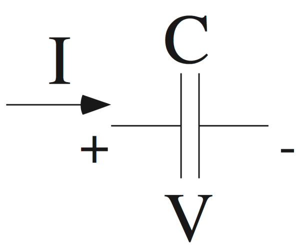
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+s\tau}$$

<u>Laplace form of Circuit elements</u>



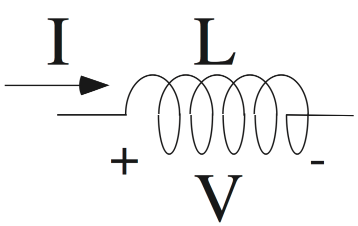
<u>Resistor</u>

$$V = RI \longrightarrow V(s) = R\,I(s)$$



<u>Capacitor</u>

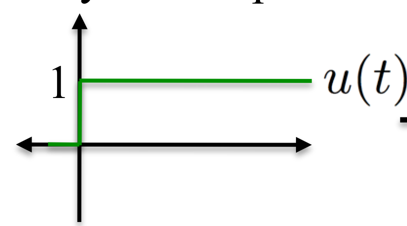
$$I = C\frac{dV}{dt} \longrightarrow I(s) = s\,C\,V(s)$$



<u>Inductor</u>

$$V = L\frac{dI}{dt} \longrightarrow V(s) = s\,L\,I(s)$$

Heavyside Step Function



$$\longrightarrow \frac{1}{s}$$

$$V_{out}(t) = (1 - e^{-t/\tau})u(t)$$

Laplace Transform → Fourier Transform

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt \qquad F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

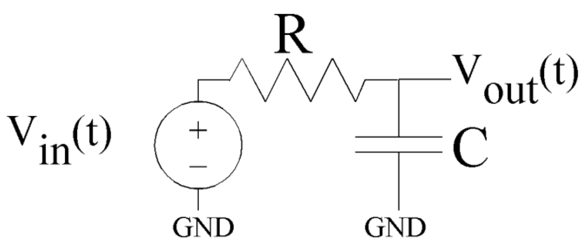
$$s = \sigma + j\omega \qquad \sigma = 0, s \to j\omega \quad j = \sqrt{-1}$$

$$f(t) \to F(s) \quad F(s) \to f(t)$$

Transform: reversible

Quantum Physics: Momentum relates to Fourier Transform of position

1st order example



$$s \to j\omega \qquad \omega = 2\pi f$$

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\omega\tau} \qquad |a + jb| = \sqrt{a^2 + b^2}$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + s\tau}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2\tau^2}}$$

$$\text{Phase} = \tan^{-1}(\omega\tau)$$

First-Order Low-Pass Filter

$$H(s) = \frac{1}{1 + s\tau}$$

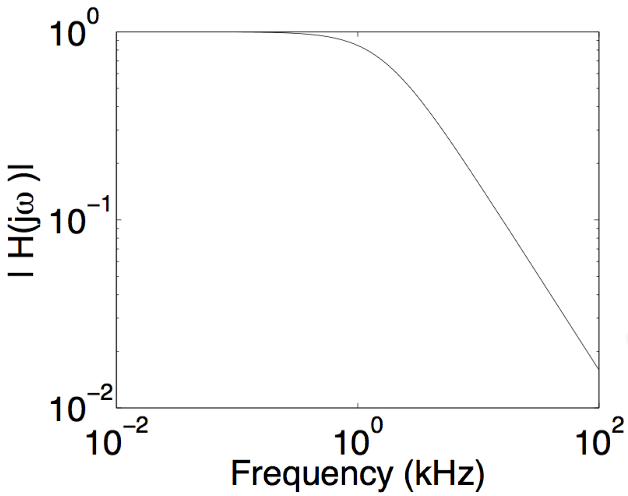
First-Order High-Pass Filter

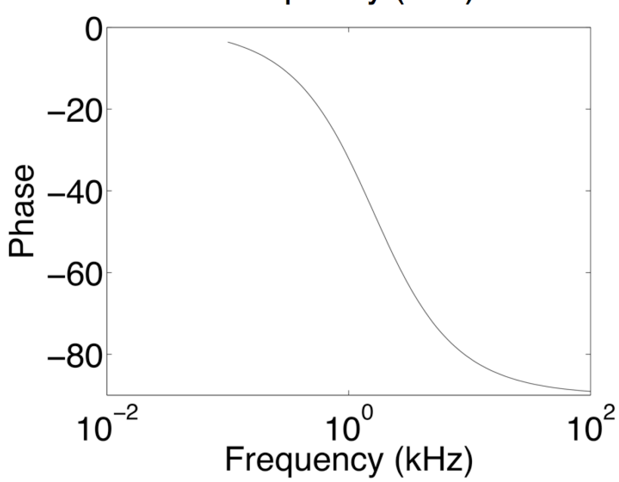
$$H(s) = \frac{s\tau}{1 + s\tau}$$

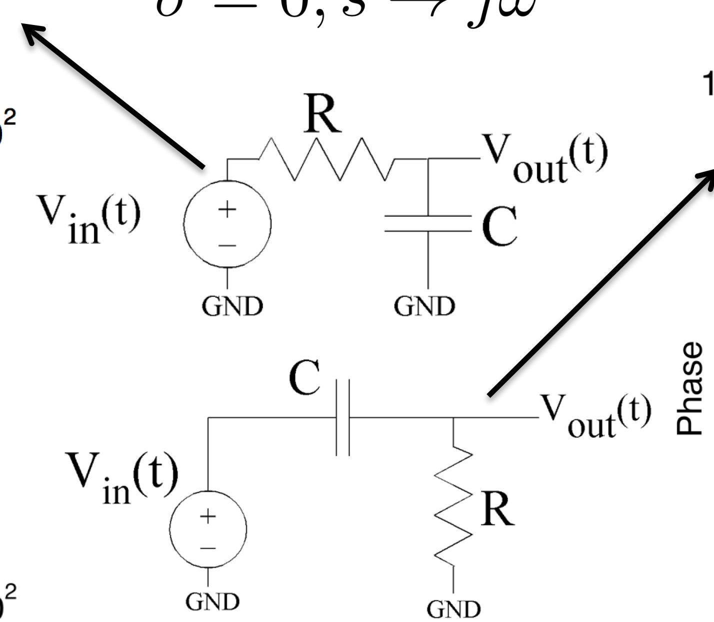
First-Order ODE:

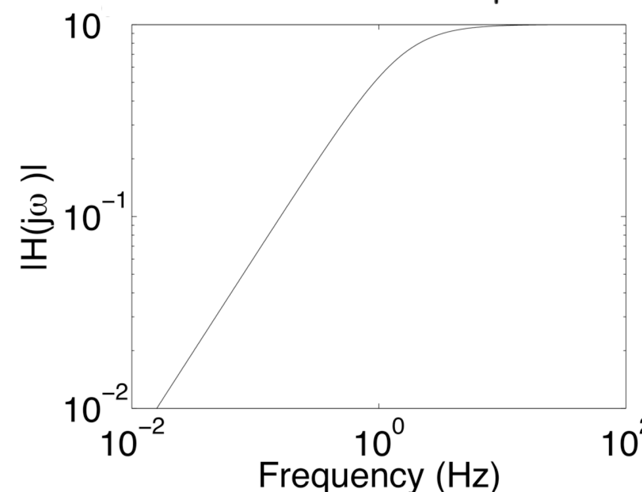
Frequency Response

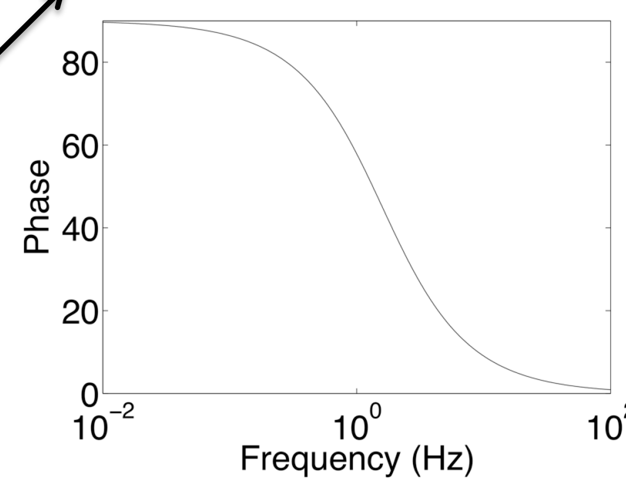
$$\sigma = 0, s \rightarrow j\omega$$

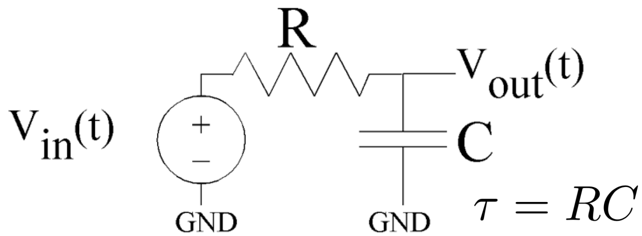








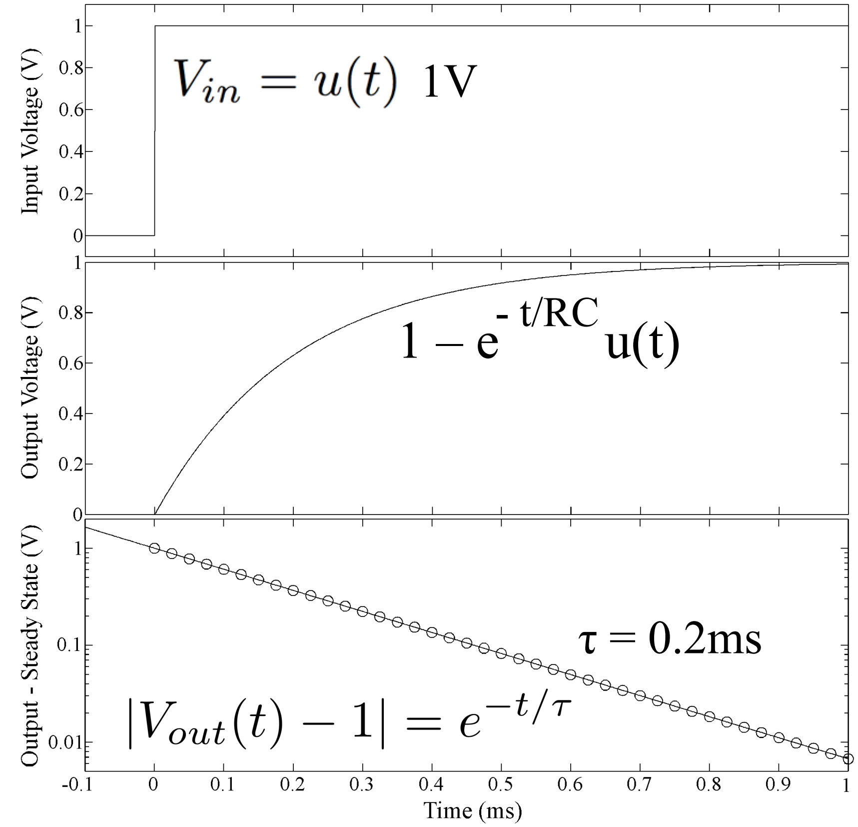


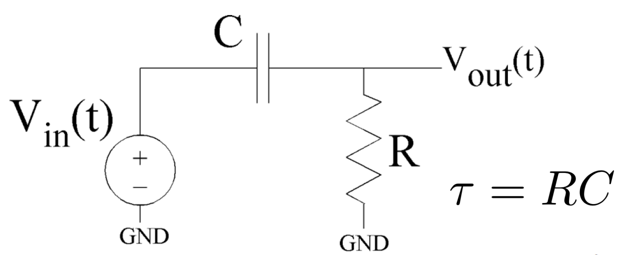


$$H(s) = \frac{1}{1+s\tau} \qquad u(t) \longrightarrow \frac{1}{s}$$

$$V_{out}(s) = \frac{1}{s(1+s\tau)} = \frac{1}{s} - \frac{\tau}{1+s\tau}$$

$$\boxed{V_{out}(t) = (1 - e^{-t/\tau})u(t)}$$

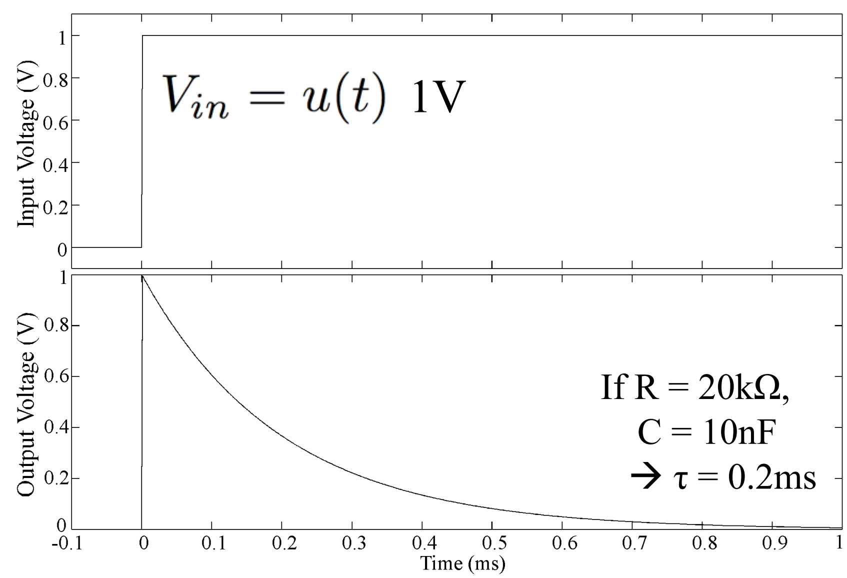




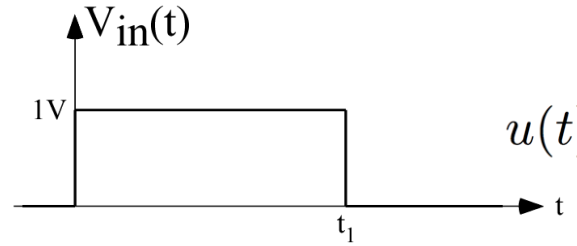
$$H(s) = \frac{s\tau}{1+s\tau} \qquad u(t) \longrightarrow \frac{1}{s}$$

$$V_{out}(s) = \frac{\tau}{1+s\tau} = \frac{1}{\frac{1}{\tau}+s}$$

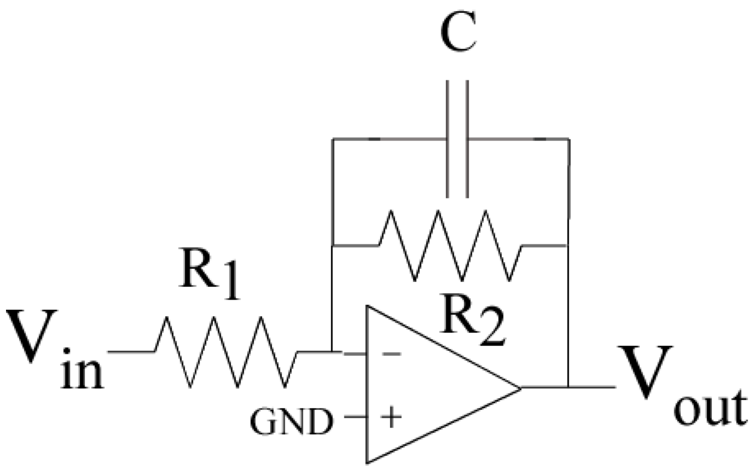
$$\boxed{V_{out}(t) = e^{-t/\tau}u(t)}$$

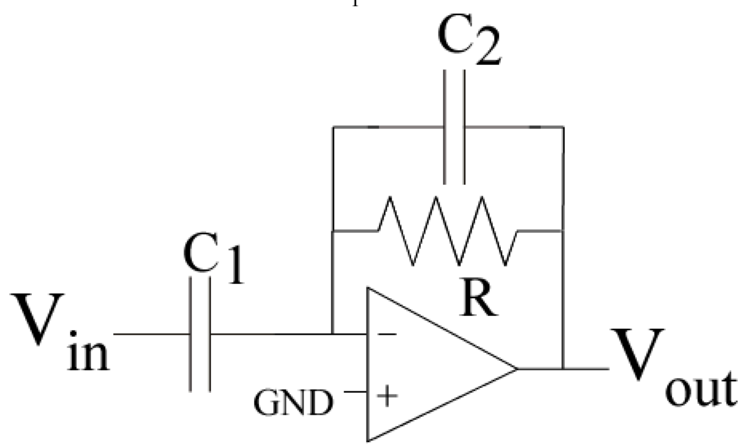


1st Order Step Response (Op-Amp)



$$u(t) \longrightarrow \frac{1}{s}$$





$$V_{in}/R_1 = -V_{out}(sC + 1/R_2)$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\frac{1}{1+s\tau} \quad \tau = R_2 C$$

$$V_{out}(s) = -\frac{R_2}{R_1}\frac{1}{s(1+s\tau)} = \frac{1}{s} - \frac{\tau}{1+s\tau}$$

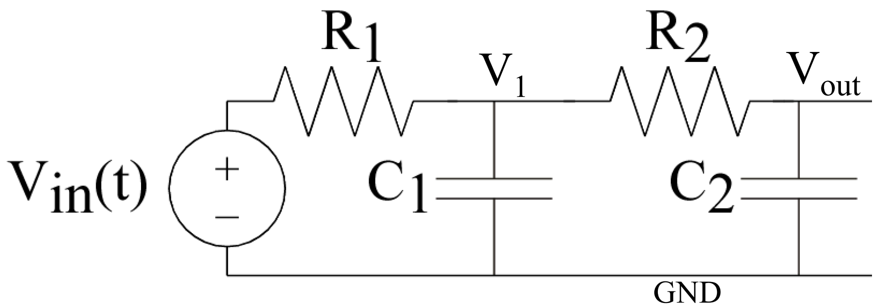
$$V_{out}(t) = -\frac{R_2}{R_1}(1 - e^{-t/\tau})u(t)$$

$$sC_1 V_{in} = -V_{out}(sC_2 + 1/R)$$

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}\frac{s\tau}{1+s\tau} \quad \tau = RC_2$$

$$V_{out}(s) = -\frac{C_1}{C_2}\frac{\tau}{1+s\tau} = \frac{1}{\frac{1}{\tau}+s}$$

$$V_{out}(t) = -\frac{C_1}{C_2}e^{-t/\tau}u(t)$$



$$V_1: \frac{V_{in} - V_1}{R_1} + \frac{V_{out} - V_1}{R_2} = sC_1 V_1$$

$$sC_1 V_1 + \frac{V_1}{R_1 // R_2} = \frac{V_{in}}{R_1} + \frac{V_{out}}{R_2}$$

$$V_{out}: \frac{V_{out} - V_1}{R_2} + sC_2 V_{out} = 0$$

$$sR_2 C_2 V_{out} + V_{out} = V_1$$

$\rightarrow$

$$sC_1(sR_2C_2V_{out} + V_{out}) + \frac{sR_2C_2V_{out} + V_{out}}{R_1//R_2} = \frac{V_{in}}{R_1} + \frac{V_{out}}{R_2}$$

$$sC_1R_1(sR_2C_2V_{out} + V_{out}) + s(R_1 + R_2)C_2V_{out} + V_{out} = V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2 R_1C_1R_2C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

$$\tau = \sqrt{R_1C_1R_2C_2} \qquad Q <= 1/2$$

<u>Specific case</u>: $R = R_1 = R_2$, $C = C_1 = C_2$ $(Q = 1/3)$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2\tau^2 + 3s\tau + 1} = \frac{1}{(s\tau + 0.382)(s\tau + 2.618)} \qquad \tau = RC$$

1V step input (0→1V)

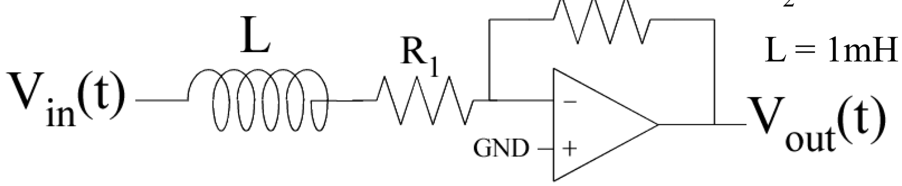
$$u(t) \longrightarrow \frac{1}{s} \quad V_{in}(s) = \frac{1V}{s}$$

$$V_{out} = \frac{1}{s(s\tau + 0.382)(s\tau + 2.618)} = \frac{1V}{s} + \frac{B}{s\tau + 0.382} + \frac{C}{s\tau + 2.618}$$

$$B = \frac{1}{s(s\tau + 2.618)} \text{ for } s\tau = -0.382 \qquad C = \frac{1}{s(s\tau + 0.382)} \text{ for } s\tau = -2.618$$

$$V_{out}(s) = \frac{1V}{s} - \frac{1.171\tau}{s\tau + 0.382} + \frac{0.171\tau}{s\tau + 2.618}$$

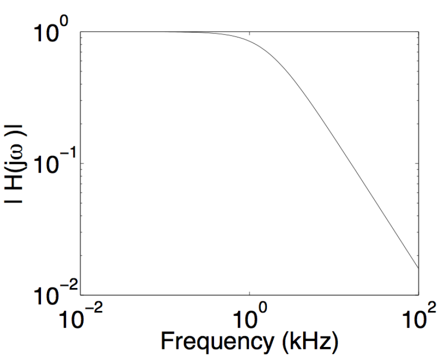
$$V_{out}(t) = \left(1 - 1.171e^{-0.382t/\tau} + 0.171e^{-2.618t/\tau}\right) u(t)$$

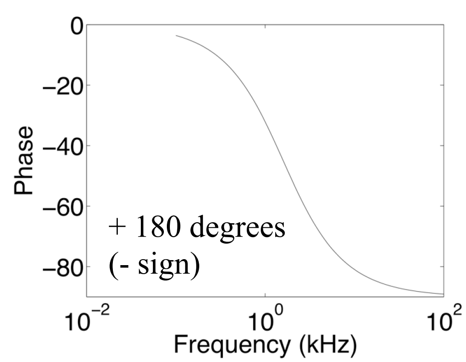


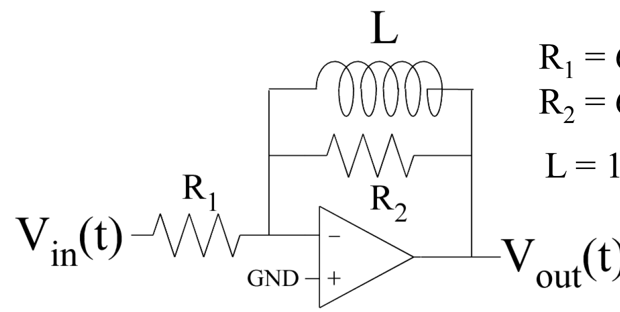
$R_1 = 6\Omega$
$R_2 = 6\Omega$
$L = 1\text{mH}$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1 + sL} = -\frac{R_2}{R_1}\frac{1}{1+s\tau} \qquad \tau = L/R_1$$

τ =1/6 ms





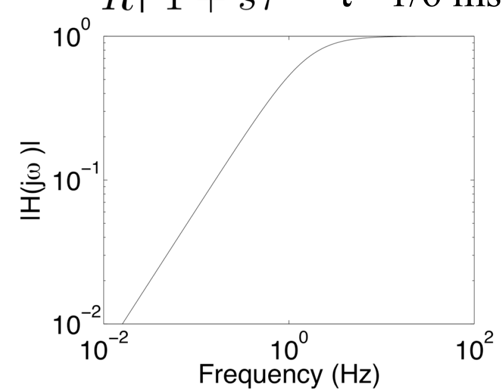


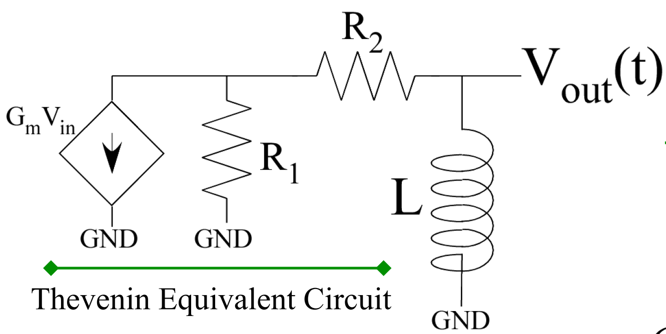
$R_1 = 6\Omega$
$R_2 = 6\Omega$
$L = 1\text{mH}$

$$\frac{V_{in}}{R_1 + sL} = -\frac{V_{out}}{R_2}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2//(sL)}{R_1}$$
$$= -\frac{R_2}{R_1}\frac{s(L/R_2)}{1+s(L/R_2)}$$
$$= -\frac{R_2}{R_1}\frac{s\tau}{1+s\tau} \qquad \tau = L/R_2$$

τ =1/6 ms





Thevenin Equivalent Circuit

$R_1 = 100k\Omega$ $\quad$ $G_m = 1/10k\Omega$

$R_2 = 100k\Omega$ $\quad$ $L = 6mH$

Becomes an R-L Circuit

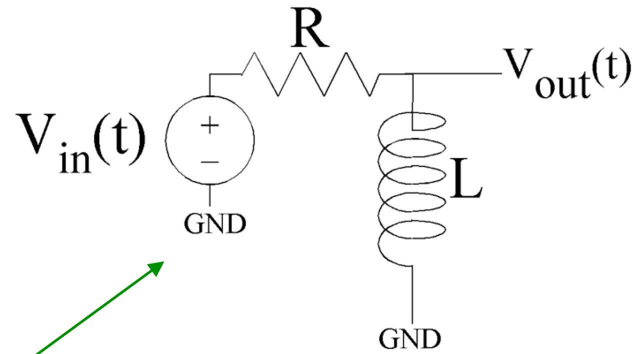
$$R_1 + R_2 \rightarrow R$$

(200kΩ)

$$G_m R_1 V_{in}(t) \rightarrow V_{in}(t)$$

($-10\ V_{in}$)

$$V_{in} = 1V u(t) \rightarrow -10V u(t)$$



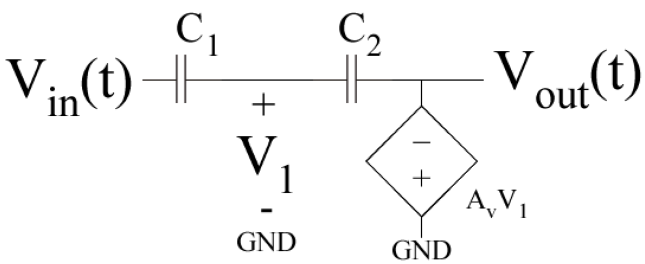
$$V_{out} = L\frac{dI}{dt} = \frac{L}{R}\frac{d(V_{in} - V_{out})}{dt}$$

$$\tau\frac{dV_{out}}{dt} + V_{out} = \tau\frac{dV_{in}}{dt} \qquad \tau = \frac{L}{R}$$

τ =30ns

$$V_{out}(s) = -\frac{10}{s + 1/\tau}$$

$$V_{out}(t) = -10e^{-t/\tau}u(t)$$



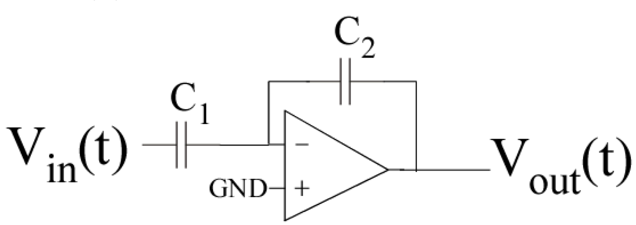
$$sC_1(V_{in} - V_1) = sC_2(V_1 - V_{out})$$

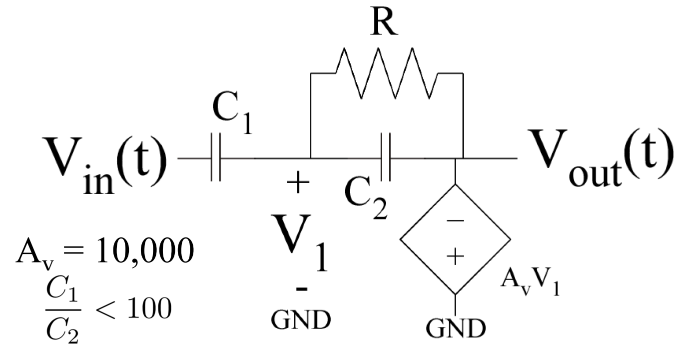
$$V_{out} = -A_v V_1$$

$$sC_1 V_{in} = -\left(sC_2 + s\frac{C_1 + C_2}{A_v}\right) V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{sC_2 + s\frac{C_1+C_2}{A_v}}{sC_1}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{C_2 + \frac{C_1+C_2}{A_v}}{C_1} \quad \text{unless } s = 0.$$





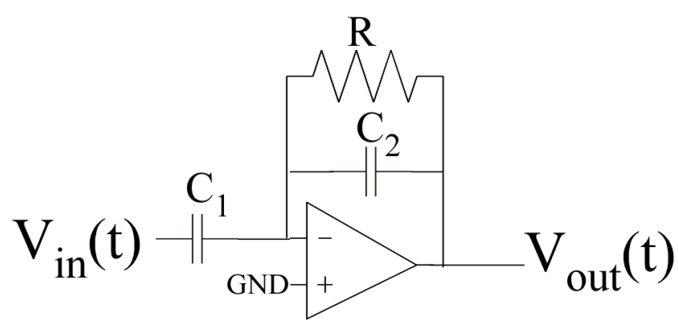
$$sC_1(V_{in} - V_1) = \frac{V_1 - V_{out}}{R} + sC_2(V_1 - V_{out})$$

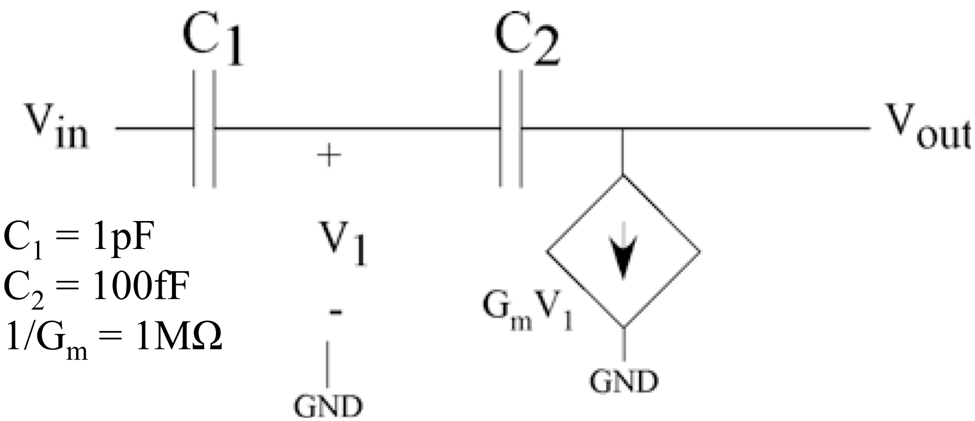
$$sC_1 V_{in} = -V_{out}\left(sC_2 + \frac{1}{R} + \frac{s(C_1 + C_2) + \frac{1}{R}}{A_v}\right)$$

$$sC_1 V_{in} = -V_{out}\left(s\left(C_2 + \frac{C_1 + C_2)}{A_v}\right) + \frac{1}{R}\left(1 + \frac{1}{A_v}\right)\right)$$

$$\frac{C_1}{C_2} < 100 \qquad \frac{V_{out}}{V_{in}} = \frac{sC_1 R}{sC_2 R + 1}$$

1st-order HPF





$C_1 = 1\text{pF}$
$C_2 = 100\text{fF}$
$1/G_m = 1\text{M}\Omega$

$$C_1\frac{d(V_{in}-V_1)}{dt} = C_2\frac{d(V_1-V_{out})}{dt} = G_m V_1$$

$$sC_1(V_{in}-V_1) = sC_2(V_1-V_{out}) = G_m V_1$$

$$\left(1+s\frac{C_1}{G_m}\right)V_1 = s\frac{C_1}{G_m}V_{in} \qquad -\left(1-s\frac{C_2}{G_m}\right)V_1 = s\frac{C_2}{G_m}V_{out}$$

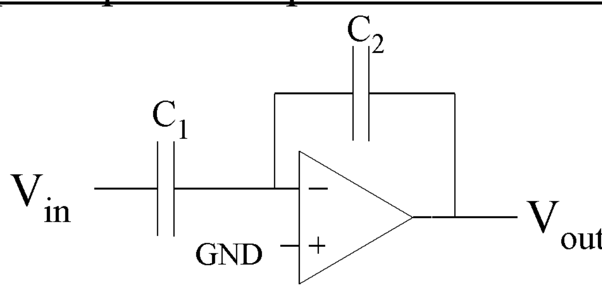
$$-s\frac{C_1}{G_m}\left(1-s\frac{C_2}{G_m}\right)V_{in} = s\frac{C_2}{G_m}\left(1+s\frac{C_1}{G_m}\right)V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{s}{s}\frac{C_1}{C_2}\frac{1-s\frac{C_2}{G_m}}{1+s\frac{C_1}{G_m}} \qquad A_v = -\frac{C_1}{C_2} \quad \tau = \frac{C_1}{G_m}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{s}{s}A_v\frac{1-s\frac{\tau}{A_v}}{1+s\tau}}$$

1st Order Transfer Function

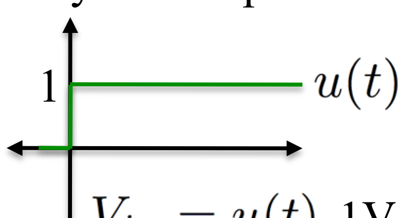
Op-Amp with Capacitive Feedback



$$C_1\frac{dV_{in}}{dt} = -C_2\frac{dV_{out}}{dt}$$

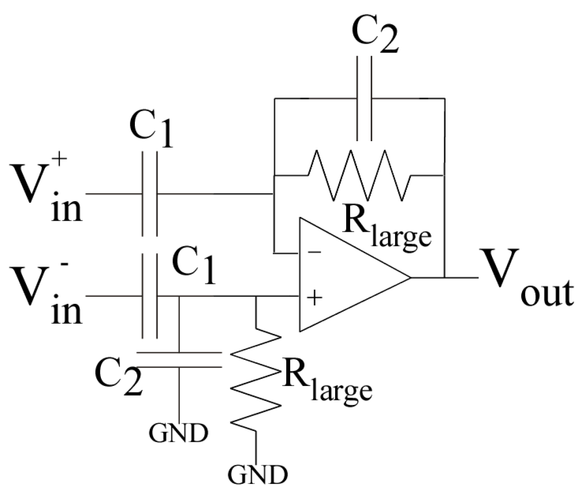
$$V_{out}(t) = -\frac{C_1}{C_2}V_{in}(t) + V_{offset}$$

Heavyside Step Function



$V_{in} = u(t)$ 1V

Differential inverting capacitor-based amplifier
(R. Harrison: Front-End Neural Amplifier)



$Z_1 = 1/sC_1$ (input) $Z_2 = R_{large}$ // $(1/sC_1)$ (feedback)

$$V_{out} = -\frac{C_1}{C_2}\frac{s\tau}{1+s\tau}(V_{in}^{+} - V_{in}^{-})$$

$\tau = R_{large}C_2$ (< 1Hz)

R_{large} sets biasing (no Q issues)

For higher frequencies: $V_{out} = -\frac{C_1}{C_2}(V_{in}^{+} - V_{in}^{-})$