Course on Linear Circuits

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Course Website:

- http://hasler.ece.gatech.edu/Courses/ECE2040/index.html
- Priority for open materials
- Short Lecture (YouTube) Nuggets

Mixture of objective & subjective

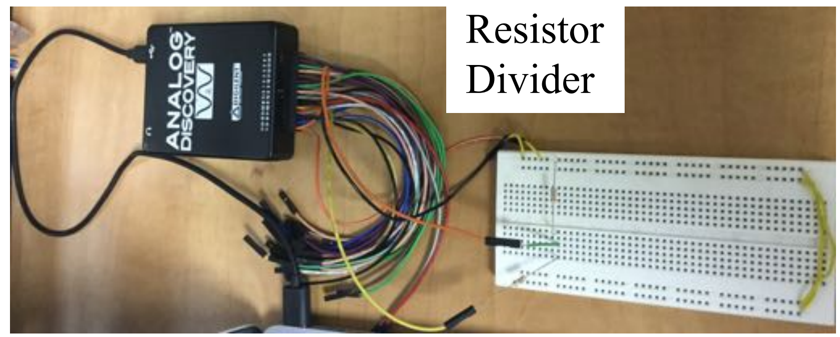
- Objective → Exams
- Subjective → Labs (often groups of 2)

Other rules

- No recording / pictures allowed in class
- Groups start in a self-organized manner
- Read the Syllabus

My Research Interests:

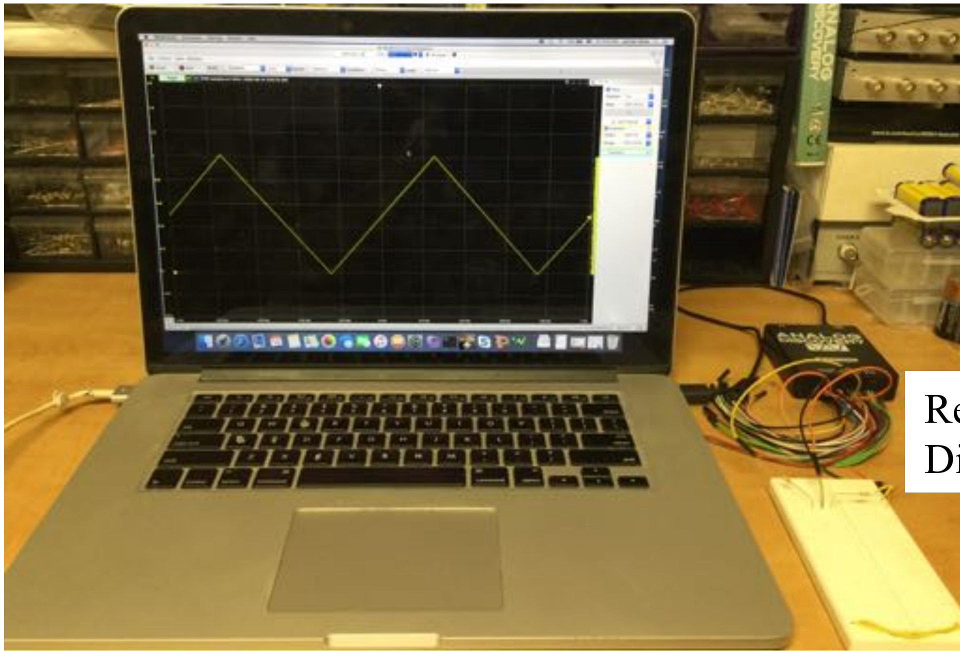
- Floating-Gate Devices & Circuits, Nonvolatile
- Wearable computing, always-on computing, etc.
- Circuits for Machine Learning, AI, Neuroscience
- Configurable, CAD tools
- Sensory interfaces



A few points of my history:

- Began as faculty at GT in 1997
- Ph.D. in Computation and Neural Systems (Caltech, 1997)
- Grew up in FL and AZ, B.S.E. & M.S. ASU (1991)
- First startup, GTronix (2002-2010), acquired by TI
- MDiv, Emory University 2020

Circuit Measurement Anywhere



Resistor
Divider

Laboratory measurements anywhere:

- Laboratory experiments can be done anywhere
- Electronics everywhere, so relatively inexpensive labs
- Utilize an acquisition system
  (e.g. scope/function generator)

Class laboratory concepts:

- Need a data acquisition system
- Work on experiments before a class session
- Focus on understanding material deeply

Circuit Graph Concepts

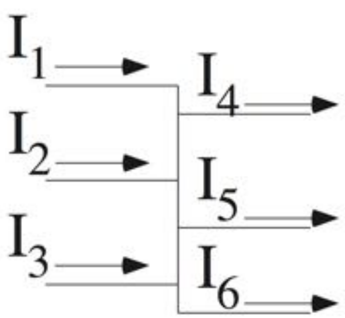
Wire

(only connection,
no R, L, C)

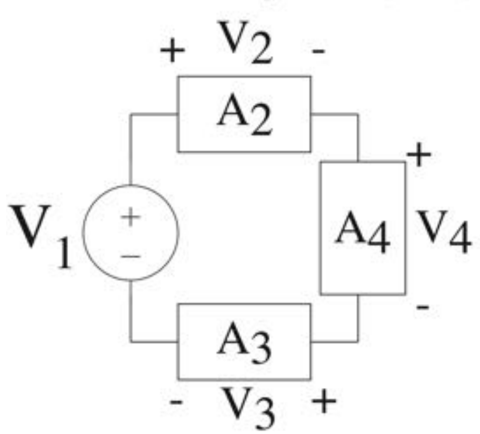
Flattened 2-Dim graph

Often: DC path to GND
(→ no charge storage)

Kirchoff's Current Law (KCL)

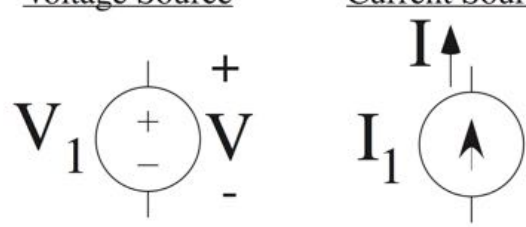


Kirchoff's Voltage Law (KVL)



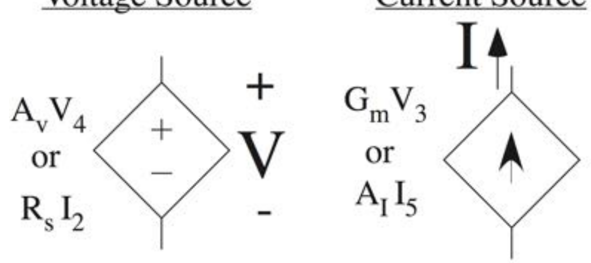
Independent Sources

Voltage Source | Current Source

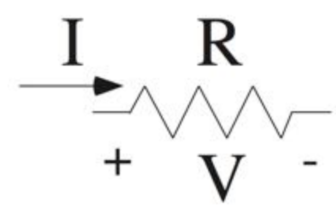


Dependent Sources

Voltage Source | Current Source

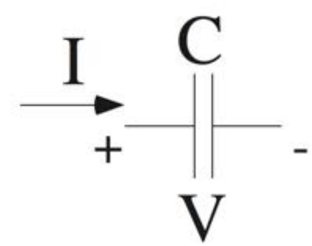


Circuit Graph Concepts



Resistor

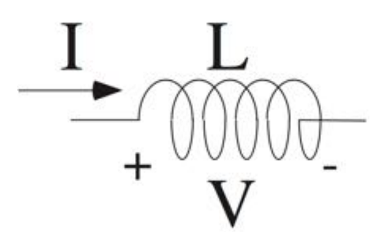
$$V = RI$$



Capacitor

$$I = C\frac{dV}{dt}$$

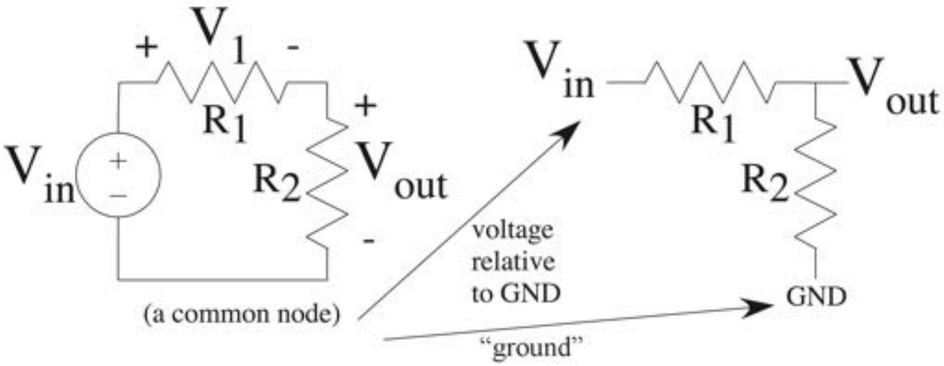
$$Q = CV$$



Inductor

$$V = L\frac{dI}{dt}$$

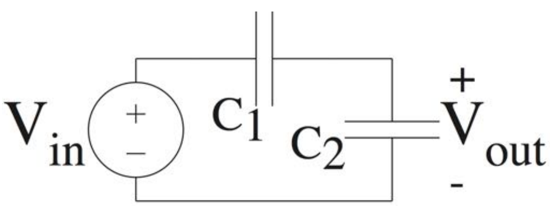
Example: Resistive Voltage Divider



$$V_{in} = V_1 + V_{out} \qquad V_{in} = \frac{R_1}{R_2} V_{out} + V_{out}$$

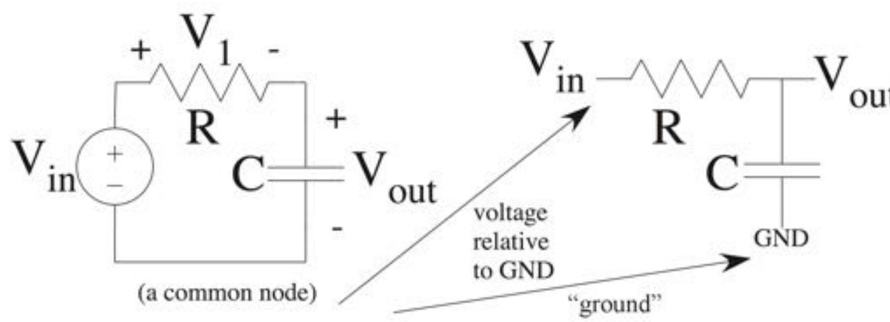
$$I = \frac{V_1}{R_1} = \frac{V_{out}}{R_2} \qquad \boxed{V_{out} = V_{in} \frac{R_2}{R_1 + R_2}}$$

Capacitive Divider?



$$V_{in} = V_1 + V_{out}$$
$$Q_1 = C_1 V_1$$
$$Q_2 = C_2 V_{out}$$
$$\ldots\ ?$$

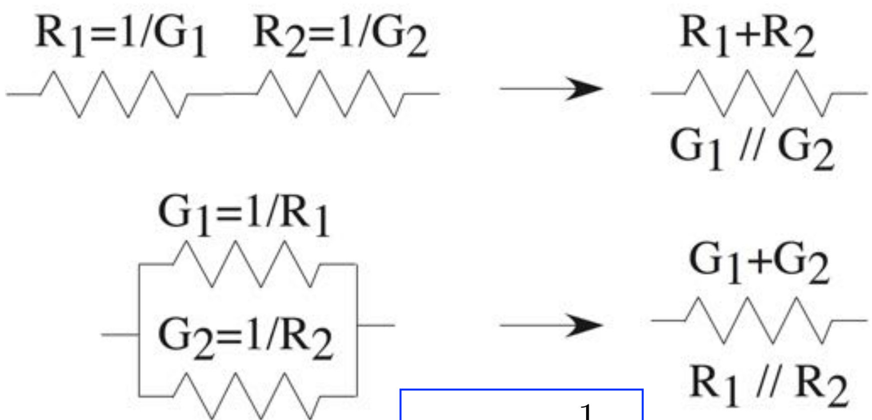
Example: RC First-Order Low-Pass Filter



$$V_{in} = V_1 + V_{out} \qquad \tau = RC \rightarrow V_1 = \tau \frac{dV_{out}}{dt}$$

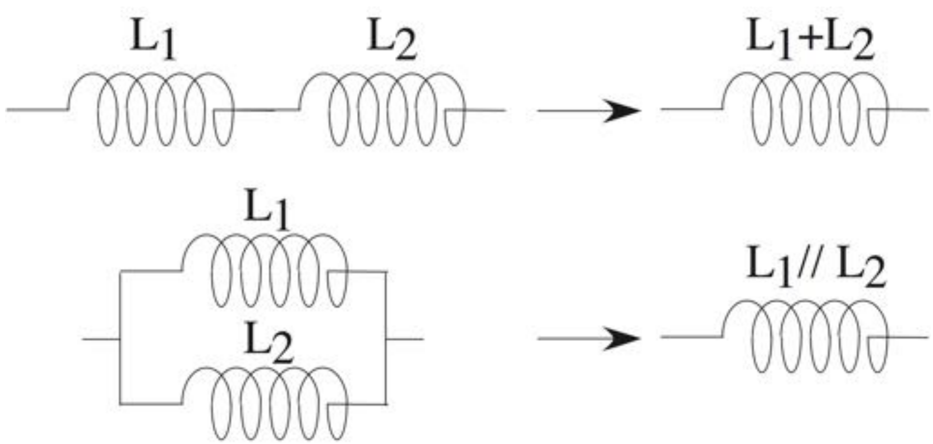
$$\frac{V_1}{R} = C \frac{dV_{out}}{dt} \qquad \boxed{\tau \frac{dV_{out}}{dt} + V_{out} = V_{in}}$$

Resistors: Serial & Parallel

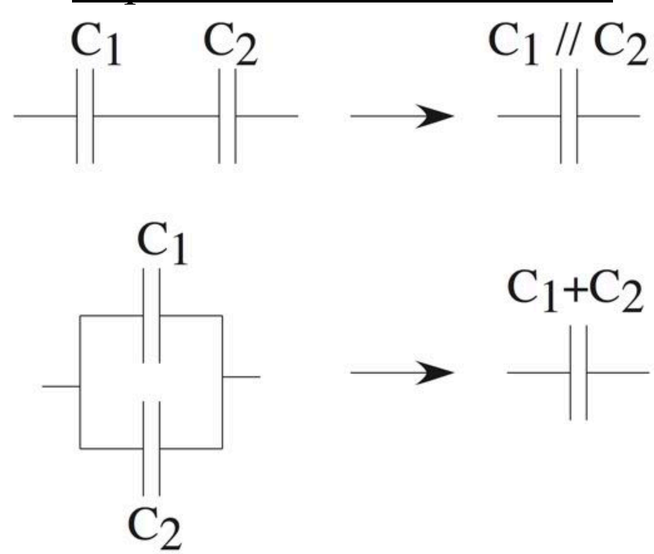


$$a//b = \frac{1}{\frac{1}{a} + \frac{1}{b}}$$

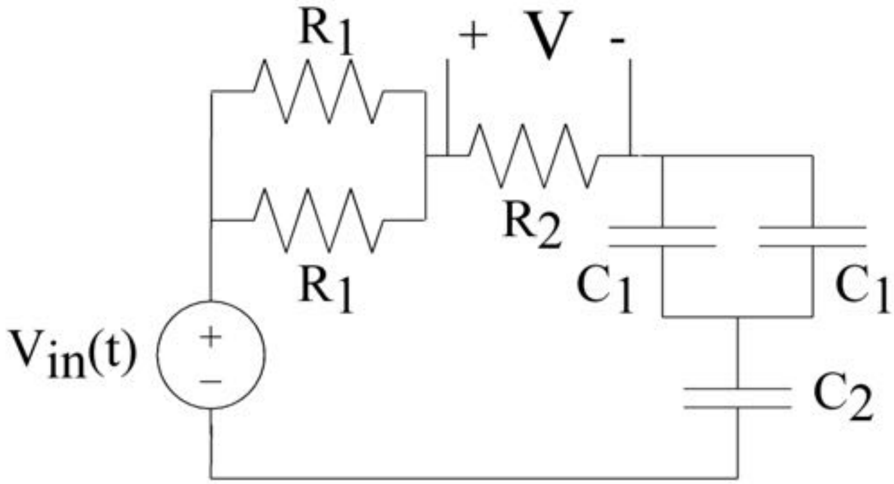
Inductors: Serial & Parallel

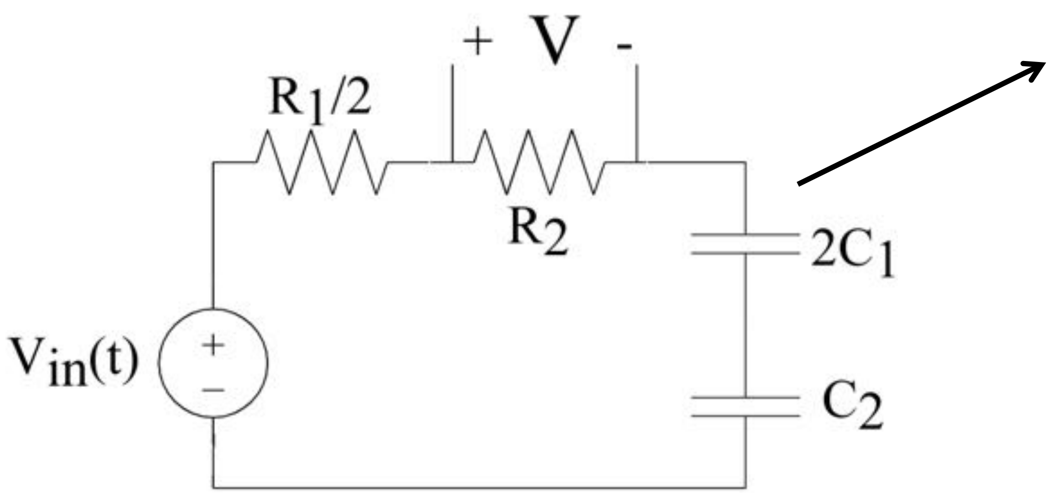


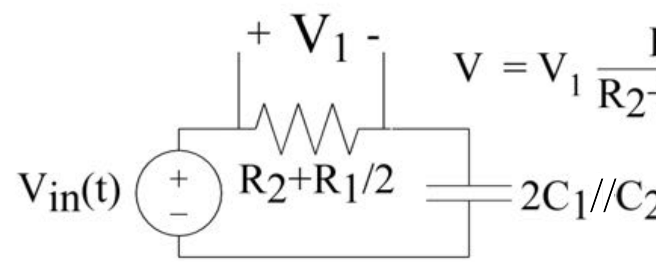
Capacitors: Serial & Parallel



Parallel-Series Simplification

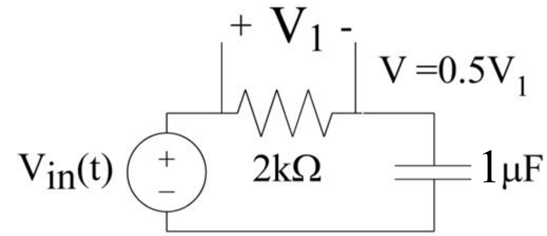






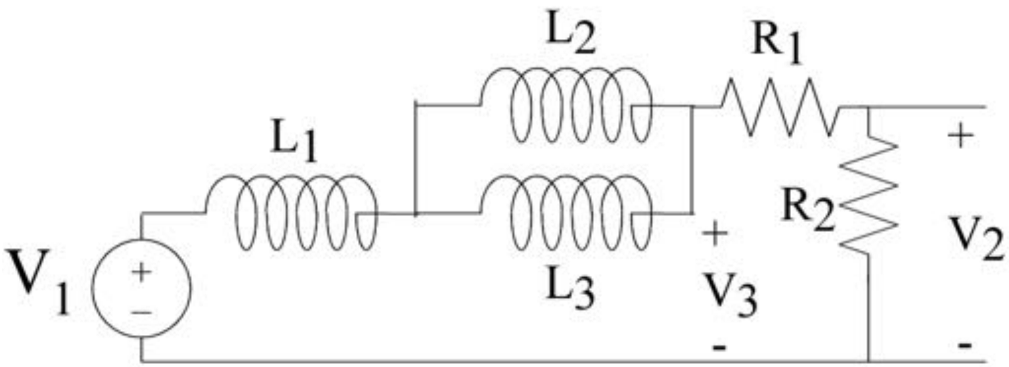
$$V = V_1 \frac{R_2}{R_2+R_1/2}$$

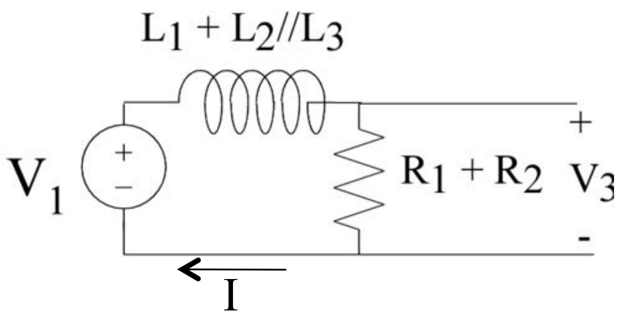
$R_1 = 2k\Omega$
$R_2 = 1k\Omega$
$C_1 = 1\mu F$
$C_2 = 2\mu F$

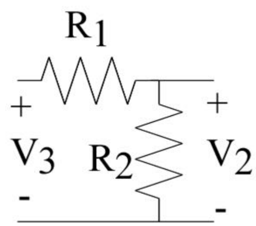


$V = 0.5V_1$

$\tau = 2ms$







$$V_1 = (L_1 + (L_2//L_3))\frac{dI}{dt} + V_3$$

$$V_3 = I\,(R_1 + R_2)$$

$$V_1 = \frac{L_1 + (L_2//L_3)}{R_1 + R_2}\frac{dV_3}{dt} + V_3$$

$$\tau = \frac{L_1 + (L_2//L_3)}{R_1 + R_2}$$

$$\tau\frac{dV_3}{dt} + V_3 = V_1$$

$$V_2 = V_3\frac{R_2}{R_1 + R_2}$$

$$\boxed{\tau\frac{dV_2}{dt} + V_2 = \frac{R_2}{R_1 + R_2}V_1}$$

$L_1 = 5\text{mH}$ $L_3 = 10\text{mH}$ $R_2 = 5\text{k}\Omega$

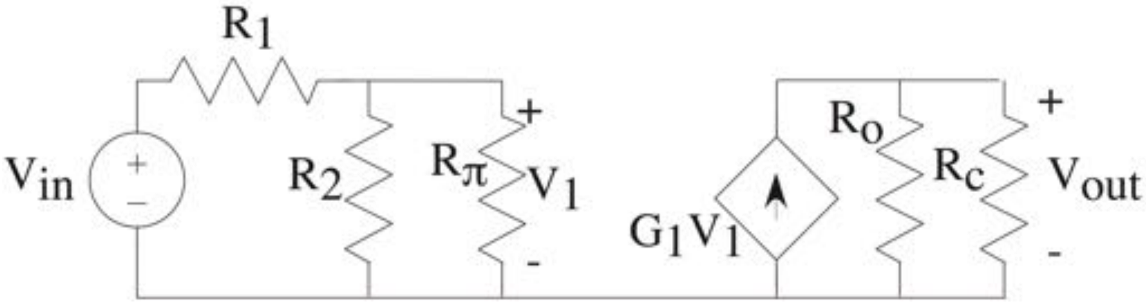
$L_2 = 10\text{mH}$ $R_1 = 15\text{k}\Omega$

$$L_1 + (L_2//L_3) = 10\text{mH}$$

$$R_1 + R_2 = 20\text{k}\Omega$$

$$\tau = 0.5\mu\text{s} \qquad \tau\frac{dV_2}{dt} + V_2 = \frac{1}{4}V_1$$

An Example Circuit Solution



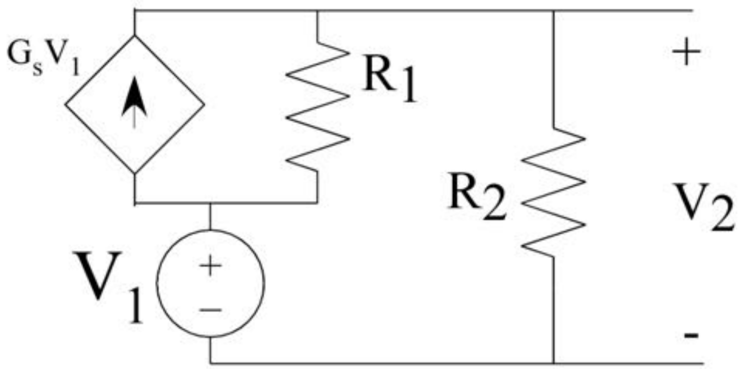
$$V_1 = V_{in} \frac{R_2//R_\pi}{R_1 + R_2//R_\pi}$$

$$V_{out} = G_1 V_1 (R_o//R_c)$$

$$V_{out} = V_{in} \frac{G_1(R_o//R_c)(R_2//R_\pi)}{R_1 + R_2//R_\pi}$$

How to think through such a circuit?

- Parallel Combination of Resistors for V_1
- Resistive voltage divider for V_1
- Current divider from the voltage-controlled I source



KCL @ V_2:

$$G_s V_1 + \frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2}$$

$$\left(G_s + \frac{1}{R_1}\right) V_1 = \frac{V_2}{R_1} + \frac{V_2}{R_2}$$

$$\left(G_s + \frac{1}{R_1}\right) V_1 = \frac{V_2}{R_1//R_2}$$

$$V_2 = (R_1//R_2)\left(G_s + \frac{1}{R_1}\right) V_1$$

$R_1 = 200\text{k}\Omega$

$R_2 = 20\text{k}\Omega$

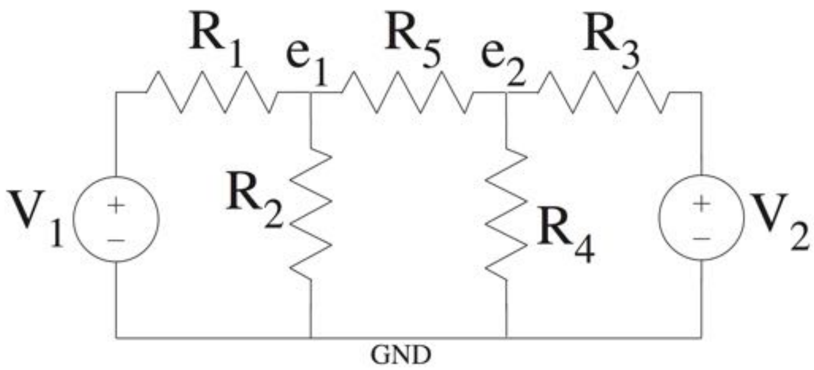
$G_s = 1 / 100\Omega$

$$G_s >> \frac{1}{R_1} \qquad R_1//R_2 \rightarrow R_2$$

$$V_2 = G_s R_2 V_1$$

$V_2 = 200\ V_1$

Node Voltage Solutions: KCL at nodes



Node 1:

$$(V_1 - e_1)G_1 + (e_2 - e_1)G_5 + (0 - e_1)G_2 = 0$$

$$V_1G_1 + e_2G_5 - e_1(G_1 + G_2 + G_5) = 0$$

Node 2:

$$(V_2 - e_2)G_3 + (e_1 - e_2)G_5 + (0 - e_2)G_4 = 0$$

$$V_2G_3 + e_1G_5 - e_2(G_3 + G_4 + G_5) = 0$$

$$\mathbf{G} = \begin{bmatrix} G_1 + G_2 + G_5 & -G_5 \\ -G_5 & G_3 + G_4 + G_5 \end{bmatrix}$$

$$\mathbf{i} = \begin{bmatrix} G_1V_1 \\ G_3V_2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

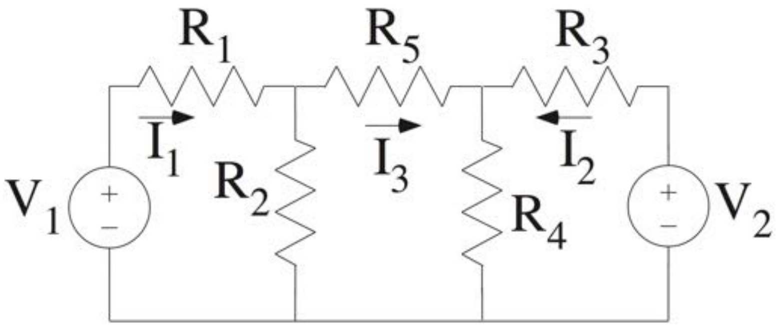
G Matrix is symmetric, positive definite

Diagonal terms = sum of conductances on the node

Off Diagonal terms = - sum of resistances between nodes

Solve: $\mathbf{G\,v} = \mathbf{i}$

KVL Equations: Solving for I



Mesh 1:

$$-V_1 + I_1 R_1 + (I_1 - I_3) R_2 = 0$$

$$-V_1 + (R_1 + R_2) I_1 - R_2 I_3 = 0$$

Mesh 2:

$$-V_2 + R_3 I_2 + R_4 (I_3 + I_2) = 0$$

$$-V_2 + R_4 I_3 + (R_3 + R_4) I_2 = 0$$

Mesh 3:

$$R_2 (I_3 - I_1) + R_5 I_3 + R_4 (I_3 + I_2) = 0$$

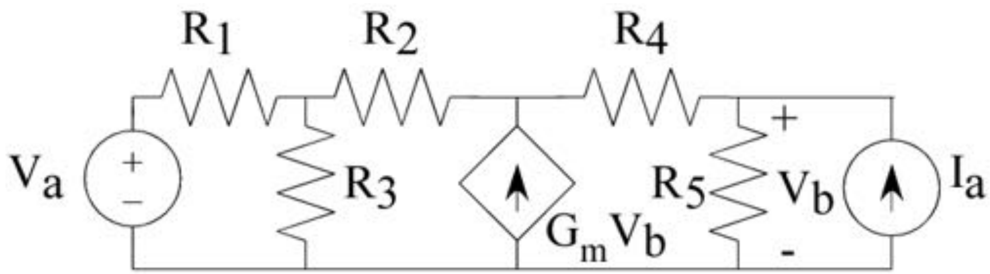
$$-R_2 I_1 + (R_2 + R_4 + R_5) I_3 + R_4 I_2 = 0$$

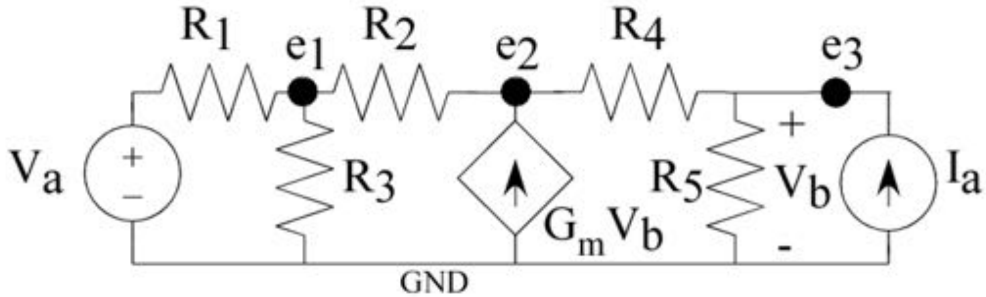
$$\mathbf{R} = \begin{bmatrix} R_1 + R_2 & 0 & -R_2 \\ 0 & R_3 + R_4 & -R_4 \\ -R_2 & -R_4 & R_2 + R_4 + R_5 \end{bmatrix}$$

$$\mathbf{i} = \begin{bmatrix} I_1 \\ -I_2 \\ I_3 \end{bmatrix}$$ (mesh currents in same direction) $$\mathbf{v} = \begin{bmatrix} V_1 \\ -V_2 \\ 0 \end{bmatrix}$$

Solve: $\mathbf{R}\,\mathbf{i} = \mathbf{v}$

R Matrix is symmetric, positive definite
Diagonal terms = sum of mesh Resistances
Off Diagonal terms = - sum of resistances between meshes





$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_4} & -\frac{1}{R_4}-G_m \\ 0 & -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \frac{V_a}{R_1} \\ 0 \\ I_a \end{bmatrix}$$

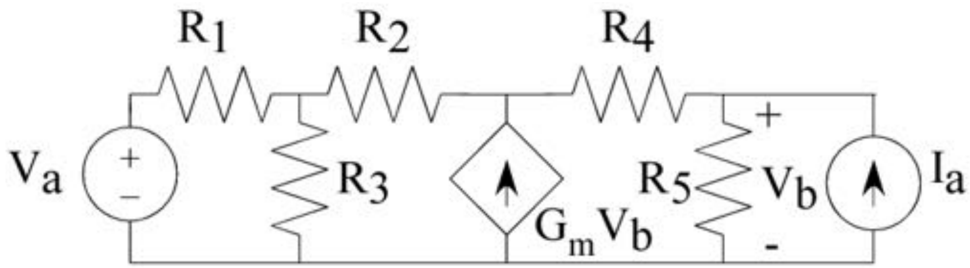
$R_1 = 1M\Omega$
$R_2 = 1M\Omega$
$R_3 = 2M\Omega$

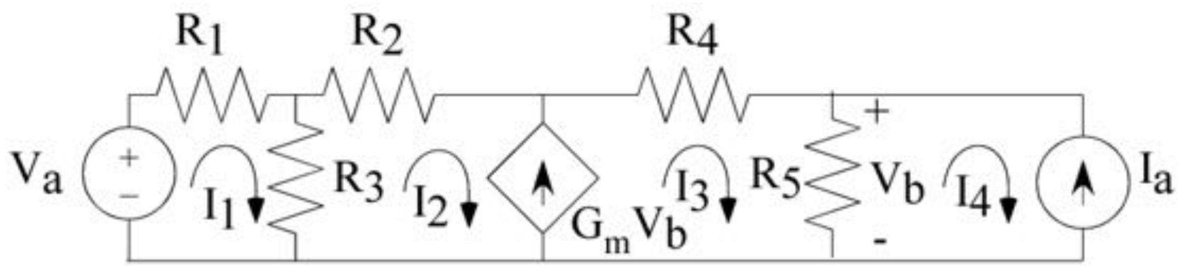
$R_4 = 1M\Omega$
$R_5 = 2M\Omega$
$G_m = 1/10k\Omega$

$V_a = 1V$
$I_a = 1\mu A$

$$\begin{bmatrix} 2.5 & -1 & 0 \\ -1 & 2 & -101 \\ 0 & -1 & 1.5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} -12.2mV \\ -1.03V \\ -20.3mV \end{bmatrix}$$





$$V_b = R_5(I_3 - I_4)$$

$$I_4 = -I_a \qquad I_3 - I_2 = GmR_5(I_3 - I_4)$$

$$I_2 + (GmR_5 - 1)I_3 - (GmR_5)I_4 = 0$$

$$\begin{bmatrix} R_1 + R_3 & -R_3 & 0 & 0 \\ -R_3 & R_2 + R_3 & R_4 + R_5 & -R_5 \\ 0 & 1 & GmR_5 - 1 & -GmR_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_a \end{bmatrix}$$

Dependent Source Constraint

Supermesh

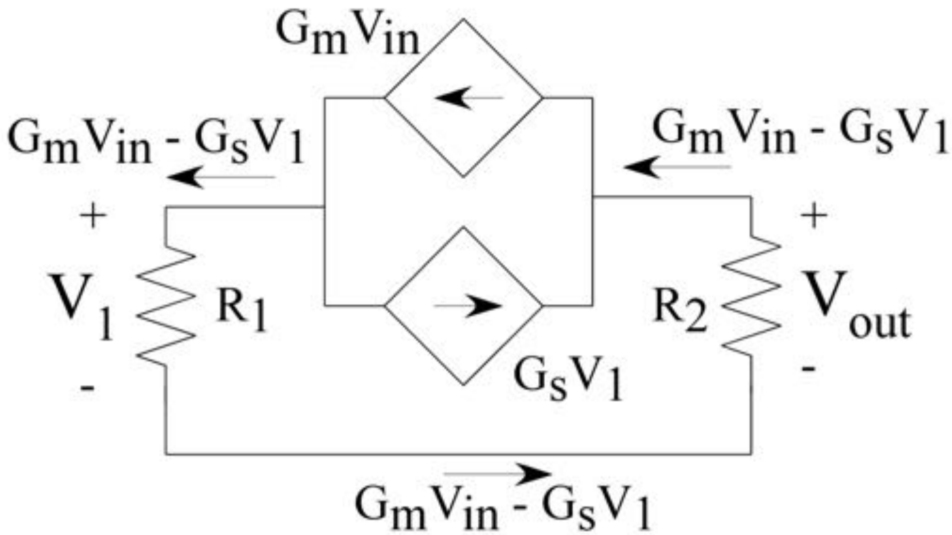
V_a = 1V, I_a = 1μA

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 73.7nA \\ 389nA \\ 1.007\mu A \\ 1\mu A \end{bmatrix}$$

R_1 = 1MΩ R_4 = 1MΩ
R_2 = 1MΩ R_5 = 2MΩ
R_3 = 2MΩ G_m = 1/10kΩ

$$\begin{bmatrix} 3 & -2 & 0 & 0 \\ -2 & 3 & 3 & -2 \\ 0 & 1 & 199 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_a \end{bmatrix}$$

I_1 through I_4: μA units



$$V_1 = R_1(G_m V_{in} - G_s V_1)$$

$$V_1 = V_{in} \frac{G_m R_1}{1 + G_s R_1}$$

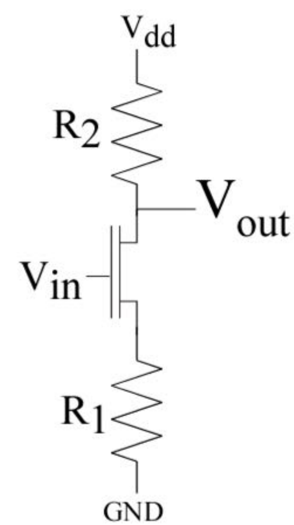
$$V_{out} = -R_2(G_s V_1 - G_m V_{in})$$

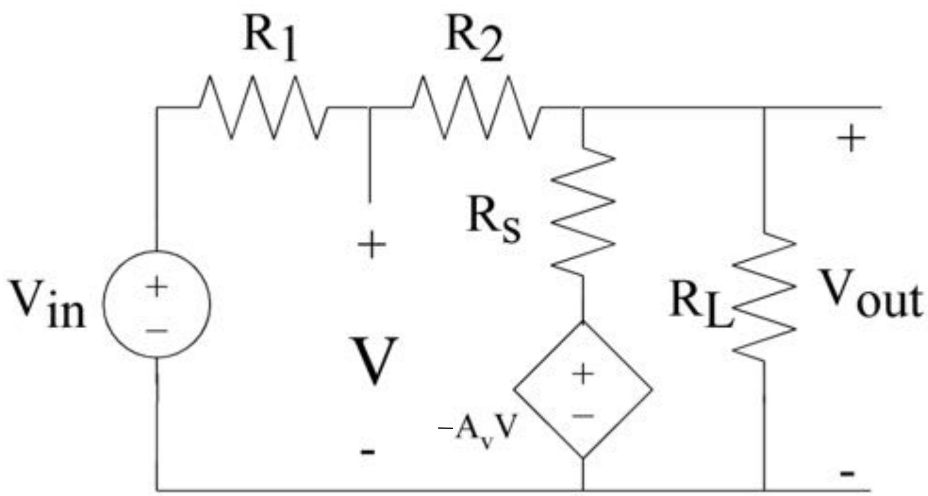
$$V_{out} = -G_m R_2\left(\frac{G_s R_1}{1 + G_s R_1} - 1\right) V_{in}$$

$$V_{out} = -V_{in} \frac{G_m R_2}{1 + G_s R_1}$$

$R_1 = 10k\Omega$
$R_2 = 100k\Omega$
$G_m = 1/200\Omega$
$G_s = 1/100\Omega$

$V_1 = (50 /101)\ V_{in}$
$\sim 0.5\ V_{in}$
$V_{out} = -(500/101)V_{in}$
$\sim -5V_{in}$





$$\frac{V_{in}-V}{R_1}=\frac{V-V_{out}}{R_2}=\frac{V_{out}+A_vV}{R_s}+\frac{V_{out}}{R_L}$$

If $R_s = 0$, $V_{out} = -A_v V$

$$\frac{R_2}{R_1}\left(V_{in}+\frac{V_{out}}{A_v}\right)=-(\frac{1}{A_v}+1)V_{out}$$

$$V_{out}\left(1+\frac{1}{A_v}\left(1+\frac{R_2}{R_1}\right)\right)=-\frac{R_2}{R_1}V_{in}$$

$$\frac{V_{out}}{V_{in}}=-\frac{R_2}{R_1}\frac{1}{1+\frac{1}{A_v}(1+\frac{R_2}{R_1})}$$

$R_1 = 10k\Omega$ $R_L = 1k\Omega$

$R_2 = 100k\Omega$ $A_v = 100{,}000$

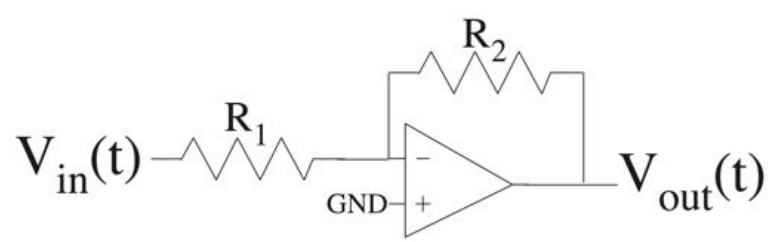
$V_{out} = -V_{in}\,(9.999) \sim -10V_{in}$

$$V\left(-\frac{A_v}{R_s}+\frac{1}{R_2}\right)=\frac{V_{out}}{R_s//R_2//R_L}$$

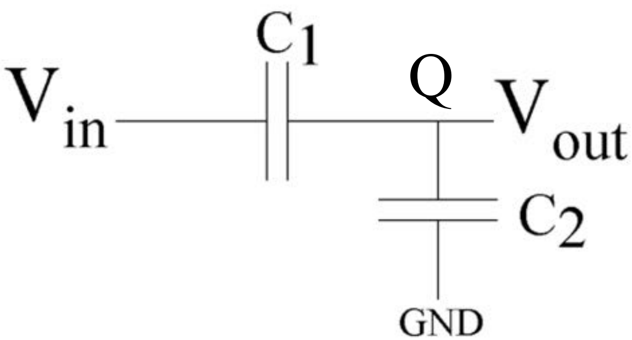
$R_s << R_2$, R_L

$-V A_v/R_s = V_{out}/R_s$

$V_{out} = -A_v V$



State Holding Divider Circuits



Maybe like Conductances: $V_{out} = V_{in} \frac{C_1}{C_1 + C_2}$?

$$V_{in} = 0, V_{out} = \frac{Q}{C_1 + C_2}$$

$$Q_1 = C_1(V_{in} - V_{out})$$

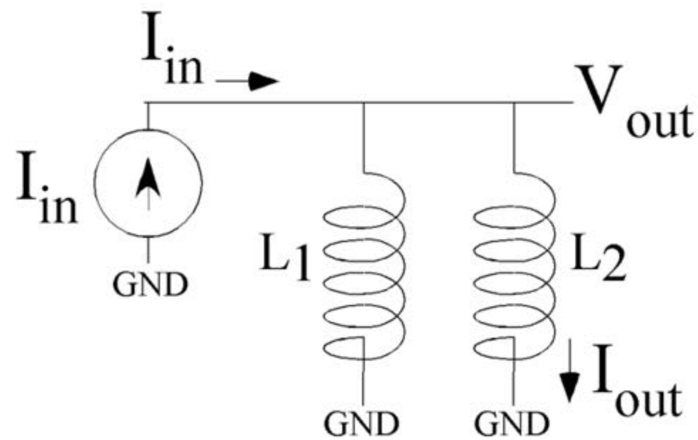
$$Q_2 = C_2 V_{out}$$

Total charge (Q - Q_1 + Q_2) at V_{out} is fixed

Q_1: Negative Plate; Q_2: Positive Plate

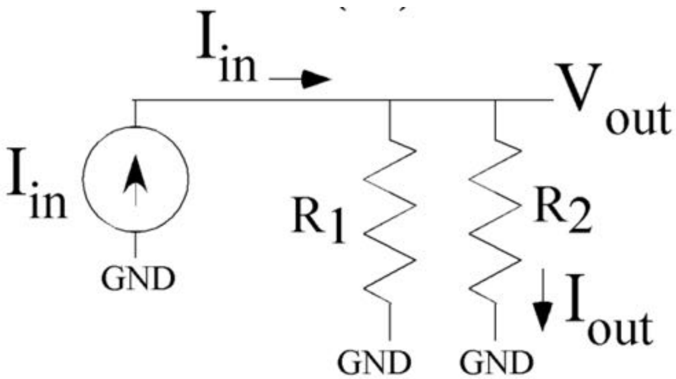
$$V_{out} = V_{in} \frac{C_1}{C_1 + C_2} + V_{offset}$$

Similar case with Inductors (current divider, offset I):



$$I_{out} = I_{in} \frac{L_1}{L_1 + L_2} + I_{offset}$$

Resistive Current Divider

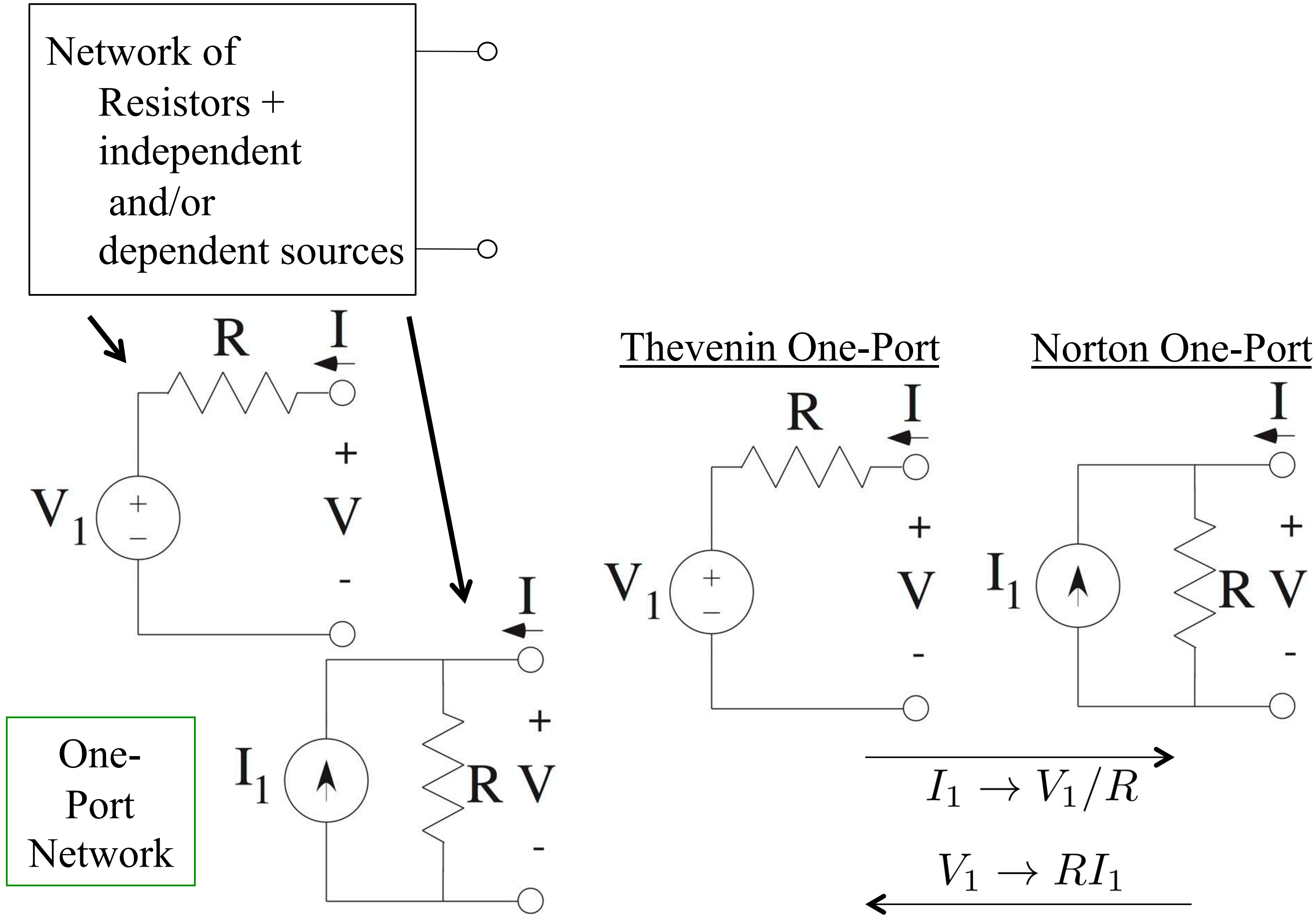


$$I_{out} = \frac{V_{out}}{R_2} = I_{in}\frac{R_1//R_2}{R_2}$$

$$a//b = \frac{1}{\frac{1}{a}+\frac{1}{b}}$$

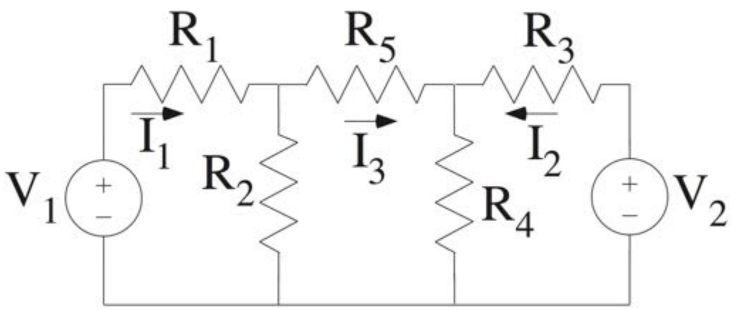
$$V_{out} = I_{in}(R_1//R_2)$$

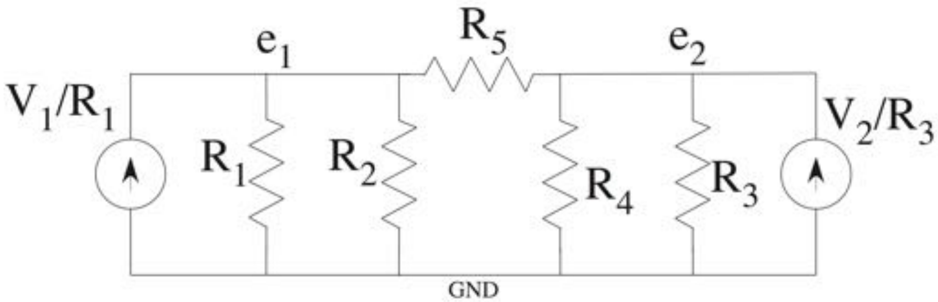
$$V_{out} = V_{in}\frac{R_1}{R_1 + R_2}$$

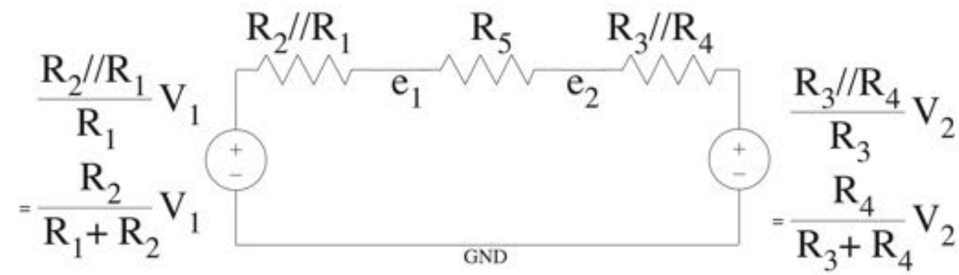


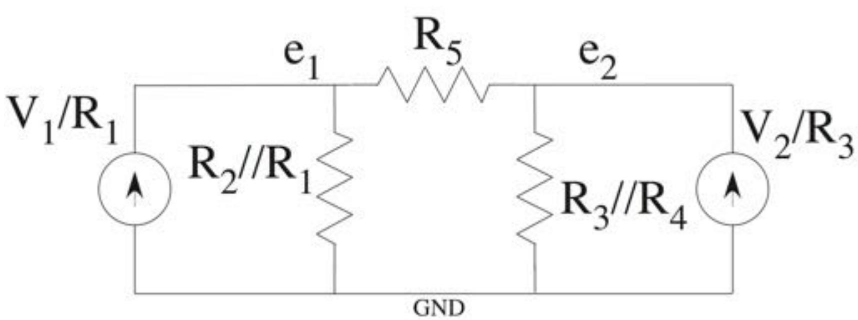
Network of Resistors + independent and/or dependent sources
R
I
+
V
-
V1
I
+
R V
-
I1
One-Port Network
Thevenin One-Port
R
I
+
V
-
V1
Norton One-Port
I
+
R V
-
I1
$I_1 \to V_1/R$
$V_1 \to RI_1$

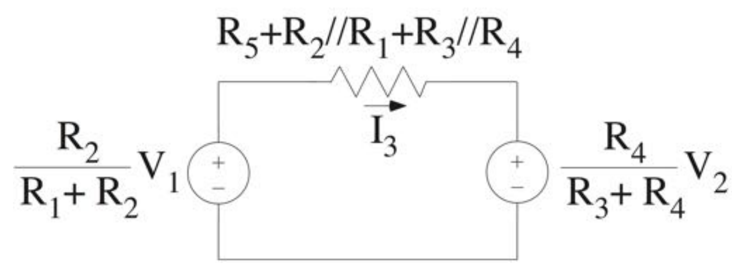
Using Thevenin-Norton Equivalents

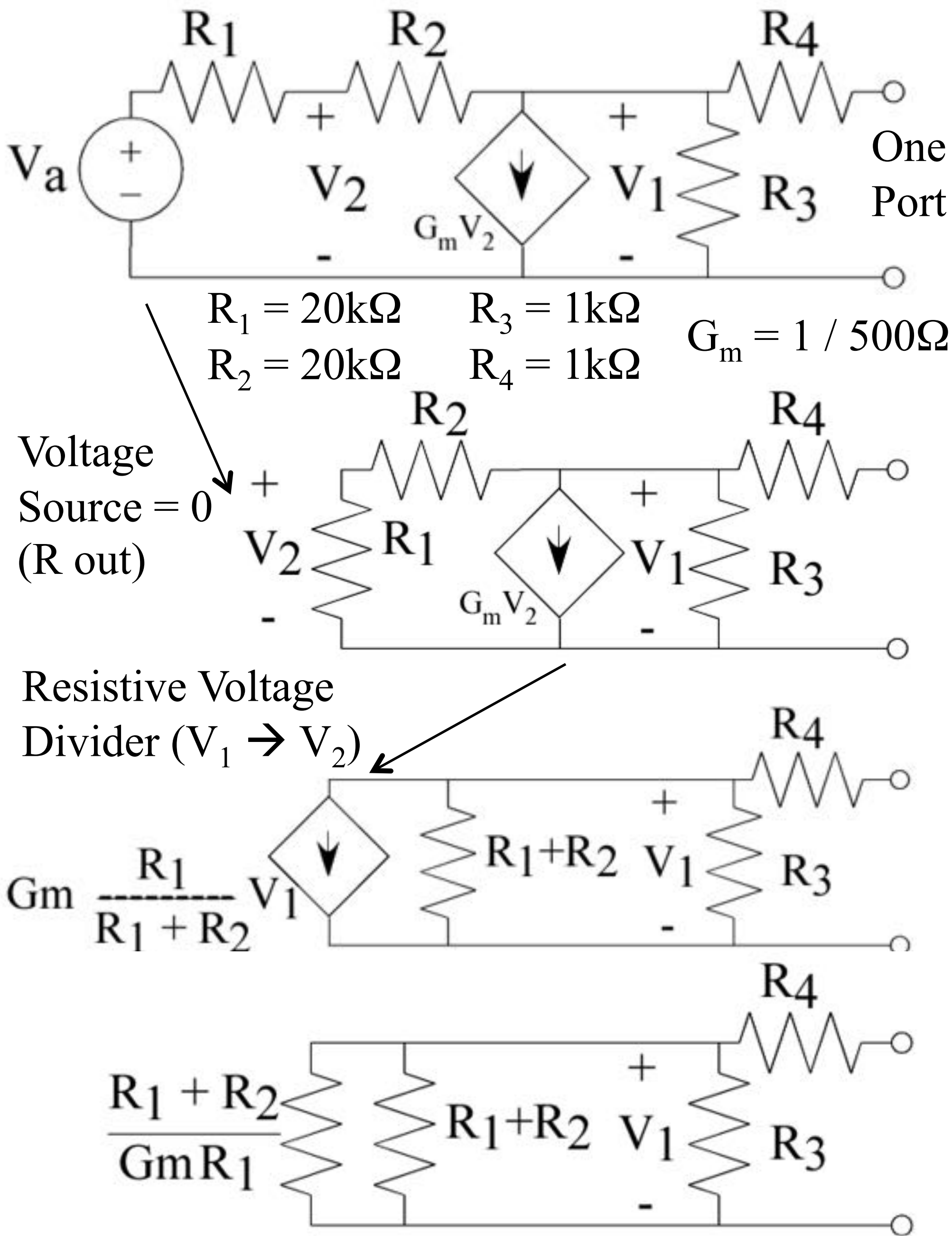












V_{eff} : Measure output. No output I
(R_4 has no effect)

KCL at V_2 & V_1

$$\frac{V_a - V_2}{R_1} = \frac{V_2 - V_1}{R_2} = \frac{V_1}{R_3} + G_m V_2$$

$$\frac{V_a}{R_1} + \frac{V_1}{R_2} = \frac{V_2}{R_1//R_2}$$

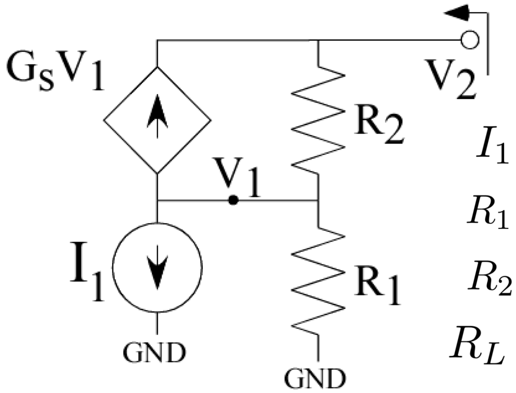
$$\longrightarrow V_a + V_1 = 2V_2$$

$$V_2\left(\frac{1}{R_2} - G_m\right) = \frac{V_1}{R_2//R_3}$$

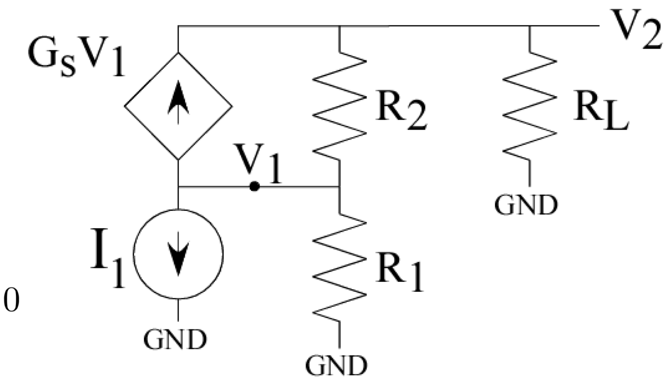
$$\longrightarrow -2V_2 = V_1$$

$$V_{eff} = V_1 = \frac{V_a}{2}$$

$$R_{eff} = R_4 + R_3//\left(\frac{R_1 + R_2}{R_1}\frac{1}{G_m}\right)//(R_1 + R_2) \longrightarrow$$ R_{eff} = 1.5kΩ



$$I_1 = 1\mu A$$
$$R_1 = 1M\Omega$$
$$R_2 = 1M\Omega$$
$$R_L = 40k\Omega$$

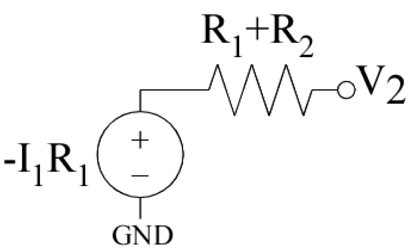


Two cases

$$G_s = 0$$

Norton-Thevenin at V_1

$$V_{eff} = -I_1 R_1 = -1V$$
$$R_{eff} = R_1 + R_2 = 2M\Omega$$
$$I_{eff} = -0.5\mu A$$



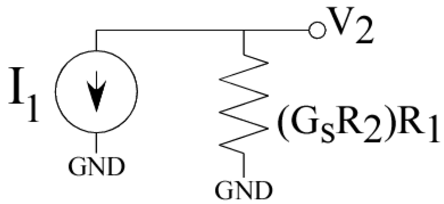
$$G_s = 1/1k\Omega$$

I_{eff}: V_2=GND (measure I @ V_2)

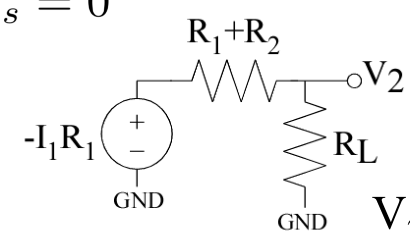
$$I_1 + V_1\left(G_s + \frac{1}{R_1 + R_2}\right) \approx I_1 + V_1 G_s = 0$$
$$V_1 = -I_1/G_s = -1mV$$
$$I_{eff} \approx 1\mu A$$

R_{eff}: I_1=0

$$R = \frac{V_2}{I} = \frac{V_2}{V_1}\frac{V_1}{I}$$
$$V_1\left(G_s + \frac{1}{R_1 + R_2}\right) = \frac{V_2}{R_2}$$
$$V_1 = V_2 \frac{1}{G_s R_2}$$
$$R = (G_s R_2) R_1 \quad (1G\Omega)$$

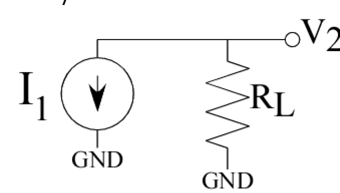


$$G_s = 0$$

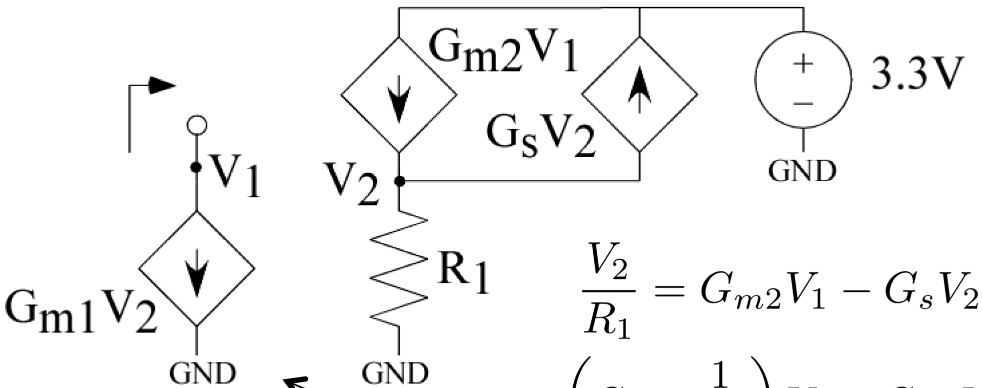


V_2 = -20mV

$$G_s = 1/1k\Omega$$



V_2 = -40mV



$$\frac{V_2}{R_1} = G_{m2}V_1 - G_sV_2$$

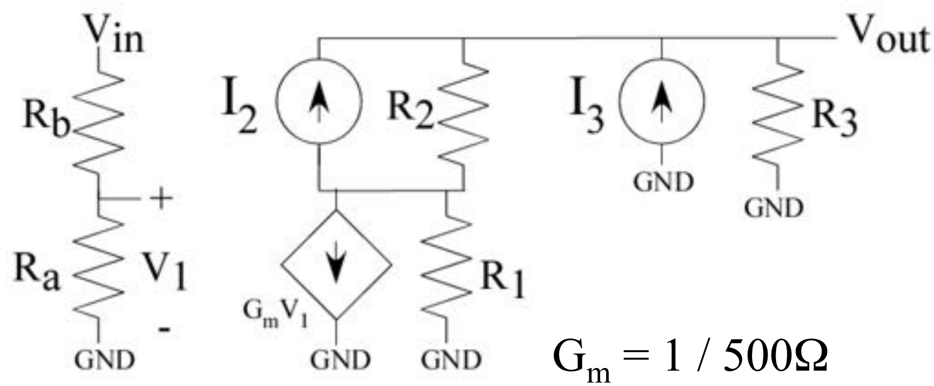
$$\left(G_s + \frac{1}{R_1}\right)V_2 = G_{m2}V_1$$

$$V_2 \approx \frac{G_{m2}}{G_s}V_1$$

Resistor: $\frac{1}{G_{m1}}\frac{G_s}{G_{m2}}$

no Thevenin voltage source

$G_{m2} = 1/2k\Omega$	$G_{m2} = 1/20k\Omega$
$G_s = 1/1k\Omega$	$G_s = 1/10k\Omega$
$G_{m1} = 1/200k\Omega$	$G_{m1} = 1/10M\Omega$
R_1=1MΩ	R_1=10MΩ
Resistor: 400kΩ	Resistor: 20MΩ

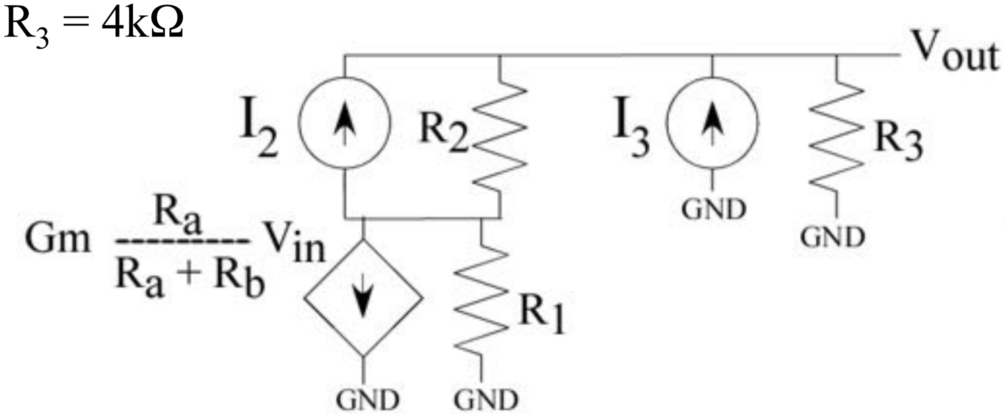


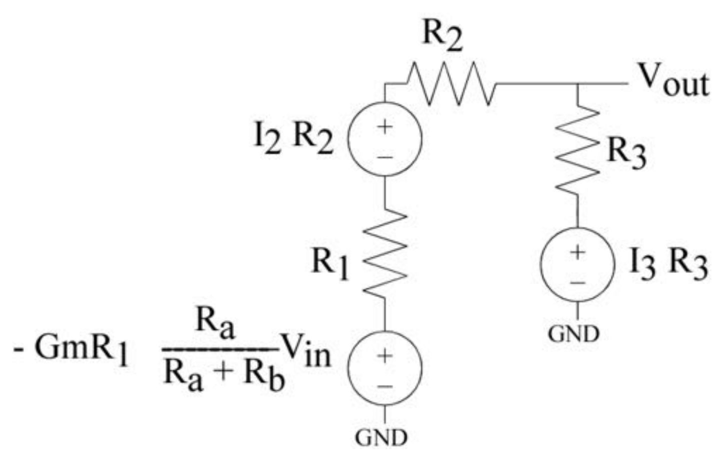
$G_m = 1 / 500\Omega$

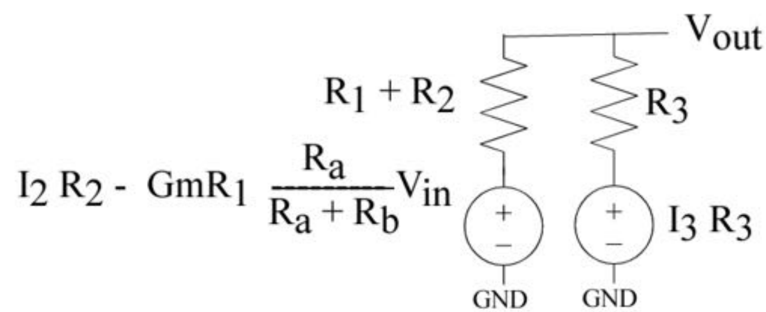
$R_1 = 2k\Omega$ $\quad R_a = 1k\Omega$ $\quad I_2 = 2mA$

$R_2 = 2k\Omega$ $\quad R_b = 1k\Omega$ $\quad I_3 = 1mA$

$R_3 = 4k\Omega$







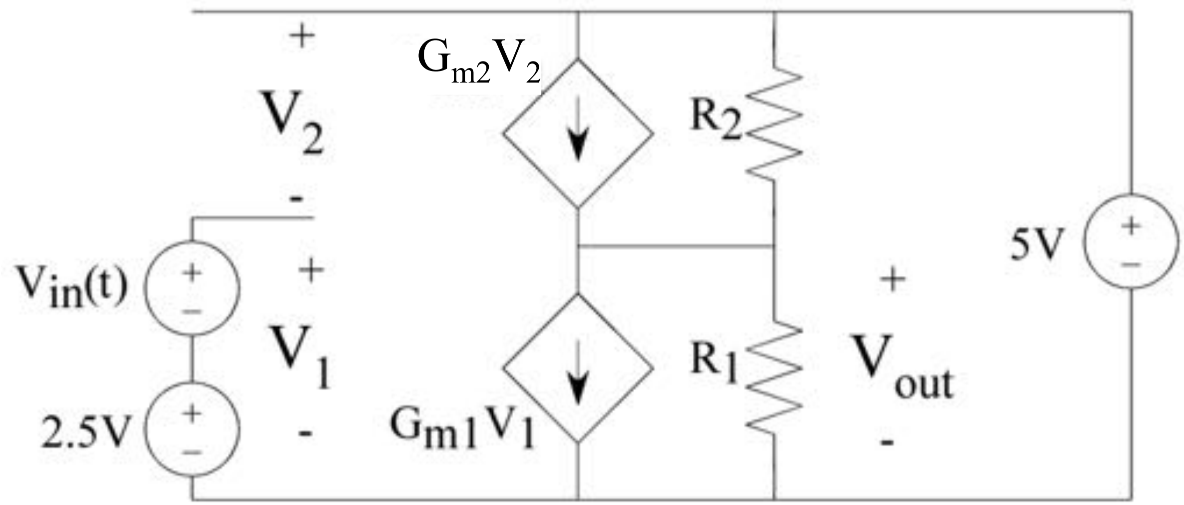
$$V_{out} = \left(I_2 R_2 - G_m R_1 \frac{R_a}{R_a + R_b} V_{in}\right)\left(\frac{R_3}{R_1 + R_2 + R_3}\right) + I_3 R_3 \left(\frac{R_1 + R_2}{R_1 + R_2 + R_3}\right)$$

$$G_m R_1 \frac{R_a}{R_a + R_b} = 4\ (1/2) = 2 \qquad I_2 R_2 = 4V$$

$$R_1 + R_2 + R_3 = 8k\Omega \qquad I_3 R_3 = 4V$$

$$V_{out} = (4V - 2V_{in})\left(\frac{1}{2}\right) + 4V\left(\frac{1}{2}\right)$$

$$V_{out} = 4V - V_{in}$$



$V_1 = V_{in} + 2.5$, $V_2 = 5 - V_1 = 2.5 - V_{in}$

Solve using superposition:

Case I: 2.5V & 5V source, Case II: V_{in} source

Case I: $V_1 = V_2 = 2.5V$

$$\frac{V_{out}}{R_1} = (G_{m2} - G_{m1})2.5V + \frac{5V - V_{out}}{R_2}$$

$$V_{out} = (R_1//R_2)\left((G_{m2} - G_{m1})2.5V + 5V/R_2\right)$$

Case II: $V_1 = -V_2 = V_{in}$

$$-(G_{m1} + G_{m2})V_{in} = \frac{V_{out}}{R_1//R_2}$$

$$\frac{V_{out}}{V_{in}} = -(G_{m1} + G_{m2})(R_1//R_2)$$

$R_1 = 100k\Omega$

$R_2 = 200k\Omega$

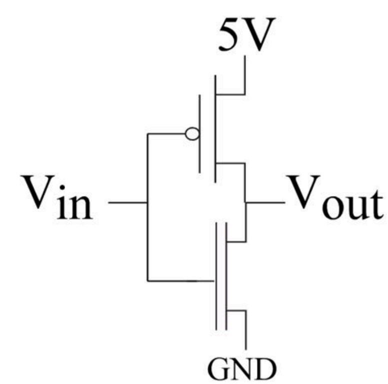
$G_{m1} = 1k\Omega$

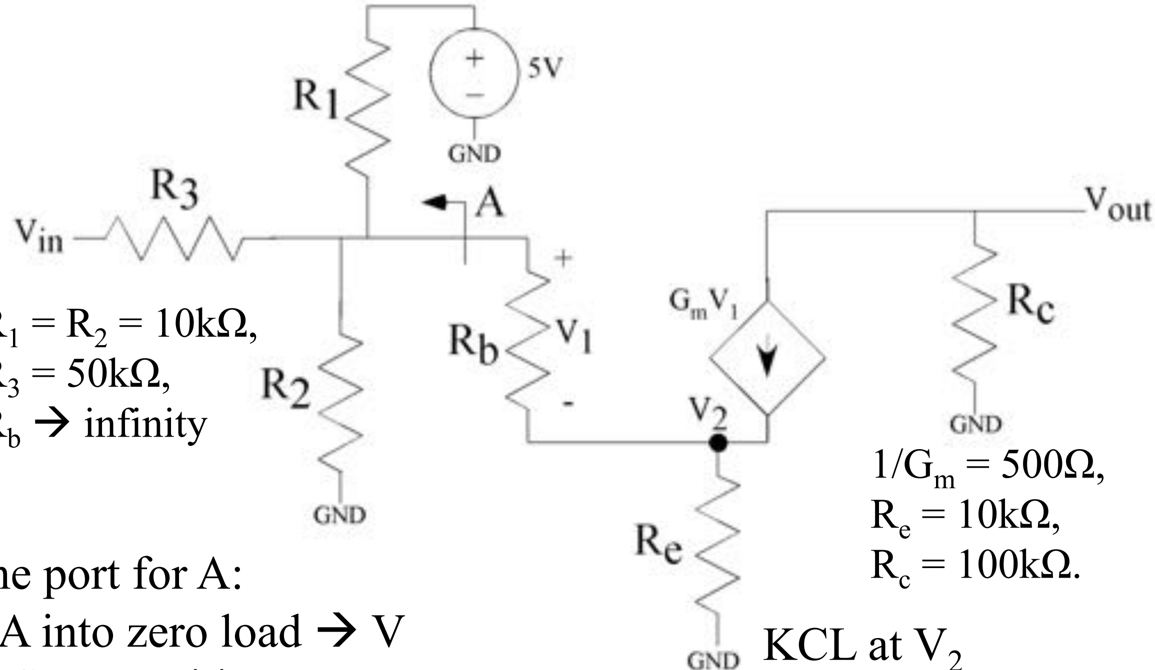
$G_{m2} = 1k\Omega$

Case I: $V_{out} = 1.67V$

Case II: $V_{out} = -133V_{in}$

CMOS Inverter





$R_1 = R_2 = 10k\Omega$,
$R_3 = 50k\Omega$,
$R_b \rightarrow$ infinity

$1/G_m = 500\Omega$,
$R_e = 10k\Omega$,
$R_c = 100k\Omega$.

One port for A:

A into zero load → V

Superposition:

$$V_A = \frac{R_1//R_2//R_3}{R_1}5V$$

$$V_A = \frac{R_1//R_2//R_3}{R_3}V_{in}$$

$$V_{eq} = (R_1//R_2//R_3)\left(\frac{V_{in}}{R_3} + \frac{5V}{R_1}\right)$$

(= 0.091 V_{in} + 2.25V)

$$R_{eq} = R_1//R_2//R_3 \quad (4.5k\Omega)$$

KCL at V_2

$$G_m V_1 = \frac{V_A - V_1}{R_e}$$

$$(1 + G_m R_e)V_1 = V_A$$

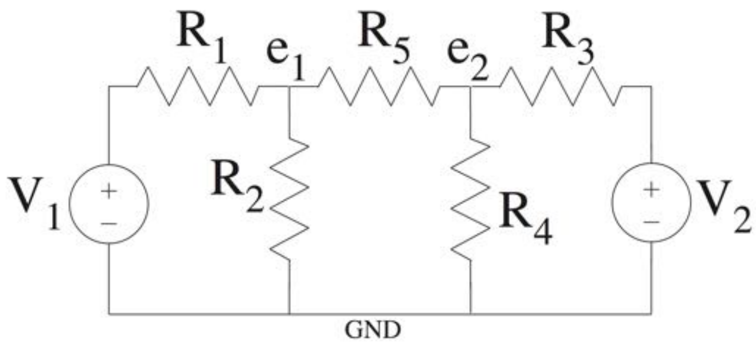
($21V_1 = V_A$)

KCL at V_{out}

$$V_{out} = G_m R_c V_1$$

($V_{out} = 200\ V_1$)

($V_{out} = 9.52V_A$)



A linear function, f( ),

f(Ax + By) = A f(x) + B f(y)

Superposition:

Can turn each source on separately, add the results

V_1 on, $V_2 = 0$: (Resistive Divider)

$$e_1 = \frac{R_2//(R_5 + R_3//R_4)}{R_1 + R_2//(R_5 + R_3//R_4)} \quad V1$$

$$e_2 = e_1 \frac{R_3//R_4}{R_5 + R_3//R_4}$$

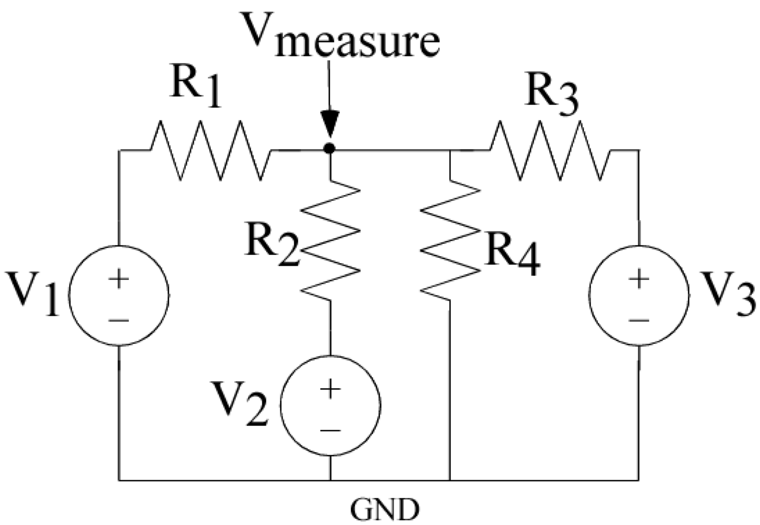
V_2 on, $V_1 = 0$:

$$e_2 = \frac{R_4//(R_5 + R_1//R_2)}{R_3 + R_4//(R_5 + R_1//R_2)} \quad V2$$

$$e_1 = e_2 \frac{R_1//R_2}{R_5 + R_1//R_2}$$

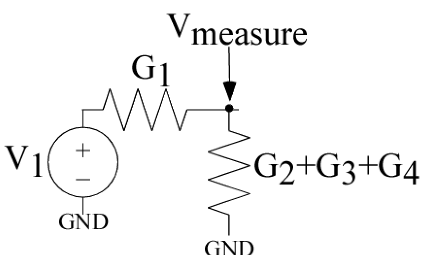
$$e_1 = \frac{R_2//(R_5 + R_3//R_4)}{R_1 + R_2//(R_5 + R_3//R_4)} V1 + \frac{R_4//(R_5 + R_1//R_2)}{R_3 + R_4//(R_5 + R_1//R_2)} \frac{R_1//R_2}{R_5 + R_1//R_2} \quad V2$$

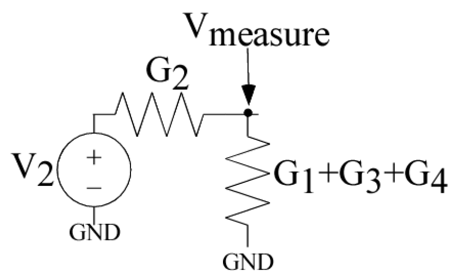
$$e_2 = \frac{R_4//(R_5 + R_1//R_2)}{R_3 + R_4//(R_5 + R_1//R_2)} V2 + \frac{R_2//(R_5 + R_3//R_4)}{R_1 + R_2//(R_5 + R_3//R_4)} \frac{R_3//R_4}{R_5 + R_3//R_4} \quad V1$$

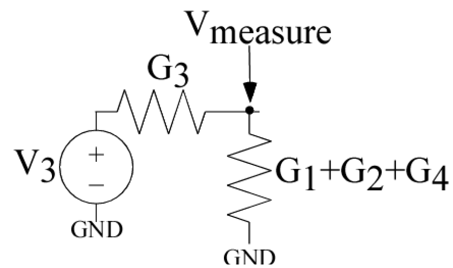


$$R_1 = 1k\Omega$$
$$R_2 = 2k\Omega$$
$$R_3 = 4k\Omega$$
$$R_4 = 4k\Omega$$

$$V_{measure} = \frac{G_1V_1 + G_2V_2 + G_3V_3}{G_1 + G_2 + G_3 + G_4}$$



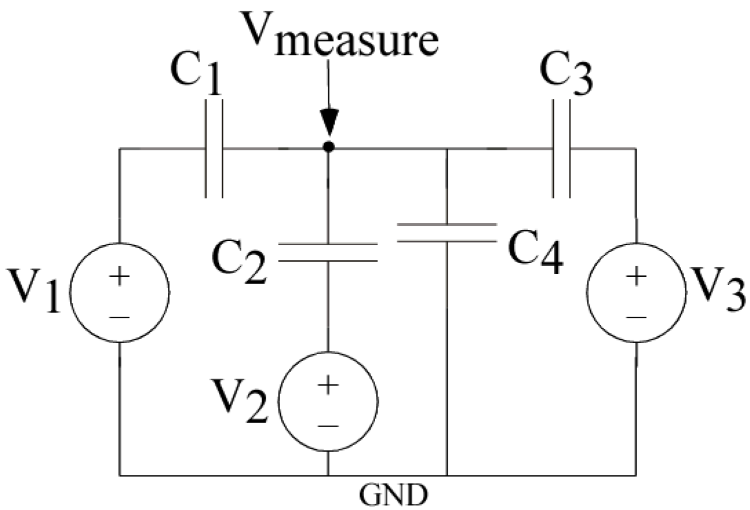




$$V_{measure} = \frac{G_1}{G_1 + G_2 + G_3 + G_4}V_1 \qquad V_{measure} = \frac{G_2}{G_1 + G_2 + G_3 + G_4}V_2 \qquad V_{measure} = \frac{G_3}{G_1 + G_2 + G_3 + G_4}V_3$$

$$V_{measure} = \frac{1}{2}V_1 + \frac{1}{4}V_2 + \frac{1}{8}V_3$$

$V_1 = V_2 = V_3 = 2V$, what is $V_{measure}$? 1.75V



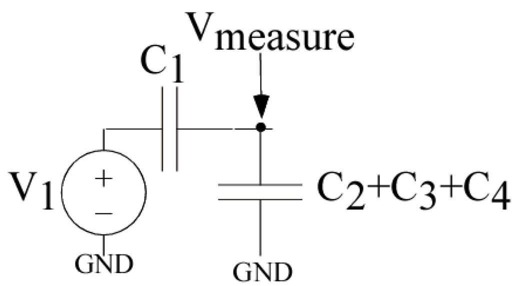
C_1=4pF

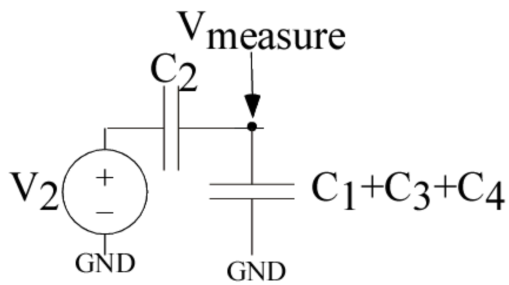
C_2=2pF

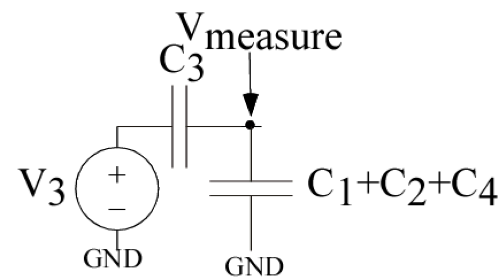
C_3=1pF

C_4=1pF

$$V_{measure} = \frac{C_1 V_1 + C_2 V_2 + C_3 V_3}{C_1 + C_2 + C_3 + C_4} + V_{offset}$$







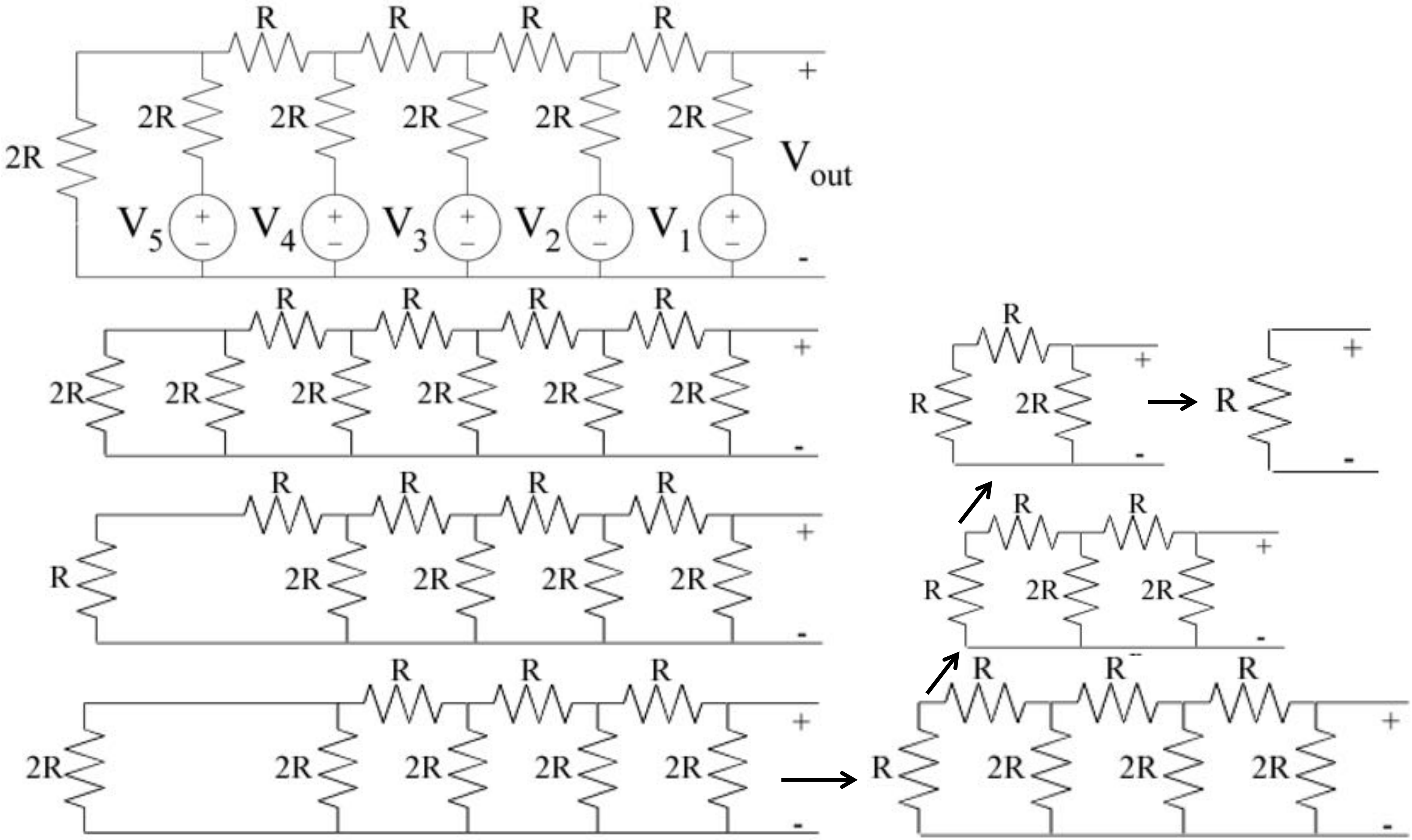
$$V_{measure} = \frac{C_1}{C_1 + C_2 + C_3 + C_4} V_1 + V_{offset}$$

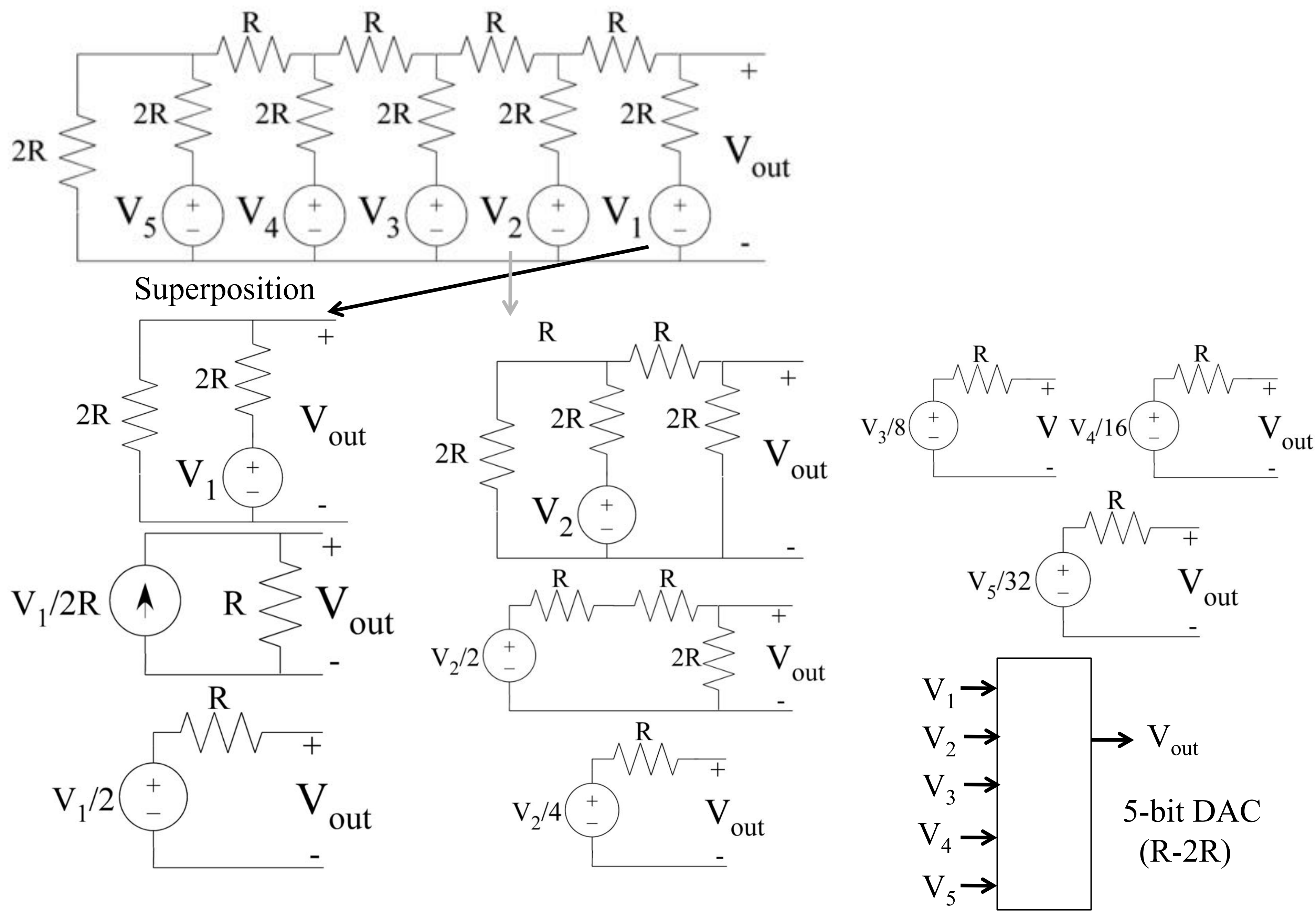
$$V_{measure} = \frac{C_2}{C_1 + C_2 + C_3 + C_4} V_2 + V_{offset}$$

$$V_{measure} = \frac{C_3}{C_1 + C_2 + C_3 + C_4} V_3 + V_{offset}$$

$$V_{measure} = \frac{1}{2} V_1 + \frac{1}{4} V_2 + \frac{1}{8} V_3 + V_{offset}$$

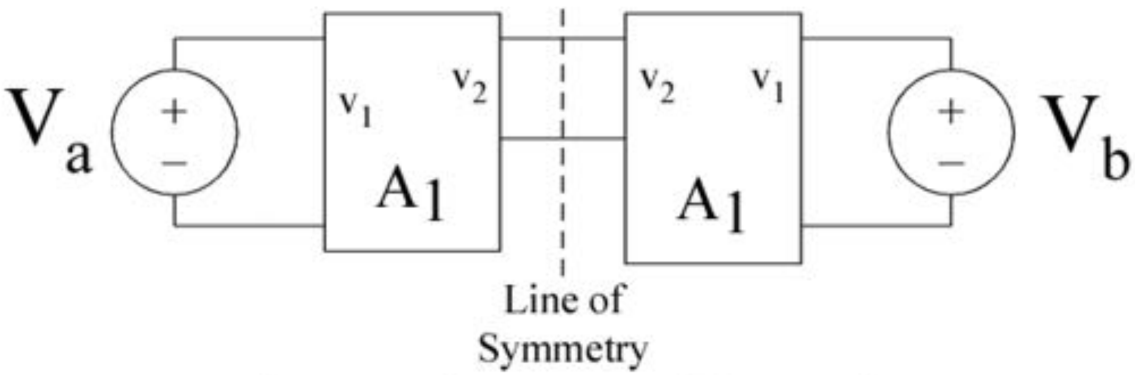
$V_{measure}$ = 0.5V when V_1, V_2, V_3 = 0V,
V_1 = V_2 = V_3 = 2V, what is $V_{measure}$? 2.25V

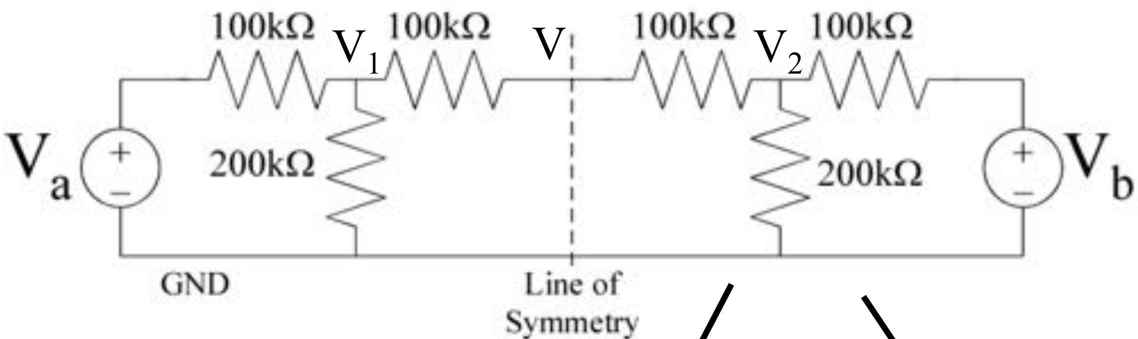

R
2R
V_out
V_5
V_4
V_3
V_2
V_1
+
-



R
2R
V5
V4
V3
V2
V1
Vout
+
-
Superposition
V1/2R
V1/2
V2/2
V2/4
V3/8
V4/16
V5/32
5-bit DAC
(R-2R)

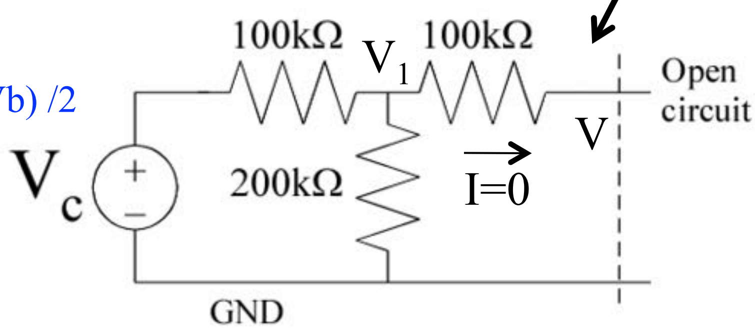
Symmetric Resistive Circuits



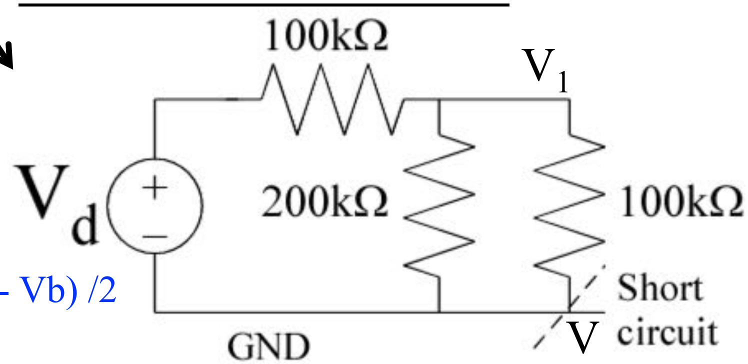


Common-Mode Circuit

(Va + Vb) /2



Differential-Mode Circuit



(Va - Vb) /2

Voltage Divider: $V = V_1 = V_2 = (2/3)\ V_c$

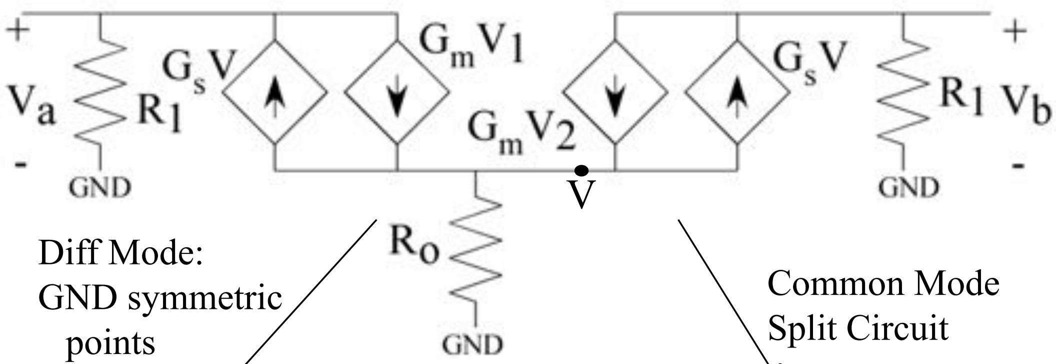
V_c = average(V_1, V_2) = ~~$(V_1 + V_2)/2$~~

(Va + Vb) /2

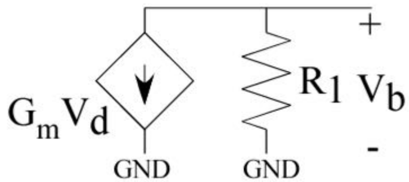
Voltage Divider: $V = V_1 = -V_2 = (2/5)\ V_d$

V_d = ~~$(V_1 - V_2)/2$~~

(Va - Vb) /2



<u>Diff Mode Half Circuit</u>

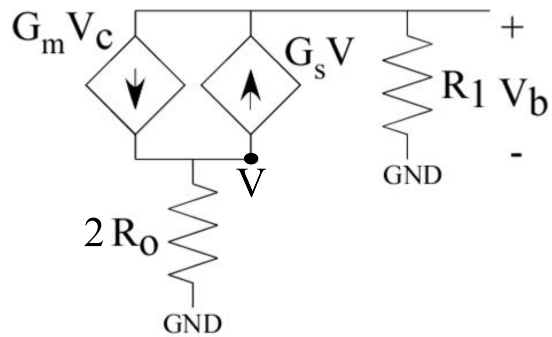


$\sout{V_b}$ Va $= -G_m R_1 V_d$

$= -5\ V_d$

$V_d = (V1-V2)/2$

<u>Common Mode Half Circuit</u>



$R_o = 10M\Omega \qquad G_m = 1/2k\Omega$

$R_1 = 10k\Omega \qquad G_s = 1/1k\Omega$

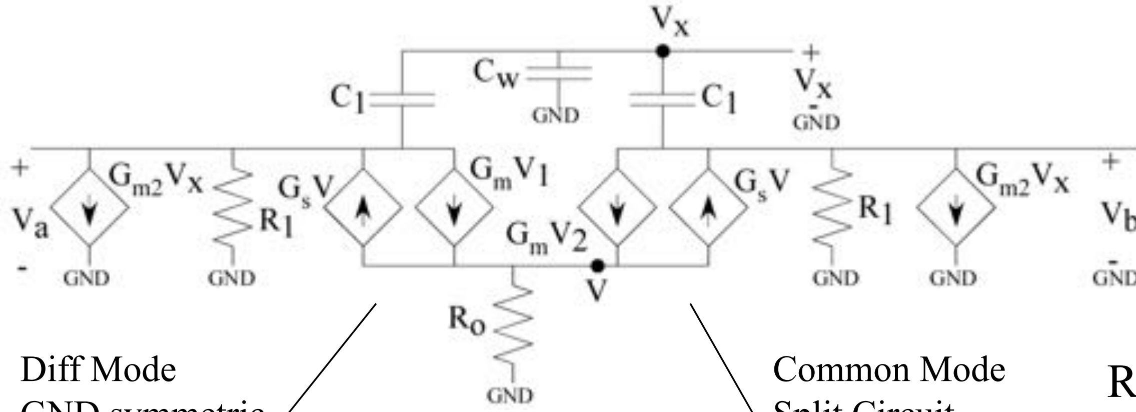
$$\frac{V}{2R_o} = G_m V_c - G_s V = -\frac{V_b}{R_1}$$

$$V\left(G_s + \frac{1}{2R_o}\right) = G_m V_c \qquad V = \frac{2R_o G_m}{2R_o G_s + 1} V_c$$

$$V_b = -\frac{R_1}{2R_o}\frac{2R_o G_m}{2R_o G_s + 1} V_c \quad \sim -0.00025\ V_c$$

$V_a = -0.00025\ V_c - 5\ V_d$

$V_b = -0.00025\ V_c + 5\ V_d$



Diff Mode
GND symmetric
points

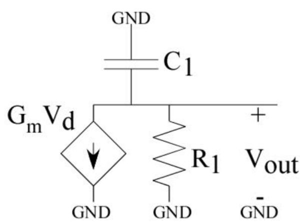
Common Mode
Split Circuit

$R_o = 10M\Omega$ $R_1 = 80k\Omega$

$C_1 = 1pF$ $C_w = 2pF$

$G_m = 1/2k\Omega$
$G_s = 1/1k\Omega$
$G_{m2} = 1/2k\Omega$

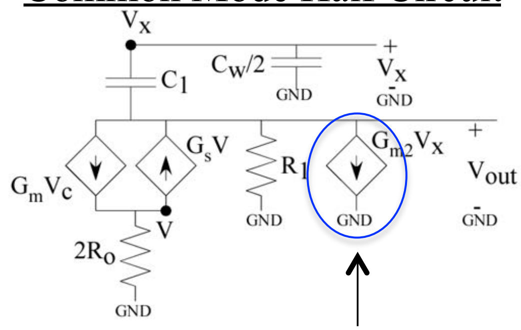
Diff Mode Half Circuit



$$V_b = -G_m R_1 V_d$$

$= -40\ V_d$

Common Mode Half Circuit



R: $\frac{1}{G_m 2}\frac{C_w/2 + C_1}{C_1} \rightarrow 4k\Omega$ << R_1

$$\frac{V}{2R_o} = G_m V_c - G_s V$$

$$V\left(G_s + \frac{1}{2R_o}\right) = G_m V_c$$

$$V = \frac{2R_o G_m}{2R_o G_s + 1} V_c$$

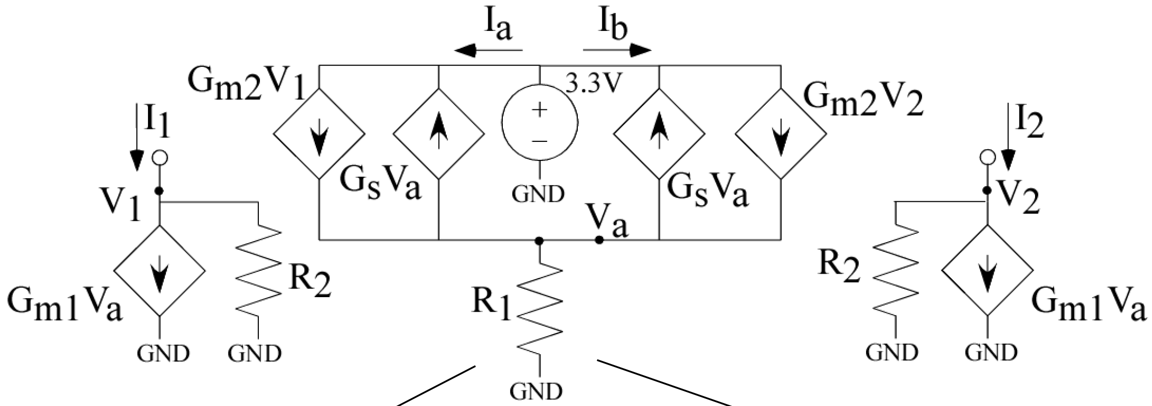
~ 0.5 V_c

$$\frac{V}{2R_o} = -\frac{V_b}{R}$$

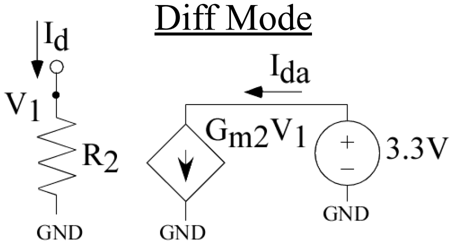
$$V_b = -\frac{R}{2R_o} V$$

$$= -\frac{R}{4R_o} V_c$$

$$= -0.0001 V_c$$



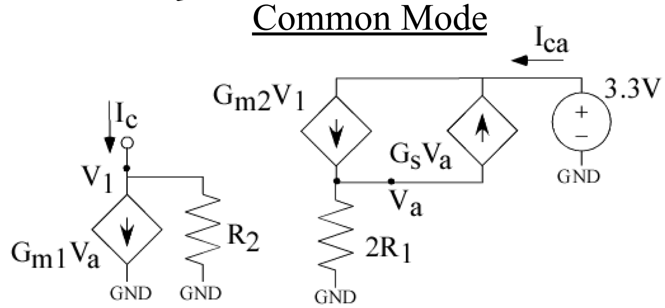
Diff Mode



$$I_{da} = G_{m2}V_1 = (G_{m2}R_2)I_d$$

$$I_{da} = 100I_d$$

Common Mode



$$G_{m2} = 1/10k\Omega$$
$$G_s = 1/5k\Omega$$
$$R_1 = 5M\Omega$$
$$R_2 = 1M\Omega$$
$$G_{m1} = 1/1k\Omega$$

$$I_c = G_{m1}V_a + \frac{V_1}{R_2}$$

$$G_{m2}V_1 = \frac{V_a}{2R_1} + G_sV_a$$

$$G_{m2}V_1 = \left(\frac{1}{2R_1} + G_s\right)V_a$$

$$I_c = V_a\left(G_{m1} + \frac{G_s}{G_{m2}}\frac{1}{R_2}\right) \leftarrow G_{m2}V_1 \approx G_sV_a$$

$$I_c \approx G_{m1}V_a \longrightarrow$$

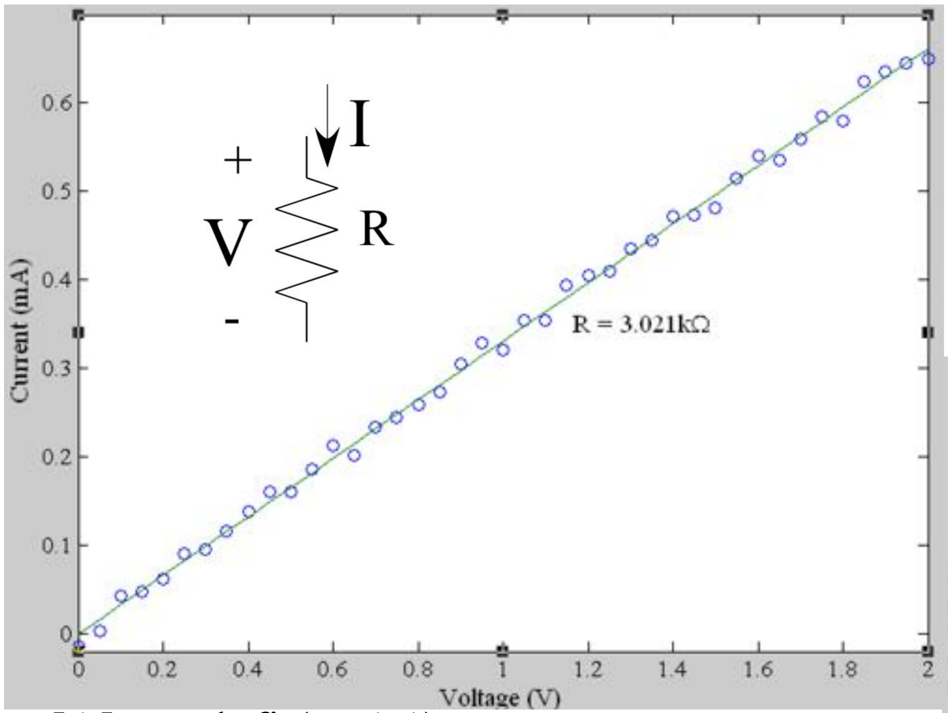
$$I_{ca} = \frac{V_a}{2R_1}$$

$$I_{ca} = \frac{I_c}{2G_{m1}R_1}$$

$$I_{ca} = 0.0001I_c$$

I_1 = 20.1µA and I_2 = 19.9µA → I_c = 20µA, I_d = 100nA → I_a = 10µA + 1nA ~ 10µA

Curve Fitting Circuit Parameters




```
[A] = polyfit(V,I1,1);
I1fit = polyval(A,V);
plot(V,I1,'o',V,I1fit);
```

```
[A] = polyfit(V,V1,1);
V1fit = polyval(A,V);
plot(V,V1,'o',V,V1fit);
```